1. (*a*) $$\begin{bmatrix} 0.1 & -0.3 & -0.4 \\ -0.5 & 0.1 & 0 \\ -0.2 & -0.3 & 0.4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \\ 6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator, $\boxed{v_2 = -8.387 \text{ V}}$

(*b*) Using a scientific calculator, the determinant is equal to $\boxed{32.}$

2. (*a*)

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 27 \\ -16 \\ -6 \end{bmatrix}$$

Solving this matrix equation using a scientific calculator,

$$\boxed{\begin{aligned} v_A &= 19.57 \\ v_B &= 18.71 \\ v_C &= -11.29 \end{aligned}}$$

(*b*) Using a scientific calculator,

$$\begin{vmatrix} 1 & 1 & 1 \\ -1 & 2 & 3 \\ 2 & 0 & 4 \end{vmatrix} = \boxed{16}$$

3.

(a) We begin by simplifying the equations prior to solution:

$$\begin{aligned} 4 &= 0.08v_1 - 0.05v_2 - 0.02v_3 \\ 8 &= -0.02v_1 - 0.025v_2 + 0.045v_3 \\ -2 &= -0.05v_1 + 0.115v_2 - 0.025v_3 \end{aligned}$$

Then, we can solve the matrix equation:

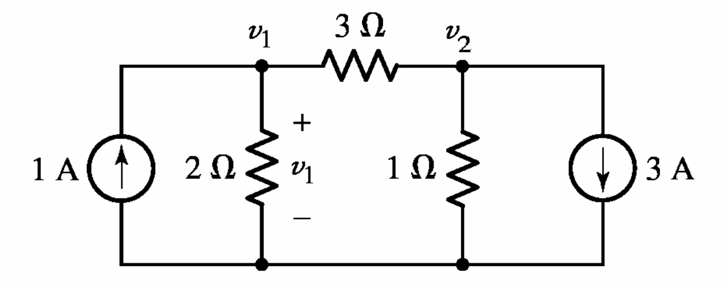
$$\begin{bmatrix} 0.08 & \text{-}0.05 & \text{-}0.02 \\ \text{-}0.02 & \text{-}0.025 & 0.045 \\ \text{-}0.05 & 0.115 & \text{-}0.025 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -2 \end{bmatrix}$$

to obtain $\boxed{v_1 = 264.3 \text{ V}, v_2 = 183.9 \text{ V and } v_3 = 397.4 \text{ V.}}$

(b) We may solve the matrix equation directly using MATLAB, but a better check is to invoke the symbolic processor:

```
>> e1 = '4 = v1/100 + (v1 - v2)/20 + (v1 - vx)/50';
>> e2 = '10 - 4 - (-2) = (vx - v1)/50 + (vx - v2)/40';
>> e3 = '-2 = v2/25 + (v2 - vx)/40 + (v2 - v1)/20';
>> a = solve(e1,e2,e3,'v1','v2','vx');
>> a.v1

ans =

82200/311

>> a.v2

ans =

57200/311

>> a.vx

ans =

123600/311
```

4. We select the bottom node as our reference terminal and define two nodal voltages:



Ref.

Next, we write the two required nodal equations:

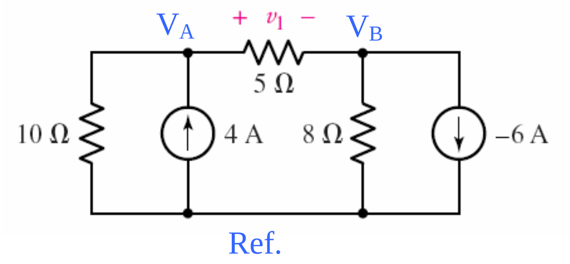
Node 1: $1 = \frac{v_1}{2} + \frac{v_1 - v_2}{3}$

Node 2: $-3 = \frac{v_2}{1} + \frac{v_2 - v_1}{3}$

Which may be simplified to: $5v_1 - 2v_2 = 6$
and $-v_1 + 4v_2 = -9$

Solving, we find that $\boxed{v_1 = 333.3 \text{ mV.}}$

5. We begin by selecting the bottom node as the reference terminal, and defining two nodal voltages V_A and V_B, as shown. (Note if we choose the upper right node, v_1 becomes a nodal voltage and falls directly out of the solution.)



We note that after completing nodal analysis, we can find v_1 as $v_1 = V_A - V_B$.

At node A: $4 = \frac{V_A}{10} + \frac{V_A - V_B}{5}$ [1]

At node B: $-(-6) = \frac{V_B}{8} + \frac{V_B - V_A}{5}$ [2]

Simplifying,

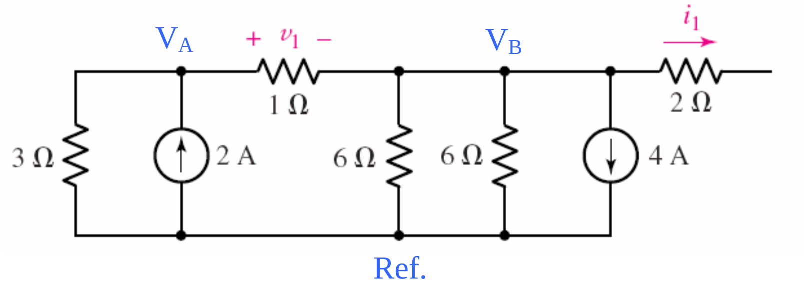
$3V_A - 2V_B = 40$ [1]

$-8V_A + 13V_B = 240$ [2]

Solving, $V_A = 43.48$ V and $V_B = 45.22$ V, so $\boxed{v_1 = -1.740 \text{ V.}}$

6. By inspection, no current flows through the 2 Ω resistor, so $i_1 = 0$.

We next designate the bottom node as the reference terminal, and define V_A and V_B as shown:



At node A: $2 = \frac{V_A}{3} + \frac{V_A - V_B}{1}$ [1]

At node B: $-2 = \frac{V_B}{6} + \frac{V_B}{6} + \frac{V_B - V_A}{1}$ [2]

Note this yields V_A and V_B, not v_1, due to our choice of reference node. So, we obtain v_1 by KVL: $v_1 = V_A - V_B$.

Simplifying Eqs. [1] and [2],

$4V_A - 3V_B = 6$ [1]
$-3V_A + 4V_B = -6$ [2]

Solving, $V_A = 0.8571$ V and $V_B = -0.8571$ V, so $\boxed{v_1 = 1.714 \text{ V.}}$

7. The bottom node has the largest number of branch connections, so we choose that as our reference node. This also makes v_P easier to find, as it will be a nodal voltage. Working from left to right, we name our nodes 1, P, 2, and 3.

NODE 1: $10 = v_1/ 20 + (v_1 - v_P)/ 40$ [1]

NODE P: $0 = (v_P - v_1)/ 40 + v_P/ 100 + (v_P - v_2)/ 50$ [2]

NODE 2: $-2.5 + 2 = (v_2 - v_P)/ 50 + (v_2 - v_3)/ 10$ [3]

NODE 3: $5 - 2 = v_3/ 200 + (v_3 - v_2)/ 10$ [4]

Simplifying,

$$
\begin{array}{lr}
60v_1 - 20v_P = 8000 & [1] \\
-50v_1 + 110\, v_P - 40v_2 = 0 & [2] \\
- v_P + 6v_2 - 5v_3 = -25 & [3] \\
-200v_2 + 210v_3 = 6000 & [4]
\end{array}
$$

Solving,

$$\boxed{v_P = 171.6 \text{ V}}$$

8. The logical choice for a reference node is the bottom node, as then v_x will automatically become a nodal voltage.

NODE 1: $4 = v_1/100 + (v_1 - v_2)/20 + (v_1 - v_x)/50$ [1]

NODE x: $10 - 4 - (-2) = (v_x - v_1)/50 + (v_x - v_2)/40$ [2]

NODE 2: $-2 = v_2/25 + (v_2 - v_x)/40 + (v_2 - v_1)/20$ [3]

Simplifying,

$4 = 0.0800v_1 - 0.0500v_2 - 0.0200v_x$ [1]
$8 = -0.0200v_1 - 0.02500v_2 + 0.04500v_x$ [2]
$-2 = -0.0500v_1 + 0.1150v_2 - 0.02500v_x$ [3]

Solving,

$$\boxed{v_x = 397.4 \text{ V.}}$$

9. Designate the node between the 3-Ω and 6-Ω resistors as node X, and the right-hand node of the 6-Ω resistor as node Y. The bottom node is chosen as the reference node.

(*a*) Writing the two nodal equations, then

NODE X: $-10 = (v_X - 240)/3 + (v_X - v_Y)/6$ [1]

NODE Y: $0 = (v_Y - v_X)/6 + v_Y/30 + (v_Y - 60)/12$ [2]

Simplifying, $-180 + 1440 = 9\,v_X - 3\,v_Y$ [1]

$10800 = -360\,v_X + 612\,v_Y$ [2]

Solving, $v_X = 181.5$ V and $v_Y = 124.4$ V

Thus, $v_1 = 240 - v_X = \boxed{58.50 \text{ V}}$ and $v_2 = v_Y - 60 = \boxed{64.40 \text{ V}}$

(*b*) The power absorbed by the 6-Ω resistor is

$$(v_X - v_Y)^2 / 6 = \boxed{543.4 \text{ W}}$$

10. Only one nodal equation is required: At the node where three resistors join,

$$0.02v_1 = (v_x - 5\, i_2) / 45 + (v_x - 100) / 30 + (v_x - 0.2\, v_3) / 50 \qquad [1]$$

This, however, is one equation in four unknowns, the other three resulting from the presence of the dependent sources. Thus, we require three additional equations:

$$i_2 = (0.2\, v_3 - v_x) / 50 \qquad [2]$$

$$v_1 = 0.2\, v_3 - 100 \qquad [3]$$

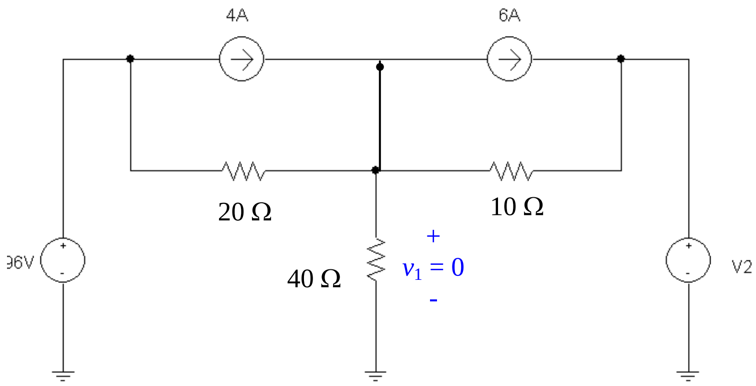
$$v_3 = 50 i_2 \qquad [4]$$

Simplifying,

$$\begin{array}{lllll} & v_1 - 0.2v_3 & & = -100 & [3] \\ & - v_3 & + 50\, i_2 & = 0 & [4] \\ -v_x & + 0.2v_3 & - 50\, i_2 & = 0 & [2] \\ 0.07556v_x - 0.02v_1 & - 0.004v_3 & - 0.111i_2 & = 33.33 & [1] \end{array}$$

Solving, we find that $\boxed{v_1 = -103..8 \text{ V}}$ and $\boxed{i_2 = -377.4 \text{ mA.}}$

11. If $v_1 = 0$, the dependent source is a short circuit and we may redraw the circuit as:



At NODE 1: $4 - 6 = v_1/40 + (v_1 - 96)/20 + (v_1 - V_2)/10$

Since $v_1 = 0$, this simplifies to

$$-2 = -96/20 - V_2/10$$

so that $\boxed{V_2 = -28 \text{ V.}}$

12. We choose the bottom node as ground to make calculation of i_5 easier. The left-most node is named "1", the top node is named "2", the central node is named "3" and the node between the 4-Ω and 6-Ω resistors is named "4."

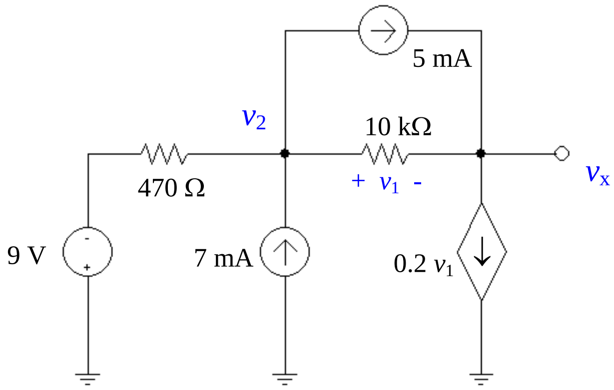
NODE 1: $-3 = v_1/2 + (v_1 - v_2)/1$ [1]
NODE 2: $2 = (v_2 - v_1)/1 + (v_2 - v_3)/3 + (v_2 - v_4)/4$ [2]
NODE 3: $3 = v_3/5 + (v_3 - v_4)/7 + (v_3 - v_2)/3$ [3]
NODE 4: $0 = v_4/6 + (v_4 - v_3)/7 + (v_4 - v_2)/4$ [4]

Rearranging and grouping terms,

$3v_1 - 2v_2 = -6$ [1]
$-12v_1 + 19v_2 - 4v_3 - 3v_4 = 24$ [2]
$-35v_2 + 71v_3 - 15v_4 = 315$ [3]
$-42v_2 - 24v_3 + 94v_4 = 0$ [4]

Solving, we find that v_3 = 6.760 V and so $\boxed{i_5 = v_3/5 = 1.352 \text{ A.}}$

13. We can redraw this circuit and eliminate the 2.2-kΩ resistor as no current flows through it:



At NODE 2: $7\times10^{-3} - 5\times10^{-3} = (v_2 + 9)/470 + (v_2 - v_x)/10\times10^{-3}$ [1]

At NODE x: $5\times10^{-3} - 0.2v_1 = (v_x - v_2)/10\times10^{3}$ [2]

The additional equation required by the presence of the dependent source and the fact that its controlling variable is not one of the nodal voltages:

$$v_1 = v_2 - v_x \qquad [3]$$

Eliminating the variable v1 and grouping terms, we obtain:

$$10{,}470\, v_2 - 470\, v_x = -89{,}518$$

and

$$1999\, v_2 - 1999\, v_x = 50$$

Solving, we find $\boxed{v_x = -8.086 \text{ V.}}$

14. We need concern ourselves with the bottom part of this circuit only. Writing a single nodal equation,

$$-4 + 2 = v/50$$

We find that $\boxed{v = -100 \text{ V.}}$

15. We choose the bottom node as the reference terminal. Then:

Node 1: $$-2 = \frac{v_1}{2} + \frac{v_1 - v_2}{1} \quad [1]$$

Node 2: $$4 = \frac{v_2 - v_1}{1} + \frac{v_2 - v_3}{2} + \frac{v_2 - v_4}{4} \quad [2]$$

Node 3: $$2 = \frac{v_3 - v_2}{2} + \frac{v_3}{5} + \frac{v_3 - v_4}{10} \quad [3]$$

Node 4: $$0 = \frac{v_4}{6} + \frac{v_4 - v_3}{10} + \frac{v_4 - v_2}{4} \quad [4]$$

Node 5: $$-1 = \frac{v_5}{2} + \frac{v_5 - v_7}{1} \quad [5]$$

Node 6: $$1 = \frac{v_6}{5} + \frac{v_6 - v_7}{2} + \frac{v_6 - v_8}{10} \quad [6]$$

Node 7: $$2 = \frac{v_7 - v_5}{1} + \frac{v_7 - v_6}{2} + \frac{v_7 - v_8}{4} \quad [7]$$

Node 8: $$0 = \frac{v_8}{6} + \frac{v_8 - v_6}{10} + \frac{v_8 - v_7}{4} \quad [8]$$

Note that Eqs. [1-4] may be solved independently of Eqs. [5-8].

Simplifying,

$$\begin{array}{rrrrrl} 3v_1 & -2v_2 & & & =-4 & [1] \\ -4v_1 & +7v_2 & -2v_3 & -v_4 & =16 & [2] \\ & -5v_2 & +8v_3 & -v_4 & =20 & [3] \\ & -15v_2 & -6v_3 & +31v_4 & =0 & [4] \end{array} \quad \text{to yield} \quad \boxed{\begin{array}{l} v_1 = 3.370\text{ V} \\ v_2 = 7.055\text{ V} \\ v_3 = 7.518\text{ V} \\ v_4 = 4.869\text{ V} \end{array}}$$

and

$$\begin{array}{rrrrrl} 3v_5 & & -2v_7 & & =-2 & [5] \\ & 8v_6 & -5v_7 & -v_8 & =10 & [6] \\ -4v_5 & -2v_6 & +7v_7 & -v_8 & =8 & [7] \\ & -6v_6 & -15v_7 & +31v_8 & =0 & [8] \end{array} \quad \text{to yield} \quad \boxed{\begin{array}{l} v_5 = 1.685\text{ V} \\ v_6 = 3.759\text{ V} \\ v_7 = 3.527\text{ V} \\ v_8 = 2.434\text{ V} \end{array}}$$

16. We choose the center node for our common terminal, since it connects to the largest number of branches. We name the left node "A", the top node "B", the right node "C", and the bottom node "D". We next form a supernode between nodes A and B.

At the supernode: $5 = (V_A - V_D)/10 + V_A/20 + (V_B - V_C)/12.5$ [1]

At node C: $V_C = 150$ [2]

At node D: $-10 = V_D/25 + (V_D - V_A)/10$ [3]

Our supernode-related equation is $V_B - V_A = 100$ [4]

Simplifiying and grouping terms,

$$0.15\,V_A + 0.08\,V_B - 0.08\,V_C - 0.1\,V_D = 5 \quad [1]$$
$$V_C = 150 \quad [2]$$
$$-25\,V_A + 35\,V_D = -2500 \quad [3]$$
$$-V_A + V_B = 100 \quad [4]$$

Solving, we find that $V_D = -63.06$ V. Since $v_4 = -V_D$,

$$\boxed{v_4 = 63.06 \text{ V.}}$$

17. Choosing the bottom node as the reference terminal and naming the left node "1", the center node "2" and the right node "3", we next form a supernode about nodes 1 and 2, encompassing the dependent voltage source.

At the supernode, $5 - 8 = (v_1 - v_2)/ 2 + v_3/ 2.5$ [1]
At node 2, $8 = v_2 / 5 + (v_2 - v_1)/ 2$ [2]

Our supernode equation is $v_1 - v_3 = 0.8\, v_A$ [3]
Since $v_A = v_2$, we can rewrite [3] as $v_1 - v_3 = 0.8v_2$

Simplifying and collecting terms,

$$0.5\, v_1 - 0.5\, v_2 + 0.4\, v_3 = -3 \quad [1]$$
$$-0.5\, v_1 + 0.7\, v_2 = 8 \quad [2]$$
$$v_1 - 0.8\, v_2 - v_3 = 0 \quad [3]$$

(*a*) Solving for $v_2 = v_A$, we find that $\boxed{v_A = 25.91\ \text{V}}$

(*b*) The power absorbed by the 2.5-Ω resistor is
$(v_3)^2/ 2.5 = (-0.4546)^2/ 2.5$ $\boxed{= 82.66\ \text{mW.}}$

18. Selecting the bottom node as the reference terminal, we name the left node "1", the middle node "2" and the right node "3."

NODE 1: $5 = (v_1 - v_2)/\,20 + (v_1 - v_3)/\,50$ [1]

NODE 2: $v_2 = 0.4\, v_1$ [2]

NODE 3: $0.01\, v_1 = (v_3 - v_2)/\,30 + (v_3 - v_1)/\,50$ [3]

Simplifying and collecting terms, we obtain

$$0.07\, v_1 - 0.05\, v_2 - 0.02\, v_3 = 5 \quad [1]$$
$$0.4\, v_1 - v_2 = 0 \quad [2]$$
$$-0.03\, v_1 - 0.03333\, v_2 + 0.05333\, v_3 = 0 \quad [3]$$

Since our choice of reference terminal makes the controlling variable of both dependent sources a nodal voltage, we have no need for an additional equation as we might have expected.

Solving, we find that $\boxed{v_1 = 148.2 \text{ V},}$ $v_2 = 59.26$ V, and $v_3 = 120.4$ V.

The power supplied by the dependent current source is therefore

$$(0.01\, v_1) \bullet v_3 = \boxed{177.4 \text{ W.}}$$

19. At node x: $v_x/4 + (v_x - v_y)/2 + (v_x - 6)/1 = 0$ [1]

At node y: $(v_y - kv_x)/3 + (v_y - v_x)/2 = 2$ [2]

Our additional constraint is that $v_y = 0$, so we may simplify Eqs. [1] and [2]:

$14\,v_x = 48$ [1]

$-2k\,v_x - 3\,v_x = 12$ [2]

Since Eq. [1] yields $v_x = 48/14 = 3.429$ V, we find that

$$k = (12 + 3\,v_x)/(-2\,v_x) = \boxed{-3.250}$$

20. Choosing the bottom node joining the 4-Ω resistor, the 2-A current sourcee and the 4-V voltage source as our reference node, we next name the other node of the 4-Ω resistor node "1", and the node joining the 2-Ω resistor and the 2-A current source node "2." Finally, we create a supernode with nodes "1" and "2."

At the supernode: $-2 = v_1/4 + (v_2 - 4)/2$ [1]
Our remaining equations: $v_1 - v_2 = -3 - 0.5i_1$ [2]
and $i_1 = (v_2 - 4)/2$ [3]

Equation [1] simplifies to $v_1 + 2v_2 = 0$ [1]
Combining Eqs. [2] and [3, $4v_1 - 3v_2 = -8$ [4]

Solving these last two equations, we find that $v_2 = 727.3$ mV. Making use of Eq. [3], we therefore find that

$$\boxed{i_1 = -1.636 \text{ A.}}$$

21. We first number the nodes as 1, 2, 3, 4, and 5 moving left to right. We next select node 5 as the reference terminal. To simplify the analysis, we form a supernode from nodes 1, 2, and 3.

At the supernode,

$$-4 - 8 + 6 = v_1/40 + (v_1 - v_3)/10 + (v_3 - v_1)/10 + v_2/50 + (v_3 - v_4)/20 \qquad [1]$$

Note that since both ends of the 10-Ω resistor are connected to the supernode, the related terms cancel each other out, and so could have been ignored.

At node 4: $v_4 = 200$ [2]

Supernode KVL equation: $v_1 - v_3 = 400 + 4v_{20}$ [3]

Where the controlling voltage $v_{20} = v_3 - v_4 = v_3 - 200$ [4]

Thus, Eq. [1] becomes $-6 = v_1/40 + v_2/50 + (v_3 - 200)/20$ or, more simply,

$$4 = v_1/40 + v_2/50 + v_3/20 \qquad [1']$$

and Eq. [3] becomes $v_1 - 5v_3 = -400$ [3']

Eqs. [1'], [3'], and [5] are not sufficient, however, as we have four unknowns. At this point we need to seek an additional equation, possibly in terms of v_2. Referring to the circuit,

$$v_1 - v_2 = 400 \qquad [5]$$

Rewriting as a matrix equation,

$$\begin{bmatrix} 1/40 & 1/50 & 1/20 \\ 1 & 0 & -5 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -400 \\ 400 \end{bmatrix}$$

Solving, we find that
$v_1 = 145.5$ V, $v_2 = -254.5$ V, and $v_3 = 109.1$ V. Since $v_{20} = v_3 - 200$, we find that

$$\boxed{v_{20} = -90.9 \text{ V.}}$$

22. We begin by naming the top left node "1", the top right node "2", the bottom node of the 6-V source "3" and the top node of the 2-Ω resistor "4." The reference node has already been selected, and designated using a ground symbol.

By inspection, $\boxed{v_2 = 5 \text{ V.}}$

Forming a supernode with nodes 1 & 3, we find

At the supernode: $-2 = v_3/\,1 + (v_1 - 5)/\,10$ [1]

At node 4: $2 = v_4/\,2 + (v_4 - 5)/\,4$ [2]

Our supernode KVL equation: $v_1 - v_3 = 6$ [3]

Rearranging, simplifying and collecting terms,

$$v_1 + 10\,v_3 = -20 + 5 = -15 \qquad [1]$$

and

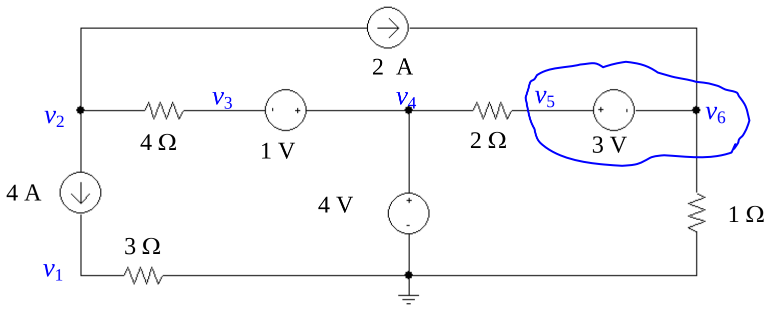
$$v_1 - v_3 = 6 \qquad [2]$$

Eq. [3] may be directly solved to obtain $\boxed{v_4 = 4.333 \text{ V.}}$

Solving Eqs. [1] and [2], we find that

$$\boxed{v_1 = 4.091 \text{ V}} \quad \text{and} \quad \boxed{v_3 = -1.909 \text{ V.}}$$

23. We begin by selecting the bottom node as the reference, naming the nodes as shown below, and forming a supernode with nodes 5 & 6.



By inspection, $v_4 = 4$ V.

By KVL, $v_3 - v_4 = 1$ so $v_3 = -1 + v_4 = -1 + 4$ or $v_3 = 3$ V.

At the supernode, $2 = v_6/1 + (v_5 - 4)/2$ [1]

At node 1, $4 = v_1/3$ therefore, $v_1 = 12$ V.

At node 2, $-4 - 2 = (v_2 - 3)/4$

Solving, we find that $v_2 = -21$ V

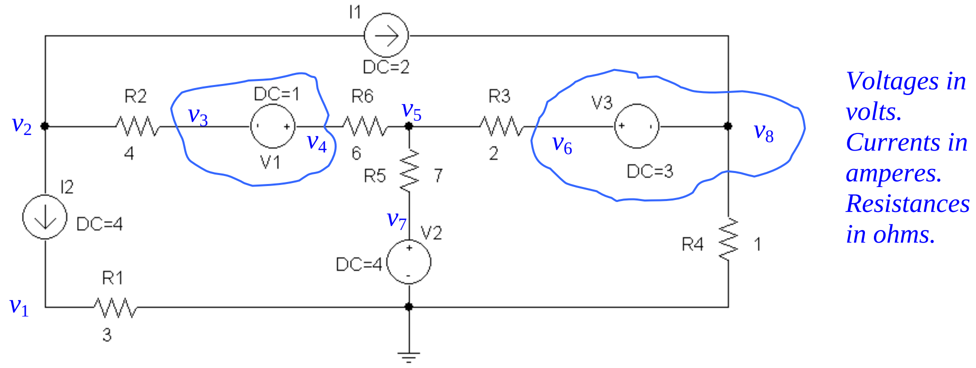
Our supernode KVL equation is $v_5 - v_6 = 3$ [2]

Solving Eqs. [1] and [2], we find that

$$v_5 = 4.667 \text{ V} \quad \text{and} \quad v_6 = 1.667 \text{ V}.$$

The power supplied by the 2-A source therefore is $(v_6 - v_2)(2) = \boxed{45.33 \text{ W.}}$

24. We begin by selecting the bottom node as the reference, naming each node as shown below, and forming two different supernodes as indicated.



By inspection, $v_7 = 4$ V and $v_1 = (3)(4) = 12$ V.

At node 2: $-4 - 2 = (v_2 - v_3)/4$ or $v_2 - v_3 = -24$ [1]

At the 3-4 supernode:
$0 = (v_3 - v_2)/4 + (v_4 - v_5)/6$ or $-6v_2 + 6v_3 + 4v_4 - 4v_5 = 0$ [2]

At node 5:
$0 = (v_5 - v_4)/6 + (v_5 - 4)/7 + (v_5 - v_6)/2$ or $-14v_4 + 68v_5 - 42v_6 = 48$ [3]

At the 6-8 supernode: $2 = (v_6 - v_5)/2 + v_8/1$ or $-v_5 + v_6 + 2v_8 = 4$ [4]

3-4 supernode KVL equation: $v_3 - v_4 = -1$ [5]
6-8 supernode KVL equation: $v_6 - v_8 = 3$ [6]

Rewriting Eqs. [1] to [6] in matrix form,

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -6 & 6 & 4 & -4 & 0 & 0 \\ 0 & 0 & -14 & 68 & -42 & 0 \\ 0 & 0 & 0 & -1 & 1 & 2 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_8 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ 48 \\ 4 \\ -1 \\ 3 \end{bmatrix}$$

Solving, we find that
$v_2 = -68.9$ V, $v_3 = -44.9$ V, $v_4 = -43.9$ V, $v_5 = -7.9$ V, $v_6 = 700$ mV, $v_8 = -2.3$ V.

The power generated by the 2-A source is therefore $(v_8 - v_6)(2) = \boxed{133.2 \text{ W.}}$

25. With the reference terminal already specified, we name the bottom terminal of the 3-mA source node "1," the left terminal of the bottom 2.2-kΩ resistor node "2," the top terminal of the 3-mA source node "3," the "+" reference terminal of the 9-V source node "4," and the "-" terminal of the 9-V source node "5."

Since we know that 1 mA flows through the top 2.2-kΩ resistor, $v_5 = -2.2$ V.
Also, we see that $v_4 - v_5 = 9$, so that $v_4 = 9 - 2.2 = 6.8$ V.
Proceeding with nodal analysis,

At node 1: $-3\times10^{-3} = v_1/ 10\times10^3 + (v_1 - v_2)/ 2.2\times10^3$ [1]

At node 2: $0 = (v_2 - v_1)/ 2.2\times10^3 + (v_2 - v_3)/ 4.7\times10^3$ [2]

At node 3: $1\times10^3 + 3\times10^3 = (v_3 - v_2)/ 4.7\times10^3 + v_3/3.3\times10^3$ [3]

Solving, $\boxed{v_1 = -8.614 \text{ V}, v_2 = -3.909 \text{ V and } v_3 = 6.143 \text{ V.}}$

Note that we could also have made use of the supernode approach here.

26. Mesh 1: $-4 + 400i_1 + 300i_1 - 300i_2 - 1 = 0$ or $700i_1 - 300i_2 = 5$
Mesh 2: $1 + 500i_2 - 300i_1 + 2 - 2 = 0$ or $-300i_1 + 500i_2 = -3.2$

Solving, $\boxed{i_1 = 5.923 \text{ mA} \quad \text{and} \quad i_2 = -2.846 \text{ mA.}}$

27. (a) Define a clockwise mesh current i_1 in the left-most mesh; a clockwise mesh current i_2 in the central mesh, and note that i_y can be used as a mesh current for the remaining mesh.

Mesh 1: $-10 + 7i_1 - 2i_2 = 0$
Mesh 2: $-2i_1 + 5i_2 = 0$
Mesh y: $-2i_2 + 9i_y = 0$

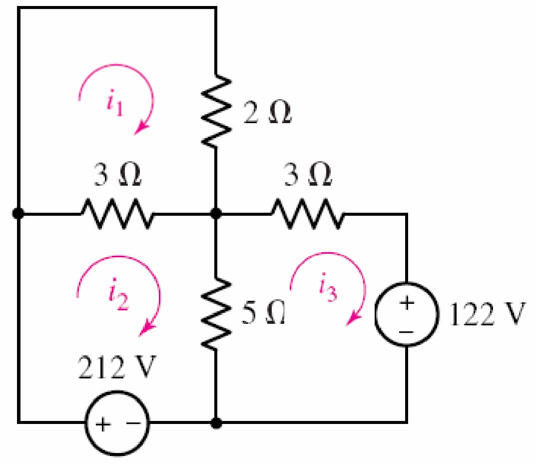
Solve the resulting matrix equation:

$$\begin{bmatrix} 7 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & -2 & 9 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_y \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

to find that i_1 = 1.613 A, and $\boxed{i_y = 143.4 \text{ mA.}}$

(b) The power supplied by the 10 V source is $(10)(i_1) = 10(1.613) = \boxed{16.13 \text{ W.}}$

28. Define three mesh currents as shown:



(a) The current through the 2 Ω resistor is i_1.

Mesh 1: $5i_1 - 3i_2 = 0$ or $5i_1 - 3i_2 = 0$
Mesh 2: $-212 + 8i_2 - 3i_1 = 0$ or $-3i_1 + 8i_2 = 212$
Mesh 3: $8i_3 - 5i_2 + 122 = 0$ or $-5i_2 + 8i_3 = -122$

Solving, $\boxed{i_1 = 20.52 \text{ A},}$ $i_2 = 34.19$ A and $i_3 = 6.121$ A.

(b) The current through the 5 Ω resistor is i_3, or $\boxed{6.121 \text{ A.}}$

*** Note: since the problem statement did not specify a direction, only the current magnitude is relevant, and its sign is arbitrary.

29. We begin by defining three clockwise mesh currents i_1, i_2 and i_3 in the left-most, central, and right-most meshes, respectively. Then,

(a) Note that $i_x = i_2 - i_3$.

Mesh 1: $i_1 = 5$ A (by inspection)
Mesh 3: $i_3 = -2$ A (by inspection)

Mesh 2: $-25i_1 + 75i_2 - 20i_3 = 0$, or, making use of the above,

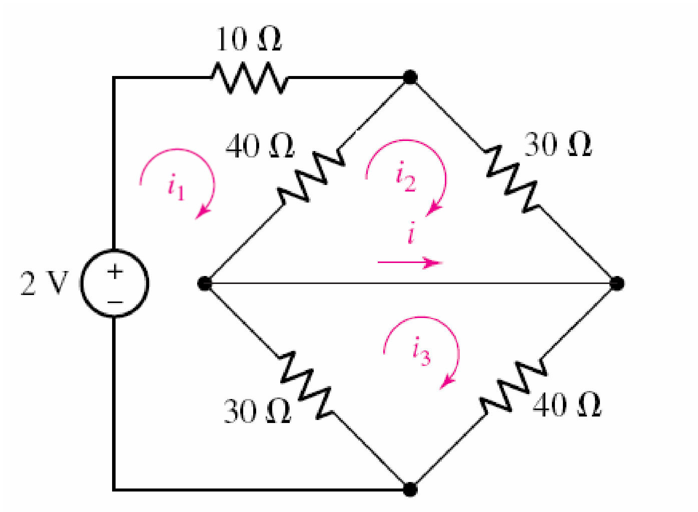
$-125 + 75i_2 + 40 = 0$ so that $i_2 = 1.133$ A.

Thus, $i_x = i_2 - i_3 = 1.133 - (-2) = \boxed{3.133 \text{ A.}}$

(b) The power absorbed by the 25 Ω resistor is

$P_{25\Omega} = 25\,(i_1 - i_2)^2 = 25\,(5 - 1.133)^2 = \boxed{373.8 \text{ W.}}$

30. Define three mesh currents as shown. Then,



Mesh 1: $-2 + 80i_1 - 40i_2 - 30i_3 = 0$

Mesh 2: $-40i_1 + 70i_2 = 0$

Mesh 3: $-30i_1 + 70i_3 = 0$

Solving, $$\begin{bmatrix} 80 & -40 & -30 \\ -40 & 70 & 0 \\ -30 & 0 & 70 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$$

we find that i_2 = 25.81 mA and i_3 = 19.35 mA. Thus, $i = i_3 - i_2$ = –6.46 mA.

31. Moving from left to right, we name the bottom three meshes, mesh "1", mesh "2," and mesh "3." In each of these three meshes we define a clockwise current. The remaining mesh current is clearly 8 A. We may then write:

MESH 1: $12\,i_1 - 4\,i_2 = 100$

MESH 2: $-4\,i_1 + 9\,i_2 - 3\,i_3 = 0$

MESH 3: $-3\,i_2 + 18\,i_3 = -80$

Solving this system of three (independent) equations in three unknowns, we find that

$$i_2 = \boxed{i_x = 2.791\text{ A.}}$$

32. We define four clockwise mesh currents. The top mesh current is labeled i_4. The bottom left mesh current is labeled i_1, the bottom right mesh current is labeled i_3, and the remaining mesh current is labeled i_2. Define a voltage "v_{4A}" across the 4-A current source with the "+" reference terminal on the left.

By inspection, $i_3 = 5$ A and $i_a = i_4$.

MESH 1: $-60 + 2i_1 - 2i_4 + 6i_4 = 0$ or $2i_1 + 4i_4 = 60$ [1]

MESH 2: $-6i_4 + v_{4A} + 4i_2 - 4(5) = 0$ or $4i_2 - 6i_4 + v_{4A} = 20$ [2]

MESH 4: $2i_4 - 2i_1 + 5i_4 + 3i_4 - 3(5) - v_{4A} = 0$ or $-2i_1 + 10i_4 - v_{4A} = 15$ [3]

At this point, we are short an equation. Returning to the circuit diagram, we note that

$i_2 - i_4 = 4$ [4]

Collecting these equations and writing in matrix form, we have

$$\begin{bmatrix} 2 & 0 & 4 & 0 \\ 0 & 4 & -6 & 1 \\ -2 & 0 & 10 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_4 \\ v_{4A} \end{bmatrix} = \begin{bmatrix} 60 \\ 20 \\ 15 \\ 4 \end{bmatrix}$$

Solving, $i_1 = 16.83$ A, $i_2 = 10.58$ A, $i_4 = 6.583$ A and $v_{4A} = 17.17$ V.
Thus, the power dissipated by the 2-Ω resistor is

$$(i_1 - i_4)^2 \bullet (2) = \boxed{210.0 \text{ W}}$$

33. We begin our analysis by defining three clockwise mesh currents. We will call the top mesh current i_3, the bottom left mesh current i_1, and the bottom right mesh current i_2.

By inspection, $i_1 = 5$ A [1] and $i_2 = -0.01\, v_1$ [2]

MESH 3: $50\, i_3 + 30\, i_3 - 30\, i_2 + 20\, i_3 - 20\, i_1 = 0$

or $-20\, i_1 - 30\, i_2 + 100\, i_3 = 0$ [3]

These three equations are insufficient, however, to solve for the unknowns. It would be nice to be able to express the dependent source controlling variable v_1 in terms of the mesh currents. Returning to the diagram, it can be seen that KVL around mesh 1 will yield

$$- v_1 + 20\, i_1 - 20\, i_3 + 0.4\, v_1 = 0$$

or $v_1 = 20\, i_1 / 0.6 - 20\, i_3 / 0.6$ or $v_1 = (20(5)/ 0.6 - 20\, i_3 / 0.6$ [4]

Substituting Eq. [4] into Eq. [2] and then the modified Eq. [2] into Eq. [3], we find

$$-20(5) - 30(-0.01)(20)(5)/0.6 + 30(-0.01)(20)\, i_3 / 0.6 + 100\, i_3 = 0$$

Solving, we find that $i_3 = (100 - 50)/ 90 = 555.6$ mA

Thus, $v_1 = 148.1$ V, $i_2 = -1.481$ A, and the power generated by the dependent voltage source is

$$0.4\, v_1 (i_2 - i_1) = \boxed{-383.9 \text{ W.}}$$

34. We begin by defining four clockwise mesh currents i_1, i_2, i_3 and i_4, in the meshes of our circuit, starting at the left-most mesh. We also define a voltage v_{dep} across the dependent current source, with the "+" on the top.

By inspection, $i_1 = 2\text{A}$ and $i_4 = -5$ A.

At Mesh 2: $10\, i_2 - 10(2) + 20\, i_2 + v_{dep} = 0$ [1]

At Mesh 3: $- v_{dep} + 25\, i_3 + 5\, i_3 - 5(-5) = 0$ [2]

Collecting terms, we rewrite Eqs. [1] and [2] as

$30\, i_2 + v_{dep} = 20$ [1]

$30\, i_3 - v_{dep} = -25$ [2]

This is only two equations but three unknowns, however, so we require an additional equation. Returning to the circuit diagram, we note that it is possible to express the current of the dependent source in terms of mesh currents. (We might also choose to obtain an expression for v_{dep} in terms of mesh currents using KVL around mesh 2 or 3.)

Thus, $1.5 i_x = i_3 - i_2$ where $i_x = i_1 - i_2$ so $-0.5\, i_2 - i_3 = -3$ [3]

In matrix form,

$$\begin{bmatrix} 30 & 0 & 1 \\ 0 & 30 & -1 \\ -0.5 & -1 & 0 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \\ v_{dep} \end{bmatrix} = \begin{bmatrix} 20 \\ -25 \\ -3 \end{bmatrix}$$

Solving, we find that $i_2 = -6.333$ A so that $\boxed{i_x = i_1 - i_2 = 8.333 \text{ A.}}$

35. We define a clockwise mesh current i_1 in the bottom left mesh, a clockwise mesh current i_2 in the top left mesh, a clockwise mesh current i_3 in the top right mesh, and a clockwise mesh current i_4 in the bottom right mesh.

MESH 1: $-0.1\ v_a + 4700\ i_1 - 4700\ i_2 + 4700\ i_1 - 4700\ i_4 = 0$ [1]

MESH 2: $9400\ i_2 - 4700\ i_1 - 9 = 0$ [2]

MESH 3: $9 + 9400\ i_3 - 4700\ i_4 = 0$ [3]

MESH 4: $9400\ i_4 - 4700\ i_1 - 4700\ i_3 + 0.1\ i_x = 0$ [4]

The presence of the two dependent sources has led to the introduction of two additional unknowns (i_x and v_a) besides our four mesh currents. In a perfect world, it would simplify the solution if we could express these two quantities in terms of the mesh currents.

Referring to the circuit diagram, we see that $i_x = i_2$ (easy enough) and that $v_a = 4700\ i_3$ (also straightforward). Thus, substituting these expressions into our four mesh equations and creating a matrix equation, we arrive at:

$$\begin{bmatrix} 9400 & -4700 & -470 & -4700 \\ -4700 & 9400 & 0 & 0 \\ 0 & 0 & 9400 & -4700 \\ -4700 & 0.1 & -4700 & 9400 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 9 \\ -9 \\ 0 \end{bmatrix}$$

Solving,

$\boxed{i_1 = 239.3\ \mu\text{A},\ i_2 = 1.077\ \text{mA},\ i_3 = -1.197\ \text{mA and } i_4 = -478.8\ \mu\text{A}.}$

36. We define a clockwise mesh current i_3 in the upper right mesh, a clockwise mesh current i_1 in the lower left mesh, and a clockwise mesh current i_2 in the lower right mesh.

MESH 1: $-6 + 6 i_1 - 2 = 0$ [1]

MESH 2: $2 + 15 i_2 - 12 i_3 - 1.5 = 0$ [2]

MESH 3: $i_3 = 0.1 v_x$ [3]

Eq. [1] may be solved directly to obtain $\boxed{i_1 = 1.333 \text{ A.}}$

It would help in the solution of Eqs. [2] and [3] if we could express the dependent source controlling variable v_x in terms of mesh currents. Referring to the circuit diagram, we see that $v_x = (1)(i_1) = i_1$, so Eq. [3] reduces to

$$\boxed{i_3 = 0.1 v_x = 0.1 i_1 = 133.3 \text{ mA.}}$$

As a result, Eq. [1] reduces to $\boxed{i_2 = [-0.5 + 12(0.1333)]/ 15 = 73.31 \text{ mA.}}$

37. (a) Define a mesh current i_2 in the second mesh. Then KVL allows us to write:

MESH 1: $-9 + R\,i_1 + 47000\,i_1 - 47000\,i_2 = 0$ [1]

MESH 2: $67000\,i_2 - 47000\,i_1 - 5 = 0$ [2]

Given that i_1 = 1.5 mA, we may solve Eq. [2] to find that

$$i_2 = \frac{5 + 47(1.5)}{67}\ \text{mA} = 1.127\ \text{mA}$$

and so

$$R = \frac{9 - 47(1.5) + 47(1.127)}{1.5\times 10^{-3}} = \boxed{-5687\ \Omega.}$$

(b) This value of R is unique; no other value will satisfy **both** Eqs. [1] **and** [2].

38. Define three clockwise mesh currents i_1, i_2 and i_3. The bottom 1-kΩ resistor can be ignored, as no current flows through it.

MESH 1: $-4 + (2700 + 1000 + 5000)\, i_1 - 1000\, i_2 = 0$ [1]

MESH 2: $(1000 + 1000 + 4400 + 3000)\, i_2 - 1000\, i_1 - 4400\, i_3 + 2.2 - 3 = 0$ [2]

MESH 3: $(4400 + 4000 + 3000)\, i_3 - 4400\, i_2 - 1.5 = 0$ [3]

Combining terms,

$$8700\, i_1 - 1000\, i_2 = 4 \quad [1]$$
$$-1000\, i_1 + 9400\, i_2 - 4400\, i_3 = 0.8 \quad [2]$$
$$-4400\, i_2 + 11400\, i_3 = 1.5 \quad [3]$$

Solving,

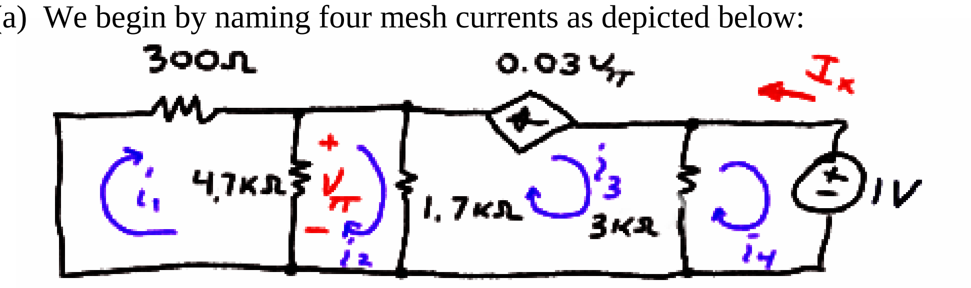
$i_1 = 487.6$ μA, $i_2 = 242.4$ μA and $i_3 = 225.1$ μA.

The power absorbed by each resistor may now be calculated:

P_{5k}	=	$5000\,(i_1)^2$	=	1.189 mW
$P_{2.7k}$	=	$2700\,(i_1)^2$	=	641.9 μW
P_{1ktop}	=	$1000\,(i_1 - i_2)^2$	=	60.12 μW
$P_{1kmiddle}$	=	$1000\,(i_2)^2$	=	58.76 μW
$P_{1kbottom}$	=	0	=	0
$P_{4.4k}$	=	$4400\,(i_2 - i_3)^2$	=	1.317 μW
P_{3ktop}	=	$3000\,(i_3)^2$	=	152.0 μW
P_{4k}	=	$4000\,(i_3)^2$	=	202.7 μW
$P_{3kbottom}$	=	$3000\,(i_2)^2$	=	176.3 μW

Check: The sources supply a total of
4(487.6) + (3 – 2.2)(242.4) + 1.5(225.1) = 2482 μW.
The absorbed powers add to 2482 μW.

39. (a) We begin by naming four mesh currents as depicted below:



Proceeding with mesh analysis, then, keeping in mind that $I_x = -i_4$,

MESH 1: $(4700 + 300)\, i_1 \; - \; 4700\, i_2 \; = \; 0$ [1]

MESH 2: $(4700 + 1700)\, i_2 - 4700\, i_1 - 1700\, i_3 \; = \; 0$ [2]

Since we have a current source on the perimeter of mesh 3, we do not require a KVL equation for that mesh. Instead, we may simply write

$i_3 \; = \; -0.03\, v_\pi$ [3*a*] where $v_\pi = 4700(i_1 - i_2)$ [3*b*]

MESH 4: $3000\, i_4 - 3000\, i_3 + 1 \; = \; 0$ [4]

Simplifying and combining Eqs. 3*a* and 3*b*,

$$\begin{aligned}
5000\, i_1 - 4700\, i_2 \qquad\qquad &= 0 \\
-4700\, i_1 + 6400\, i_2 - 1700\, i_3 \qquad &= 0 \\
-141\, i_1 + 141\, i_2 - i_3 \qquad &= 0 \\
-3000\, i_3 + 3000\, i_4 &= -1
\end{aligned}$$

Solving, we find that i_4 = -333.3 mA, so $\boxed{I_x = 333.3\ \mu\text{A}.}$

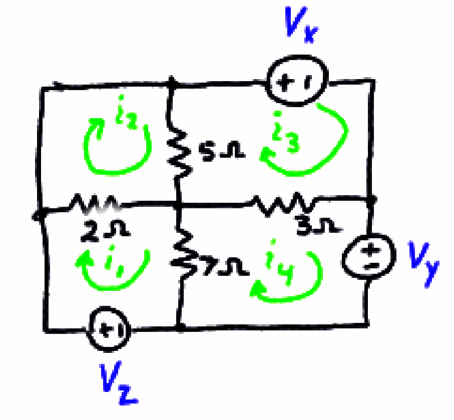
(b) At node "π" : $0.03\, v_\pi = v_\pi / 300 + v_\pi / 4700 + v_\pi /1700$

Solving, we find that $v_\pi = 0$, therefore no current flows through the dependent source.

Hence, $\boxed{I_x = 333.3\ \mu\text{A as found in part (a).}}$

(c) $\boxed{V_s / I_x \text{ has units of resistance}}$. It can be thought of as the resistance "seen" by the voltage source V_s…. more on this in Chap. 5….

40. We begin by naming each mesh and the three undefined voltage sources as shown below:



MESH 1: $-V_z + 9i_1 - 2i_2 \quad - 7i_4 = 0$

MESH 2: $-2i_1 + 7i_2 - 5i_3 = 0$

MESH 3: $V_x \quad - 5i_2 + 8i_3 - 3i_4 = 0$

MESH 4: $V_y - 7i_1 \quad - 3i_3 + 10i_4 = 0$

Rearranging and setting $i_1 - i_2 = 0$, $i_2 - i_3 = 0$, $i_1 - i_4 = 0$ and $i_4 - i_3 = 0$,

$$\begin{aligned}
9i_1 - 2i_2 \quad -7i_4 &= V_z \\
-2i_1 + 7i_2 - 5i_3 &= 0 \\
-5i_2 + 8i_3 - 3i_4 &= -V_x \\
-7i_1 \quad -3i_3 + 10i_4 &= -V_y
\end{aligned}$$

Since $i_1 = i_2 = i_3 = i_4$, these equations produce:

$$\begin{aligned}
V_z &= 0 \\
0 &= 0 \\
-V_x &= 0 \\
-V_y &= 0
\end{aligned}$$

This is a unique solution. Therefore, the request that nonzero values be found cannot be satisfied.

41. The "supermesh" concept is not required (or helpful) in solving this problem, as there are no current sources shared between meshes. Starting with the left-most mesh and moving right, we define four clockwise mesh currents i_1, i_2, i_3 and i_4. By inspection, we see that i_1 = 2 mA.

MESH 2: $-10 + 5000i_2 + 4 + 1000i_3 = 0$ [1]

MESH 3: $-1000i_3 + 6 + 10{,}000 - 10{,}000i_4 = 0$ [2]

MESH 4: $i_4 = -0.5i_2$ [3]

Reorganising, we find

$$\begin{aligned} 5000\,i_2 + 1000\,i_3 \qquad\qquad &= 6 \quad [1] \\ 9000\,i_3 - 10{,}000\,i_4 &= -6 \quad [2] \\ 0.5\,i_2 \qquad\qquad + \quad i_4 &= 0 \quad [3] \end{aligned}$$

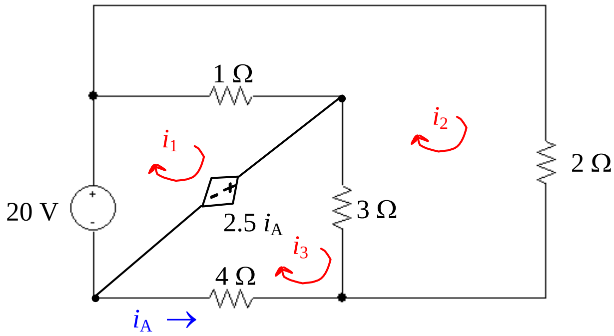
We could either subtitute Eq. [3] into Eq. [2] to reduce the number of equations, or simply go ahead and solve the system of Eqs. [1-3]. Either way, we find that

$$i_1 = 2 \text{ mA},\ i_2 = 1.5 \text{ mA},\ i_3 = -1.5 \text{ mA and } i_4 = -0.75 \text{ mA.}$$

The power generated by each source is:

P_{2mA}	$= 5000(i_1 - i_2)(i_1)$	= 5 mW
P_{4V}	$= 4\,(-i_2)$	= -6 mW
P_{6V}	$= 6\,(-i_3)$	= 9 mW
P_{depV}	$= 1000\,i_3\,(i_3 - i_2)$	= 4.5 mW
P_{depI}	$= 10{,}000(i_3 - i_4)(0.5\,i_2)$	= -5.625 mW

42. This circuit does not require the supermesh technique, as it does not contain any current sources. Redrawing the circuit so its planar nature and mesh structure are clear,



MESH 1: $-20 + i_1 - i_2 + 2.5\, i_A = 0$ [1]

MESH 2: $2\, i_2 + 3\, i_2 + i_2 - 3\, i_3 - i_1 = 0$ [2]

MESH 3: $-2.5\, i_A + 7\, i_3 - 3\, i_2 = 0$ [3]

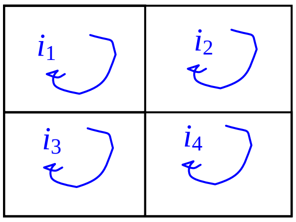
Combining terms and making use of the fact that $i_A = -i_3$,

$$i_1 - i_2 - 2.5\, i_3 = 20 \quad [1]$$
$$-i_1 + 6i_2 - 3\, i_3 = 0 \quad [2]$$
$$-3\, i_2 + 9.5\, i_3 = 0 \quad [3]$$

Solving, $i_1 = 30.97$ A, $i_2 = 6.129$ A, and $i_3 = 1.936$ A. Since $i_A = -i_3$,

$$\boxed{i_A = -1.936 \text{ A.}}$$

43. Define four mesh currents



By inspection, $i_1 = -4.5$ A.

We form a supermesh with meshes 3 and 4 as defined above.

MESH 2: $2.2 + 3\, i_2 + 4\, i_2 + 5 - 4\, i_3 = 0$ [1]

SUPERMESH: $3\, i_4 + 9\, i_4 - 9\, i_1 + 4\, i_3 - 4\, i_2 + 6\, i_3 + i_3 - 3 = 0$ [2]

Supermesh KCL equation: $i_4 - i_3 = 2$ [3]

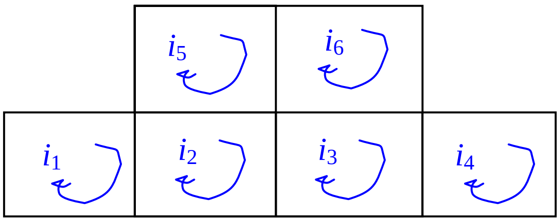
Simplifying and combining terms, we may rewrite these three equations as:

$$\begin{aligned} 7\, i_2 - 4\, i_3 \qquad\qquad &= -7.2 \qquad [1] \\ -4\, i_2 + 11\, i_3 + 12\, i_4 &= -37.5 \qquad [2] \\ -\, i_3 \quad + i_4 &= 2 \qquad [3] \end{aligned}$$

Solving, we find that $i_2 = -2.839$ A, $i_3 = -3.168$ A, and $i_4 = -1.168$ A.

The power supplied by the 2.2-V source is then $2.2\,(i_1 - i_2) = \boxed{-3.654 \text{ W.}}$

44. We begin by defining six mesh currents as depicted below:



- We form a supermesh with meshes 1 and 2 since they share a current source.
- We form a *second* supermesh with meshes 3 and 4 since they also share a current source.

1, 2 Supermesh:
$(4700 + 1000 + 10{,}000)\ i_1 - 2200\ i_5 + (2200 + 1000 + 4700)\ i_2 - 1000\ i_3 = 0$ [1]

3, 4 Supermesh:
$(4700 + 1000 + 2200)\ i_3 - 1000\ i_2 - 2200\ i_6 + (4700 + 10{,}000 + 1000)\ i_4 = 0$ [2]

MESH 5: $(2200 + 4700)\ i_5 - 2200\ i_2 + 3.2 - 1.5 = 0$ [3]

MESH 6: $1.5 + (4700 + 4700 + 2200)\ c - 2200\ i_3 = 0$ [4]

1, 2 Supermesh KCL equation: $i_1 - i_2 = 3\times10^{-3}$ [5]

3, 4 Supermesh KCL equation: $i_4 - i_3 = 2\times10^{-3}$ [6]

We can simplify these equations prior to solution in several ways. Choosing to retain six equations,

$$
\begin{array}{lllllllll}
15{,}700\ i_1 & + 7900\ i_2 & - 1000\ i_3 & & -2200\ \mathrm{i5} & & = 0 & [1] \\
& - 1000\ i_2 & + 7900\ i_3 & + 15{,}700\ i_4 & & -2200\ i_6 & = 0 & [2] \\
& - 2200\ i_2 & & & + 6900\ i_5 & & = -1.7 & [3] \\
& & - 2200\ i_3 & & & + 11{,}600\ i_6 & = -1.5 & [4] \\
i_1 & - i_2 & & & & & = 3\times10^{-3} & [5] \\
& & - i_3 & + i_4 & & & = 2\times10^{-3} & [6]
\end{array}
$$

Solving, we find that $i_4 = 540.8$ mA. Thus, the voltage across the 2-mA source is

$$(4700 + 10{,}000 + 1000)\ (540.8\times10^{-6}) = \boxed{8.491\ \text{V}}$$

45. We define a mesh current i_a in the left-hand mesh, a mesh current i_1 in the top right mesh, and a mesh current i_2 in the bottom right mesh (all flowing clockwise).

The left-most mesh can be analysed separately to determine the controlling voltage v_a, as KCL assures us that no current flows through either the 1-Ω or 6-Ω resistor.

Thus, $-1.8 + 3\, i_a - 1.5 + 2\, i_a = 0$, which may be solved to find $i_a = 0.66$ A. Hence, $v_a = 3\, i_a = 1.98$ V.

Forming one supermesh from the remaining two meshes, we may write:

$$-3 + 2.5\, i_1 + 3\, i_2 + 4\, i_2 = 0$$

and the supermesh KCL equation: $i_2 - i_1 = 0.5\, v_a = 0.5(1.98) = 0.99$

Thus, we have two equations to solve:

$$2.5\, i_1 + 7\, i_2 = 3$$
$$-i_1 + i_2 = 0.99$$

Solving, we find that $i_1 = -413.7$ mA and the voltage across the 2.5-Ω resistor (arbitrarily assuming the left terminal is the "+" reference) is $2.5\, i_1 = \boxed{-1.034 \text{ V}}$.

46. There are only three meshes in this circuit, as the botton 22-mΩ resistor is not connected connected at its left terminal. Thus, we define three mesh currents, i_1, i_2, and i_3, beginning with the left-most mesh.

We next create a supermesh from meshes 1 and 2 (note that mesh 3 is independent, and can be analysed separately).

Thus, $-11.8 + 10\times10^{-3}\, i_1 + 22\times10^{-3}\, i_2 + 10\times10^{-3}\, i_2 + 17\times10^{-3}\, i_1 = 0$

and applying KCL to obtain an equation containing the current source,

$$i_1 - i_2 = 100$$

Combining terms and simplifying, we obtain

$$\begin{aligned} 27\times10\text{-}3\, i_1 + 32\times10^{-3}\, i_2 &= 11.8 \\ i_1 - i_2 &= 100 \end{aligned}$$

Solving, we find that $\boxed{i_1 = 254.2 \text{ A}}$ and $\boxed{i_2 = 154.2 \text{ A.}}$

The final mesh current is easily found: $\boxed{i_3 = 13\times10^3/\,(14 + 11.6 + 15) = 320.2 \text{ A.}}$

47. MESH 1: $-7 + i_1 - i_2 = 0$ [1]
MESH 2: $i_2 - i_1 + 2i_2 + 3i_2 - 3i_3 = 0$ [2]
MESH 3: $3i_3 - 3i_2 + \mathrm{x}i_3 + 2i_3 - 7 = 0$ [3]

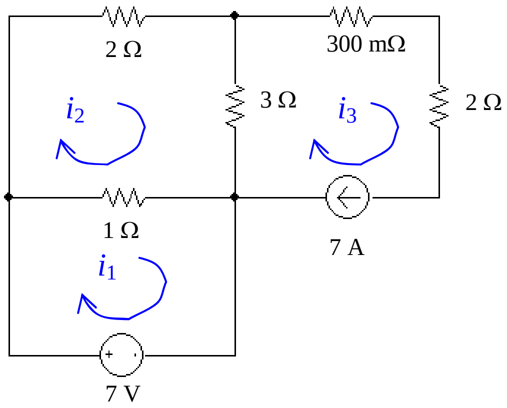
Grouping terms, we find that

$$i_1 - i_2 = 7 \quad [1]$$
$$-i_1 + 6i_2 - 3i_3 = 0 \quad [2]$$
$$-3i_2 + (5 + x)i_3 = 7 \quad [3]$$

This, unfortunately, is four unknowns but only three equations. However, we have not yet made use of the fact that we are trying to obtain $i_2 = 2.273$ A. Solving these “four” equations, we find that

$$\boxed{x = (7 + 3\, i_2 - 5\, i_3)/\, i_3 = 4.498\ \Omega.}$$

48. We begin by redrawing the circuit as instructed, and define three mesh currents:



By inspection, $i_3 = 7$ A.

MESH 1: $-7 + i_1 - i_2 = 0$ or $i_1 - i_2 = 7$ [1]

MESH 2: $(1 + 2 + 3)\, i_2 - i_1 - 3(7) = 0$ or $-i_1 + 6i_2 = 21$ [2]

There is no need for supermesh techniques for this situation, as the only current source lies on the outside perimeter of a mesh- it is not shared between meshes.

Solving, we find that $\boxed{i_1 = 12.6 \text{ A}, i_2 = 5.6 \text{ A and } i_3 = 7 \text{ A.}}$

49. (*a*) We are asked for a voltage, and have one current source and one voltage source. Nodal analysis is probably best then- the nodes can be named so that the desired voltage is a nodal voltage, or, at worst, we have one supernode equation to solve.

Name the top left node "1" and the top right node "x"; designate the bottom node as the reference terminal. Next, form a supernode with nodes "1" and "x."

At the supernode: $11 = v_1/2 + v_x/9$ [1]

and the KVL Eqn: $v_1 - v_x = 22$ [2]

Rearranging, $11(18) = 9v_1 + 2v_x$ [1]
$22 = v_1 - v_x$ [2]

Solving, $\boxed{v_x = 0}$

(*b*) We are asked for a voltage, and so may suspect that nodal analysis is preferrable; with two current sources and only one voltage source (easily dealt with using the supernode technique), nodal analysis does seem to have an edge over mesh analysis here.

Name the top left node "x," the top right node "y" and designate the bottom node as the reference node. Forming a supernode from nodes "x" and "y,"

At the supernode: $6 + 9 = v_x/10 + v_y/20$ [1]
and the KVL Eqn: $v_y - v_x = 12$ [2]

Rearranging, $15(20) = 2v_x + v_y$ [1]
and $12 = -v_x + v_y$ [2]

Solving, we find that $\boxed{v_x = 96 \text{ V.}}$

(*c*) We are asked for a voltage, but would have to subtract two nodal voltages (not much harder than invoking Ohm's law). On the other hand, the dependent current source depends on the desired unknown, which would lead to the need for another equation if invoking mesh analysis. Trying nodal analysis,

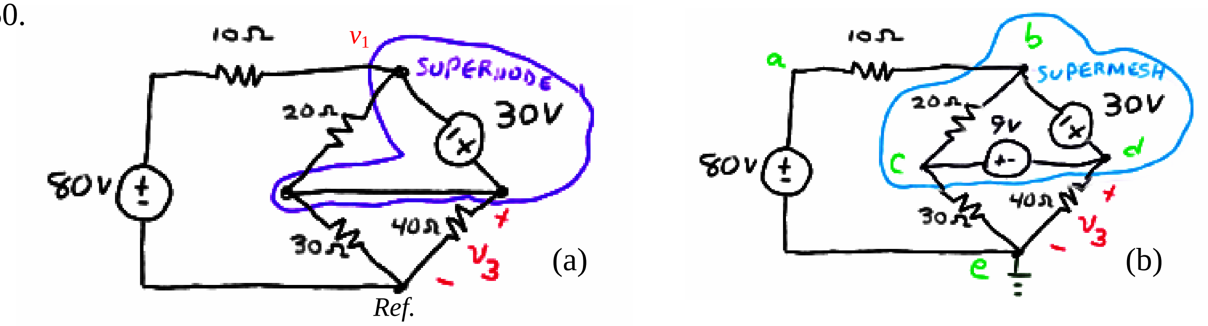
$$0.1\,v_x = (v_1 - 50)/2 + v_x/4 \qquad [1]$$

referring to the circuit we see that $v_x = v_1 - 100$. Rearranging so that we may eliminate v_1 in Eq. [1], we obtain $v_1 = v_x + 100$. Thus, Eq. [1] becomes

$$0.1\,v_x = (v_x + 100 - 50)/2 + v_x/4$$

and a little algebra yields $\boxed{v_x = -38.46 \text{ V.}}$

50.



(*a*) We begin by noting that it is a voltage that is required; no current values are requested. This is a three-mesh circuit, or a four-node circuit, depending on your perspective. Either approach requires three equations…. Except that applying the supernode technique reduces the number of needed equations by one.

At the 1, 3 supernode:

$$0 = (v_1 - 80)/10 + \cancel{(v_1 - v_3)/20} + \cancel{(v_3 - v_1)/20} + v_3/40 + v_3/30$$

and $\quad v_3 - v_1 = 30$

Both ends of the resistor are connected to the supernode, so we could actually just ignore it…

We simplify these two equations and collect terms, yielding

$$0.1\,v_1 + 0.05833\,v_3 = 8$$
$$-v_1 + v_3 = 30$$

Solving, we find that $\boxed{v_3 = 69.48 \text{ V}}$

(*b*) Mesh analysis would be straightforward, requiring 3 equations and a (trivial) application of Ohm's law to obtain the final answer. Nodal analysis, on the other hand, would require only two equations, and the desired voltage will be a nodal voltage.

At the b, c, d supernode: $\quad 0 = (v_b - 80)/10 + v_d/40 + v_c/30$

and: $\quad v_d - v_b = 30 \qquad v_c - v_d = 9$

Simplify and collect terms:

$$0.1\,v_b + 0.03333\,v_c + 0.025\,v_d = 80$$
$$-v_b + v_d = 30$$
$$v_c - v_d = 9$$

Solving, $\boxed{v_d\ (= v_3) = 67.58 \text{ V}}$

(c) We are now faced with a dependent current source whose value depends on a mesh current. Mesh analysis in this situation requires 1 supermesh, 1 KCL equation and Ohm's law. Nodal analysis requires 1 supernode, 1 KVL equation, 1 other nodal equation, and one equation to express i_1 in terms of nodal voltages. Thus, mesh analysis has an edge here. Define the left mesh as "1," the top mesh as "2", and the bottom mesh as "3."

Mesh 1: $\quad -80 + 10\,i_1 + 20\,i_1 - 20\,i_2 + 30\,i_1 - 30\,i_3 = 0$

2, 3 supermesh: $\quad 20\,i_2 - 20\,i_1 - 30 + 40\,i_3 + 30\,i_3 - 30\,i_1 = 0$

and: $\quad i_2 - i_3 = 5\,i_1$

Rewriting,

$$60\,i_1 - 20\,i_2 - 30\,i_3 = 80$$
$$-50\,i_1 + 20\,i_2 + 70\,i_3 = 30$$
$$5\,i_1 - i_2 + i_3 = 0$$

Solving, $\quad i_3 = 4.727$ A $\qquad$ so $\qquad \boxed{v_3 = 40\,i_3 = 189 \text{ V.}}$

51. This circuit consists of 3 meshes, and no dependent sources. Therefore 3 simultaneous equations and 1 subtraction operation would be required to solve for the two desired currents. On the other hand, if we use nodal analysis, forming a supernode about the 30-V source would lead to 5 – 1 – 1 = 3 simulataneous equations as well, plus several subtraction and division operations to find the currents. Thus, mesh analysis has a slight edge here.

Define three clockwise mesh currents: i_a in the left-most mesh, i_b in the top right mesh, and i_c in the bottom right mesh. Then our mesh equations will be:

Mesh *a*: $-80 + (10 + 20 + 30)\, i_a - 20\, i_b - 30\, i_c = 0$ [1]
Mesh *b*: $-30 + (12 + 20)\, i_b - 12\, i_c - 20\, i_a = 0$ [2]
Mesh *c*: $(12 + 40 + 30)\, i_c - 12\, i_b - 30\, i_a = 0$ [3]

Simplifying and collecting terms,

$60\, i_a - 20\, i_b - 30\, i_c = 80$ [1]
$-20\, i_a + 32\, i_b - 12\, i_c = 30$ [2]
$-30\, i_a - 12\, i_b + 82\, i_c = 0$ [3]

Solving, we find that i_a = 3.549 A, i_b = 3.854 A, and i_c = 1.863 A. Thus,

$$i_1 = i_a = \boxed{3.549 \text{ A}} \quad \text{and} \quad i_2 = i_a - i_c = \boxed{1.686 \text{ A.}}$$

52. Approaching this problem using nodal analysis would require 3 separate nodal equations, plus one equation to deal with the dependent source, plus subtraction and division steps to actually find the current i_{10}. Mesh analysis, on the other hand, will require 2 mesh/supermesh equations, 1 KCL equation, and one subtraction step to find i_{10}. Thus, mesh analysis has a clear edge. Define three clockwise mesh currents: i_1 in the bottom left mesh, i_2 in the top mesh, and i_3 in the bottom right mesh.

MESH 1: $i_1 = 5$ mA *by inspection* [1]

SUPERMESH:

$i_1 - i_2 = 0.4\, i_{10}$

$i_1 - i_2 = 0.4(i_3 - i_2)$

$i_1 - 0.6\, i_2 - 0.4\, i_3 = 0$ [2]

MESH 3: $-5000\, i_1 - 10000\, i_2 + 35000\, i_3 = 0$ [3]

Simplify:

$0.6\, i_2 + 0.4\, i_3 = 5\times10^{-3}$ [2]

$-10000\, i_2 + 35000\, i_3 = 25$ [3]

Solving, we find $i_2 = 6.6$ mA and $i_3 = 2.6$ mA. Since $i_{10} = i_3 - i_2$, we find that

$$\boxed{i_{10} = -4 \text{ mA.}}$$

53. For this circuit problem, nodal analysis will require 3 simultaneous nodal equations, then subtraction/ division steps to obtain the desired currents. Mesh analysis requires 1 mesh equation, 1 supermesh equation, 2 simple KCL equations and one subtraction step to determine the currents. If either technique has an edge in this situation, it's probably mesh analysis. Thus, define four clockwise mesh equations: i_a in the bottom left mesh, i_b in the top left mesh, i_c in the top right mesh, and i_d in the bottom right mesh.

At the *a*, *b*, *c* supermesh: $-100 + 6\, i_a + 20\, i_b + 4\, i_c + 10\, i_c - 10\, i_d = 0$ [1]

Mesh d: $100 + 10\, \text{id} - 10\, i_c + 24\, i_d = 0$ [2]

KCL: $-\, i_a + i_b = 2$ [3]

and $-\, i_b + i_c = 3\, i_3 = 3\, i_a$ [4]

Collecting terms & simplifying,

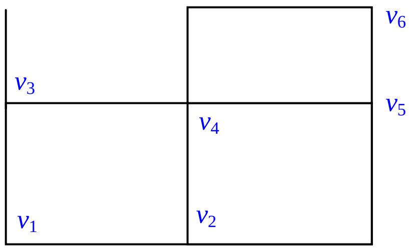
$$\begin{aligned}
6\, i_a + 20\, i_b + 14\, i_c - 10\, i_d &= 100 && [1]\\
-10\, i_c + 34\, i_d &= -100 && [2]\\
-\, i_a + i_b &= 2 && [3]\\
-3\, i_a - i_b + i_c &= 0 && [4]
\end{aligned}$$

Solving,

$i_a = 0.1206$ A, $i_b = 2.121$ A, $i_c = 2.482$ A, and $i_d = -2.211$ A. Thus,

$$i_3 = i_a = \boxed{120.6 \text{ mA}} \quad \text{and} \quad i_{10} = i_c - i_d = \boxed{4.693 \text{ A.}}$$

54. With 7 nodes in this circuit, nodal analysis will require the solution of three simultaneous nodal equations (assuming we make use of the supernode technique) and one KVL equation. Mesh analysis will require the solution of three simultaneous mesh equations (one mesh current can be found by inspection), plus several subtraction and multiplication operations to finally determine the voltage at the central node. Either will probably require a comparable amount of algebraic manoeuvres, so we go with nodal analysis, as the desired unknown is a direct result of solving the simultaneous equations. Define the nodes as:



NODE 1: $-2\times10^{-3} = (v_1 - 1.3)/\ 1.8\times10^{3} \quad \rightarrow \quad v_1 = -2.84$ V.

2, 4 Supernode:
$2.3\times10^{-3} = (v_2 - v_5)/\ 1\text{x}10^{3} + (v_4 - 1.3)/\ 7.3\times10^{3} + (v_4 - v_5)/\ 1.3\times10^{3} + v_4/\ 1.5\times10^{3}$

KVL equation: $-v_2 + v_4 = 5.2$

Node 5: $0 = (v_5 - v_2)/\ 1\text{x}10^{3} + (v_5 - v_4)/\ 1.3\text{x}10^{3} + (v_5 - 2.6)/\ 6.3\text{x}10^{3}$

Simplifying and collecting terms,

$$14.235\, v_2 + 22.39\, v_4 - 25.185\, v_5 = 35.275 \quad [1]$$
$$-v_2 + v_4 = 5.2 \quad [2]$$
$$-8.19\, v_2 - 6.3\, v_4 + 15.79\, v_5 = 3.38 \quad [3]$$

Solving, we find the voltage at the central node is $\boxed{v_4 = 3.460 \text{ V.}}$

55. Mesh analysis yields current values directly, so use that approach. We therefore define four clockwise mesh currents, starting with i_1 in the left-most mesh, then i_2, i_3 and i_4 moving towards the right.

Mesh 1: $-0.8i_x + (2 + 5)\, i_1 - 5\, i_2 = 0$ [1]

Mesh 2: $i_2 = 1$ A by inspection [2]

Mesh 3: $(3 + 4)\, i_3 - 3(1) - 4(i_4) = 0$ [3]

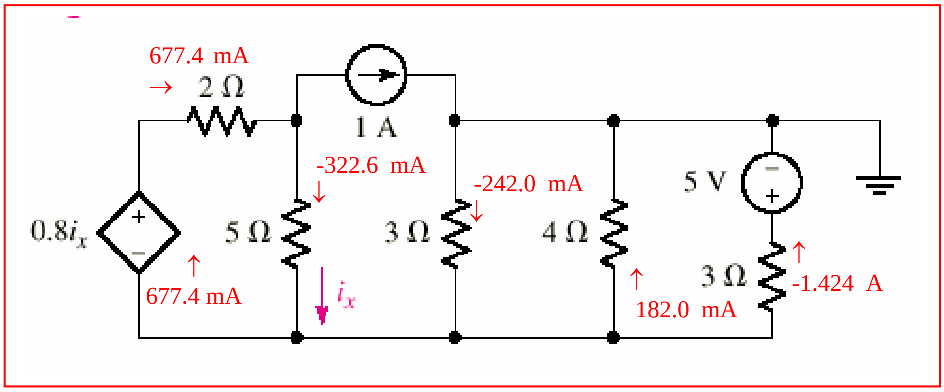
Mesh 4: $(4 + 3)\, i_4 - 4\, i_3 - 5 = 0$ [4]

Simplify and collect terms, noting that $i_x = i_1 - i_2 = i_1 - 1$

$-0.8(i_1 - 1) + 7\, i_1 - 5(1) = 0$ yields $i_1 = 677.4$ mA

Thus, [3] and [4] become: $7\, i_3 - 4\, i_4 = 3$ [3]
$-4\, i_3 + 7\, i_4 = 5$ [4]

Solving, we find that $i_3 = 1.242$ A and $i_4 = 1.424$ A. A map of individual branch currents can now be drawn:



56. If we choose to perform mesh analysis, we require 2 simultaneous equations (there are four meshes, but one mesh current is known, and we can employ the supermesh technique around the left two meshes). In order to find the voltage across the 2-mA source we will need to write a KVL equation, however. Using nodal analysis is less desirable in this case, as there will be a large number of nodal equations needed. Thus, we define four clockwise mesh currents i_1, i_2, i_3 and i_4 starting with the left-most mesh and moving towards the right of the circuit.

At the 1,2 supermesh: $2000\, i_1 + 6000\, i_2 - 3 + 5000\, i_2 = 0$ [1]

and $i_1 - i_2 = 2\times10^{-3}$ [2]

by inspection, $i_4 = -1$ mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and i_1 and i_2 are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain $2000\, i_1 + 11000\, i_2 = 3$ [1]

$1000\, i_1 - 1000\, i_2 = 2$ [2]

Solving, $i_1 = 1.923$ mA and $i_2 = -76.92$ μA.

Thus, the voltage across the 2-mA source ("+" reference at the top of the source) is

$$v = -2000\, i_1 - 6000\, (i_1 - i_2) = \boxed{-15.85 \text{ V.}}$$

57. Nodal analysis will require 2 nodal equations (one being a "supernode" equation), 1 KVL equation, and subtraction/division operations to obtain the desired current. Mesh analysis simply requires 2 "supermesh" equations and 2 KCL equations, with the desired current being a mesh current. Thus, we define four clockwise mesh currents i_a, i_b, i_c, i_d starting with the left-most mesh and proceeding to the right of the circuit.

At the *a*, *b* supermesh: $-5 + 2\, i_a + 2\, i_b + 3\, i_b - 3\, i_c = 0$ [1]

At the *c*, *d* supermesh: $3\, i_c - 3\, i_b + 1 + 4\, i_d = 0$ [2]

and $i_a - i_b = 3$ [3]

$i_c - i_d = 2$ [4]

Simplifying and collecting terms, we obtain

$$2\, i_a + 5\, i_b - 3\, i_c = 5 \quad [1]$$
$$-3\, i_b + 3\, i_c + 4\, i_d = -1 \quad [2]$$
$$i_a - i_b = 3 \quad [3]$$
$$i_c - i_d = 2 \quad [4]$$

Solving, we find i_a = 3.35 A, i_b = 350 mA, i_c = 1.15 A, and i_d = -850 mA. As $i_1 = i_b$,

$$\boxed{i_1 = 350 \text{ mA.}}$$

58. Define a voltage v_x at the top node of the current source I_2, and a clockwise mesh current i_b in the right-most mesh.

We want 6 W dissipated in the 6-Ω resistor, which leads to the requirement $i_b = 1$ A. Applying nodal analysis to the circuit,

$I_1 + I_2 = (v_x - v_1)/6 = 1$

so our requirement is $I_1 + I_2 = 1$. There is no constraint on the value of v_1 other than we are told to select a nonzero value.

Thus, $\boxed{\text{we choose } I_1 = I_2 = 500 \text{ mA and } v_1 = 3.1415 \text{ V.}}$

59. Inserting the new 2-V source with "+" reference at the bottom, and the new 7-mA source with the arrow pointing down, we define four clockwise mesh currents i_1, i_2, i_3, i_4 starting with the left-most mesh and proceeding towards the right of the circuit.

Mesh 1: $(2000 + 1000 + 5000)\, i_1 - 6000\, i_2 - 2 = 0$ [1]

2, 3 Supermesh:
$2 + (5000 + 5000 + 1000 + 6000)\, i_2 - 6000\, i_1 + (3000 + 4000 + 5000)\, i_3 - 5000\, i_4 = 0$ [2]

and $i_2 - i_3 = 7\times10^{-3}$ [3]

Mesh 4: $i_4 = -1$ mA by inspection [4]

Simplifying and combining terms,

$$
\begin{aligned}
8000\, i_1 - 6000\, i_2 \qquad\qquad &= 2 \quad [1] \\
1000\, i_2 - 1000\, i_3 &= 7 \quad [4] \\
-6000\, i_1 + 17000\, i_2 + 12000\, i_3 &= -7 \quad [2]
\end{aligned}
$$

Solving, we find that

$$\boxed{i_1 = 2.653 \text{ A},\ i_2 = 3.204 \text{ A},\ i_3 = -3.796 \text{ A},\ i_4 = -1 \text{ mA}}$$

60. This circuit is easily analyzed by mesh analysis; it's planar, and after combining the 2A and 3 A sources into a single 1 A source, supermesh analysis is simple.

First, define clockwise mesh currents i_x, i_1, i_2 and i_3 starting from the left-most mesh and moving to the right. Next, combine the 2 A and 3 A sources temporarily into a 1 A source, arrow pointing upwards. Then, define four nodal voltages, V_1, V_2, V_3 and V_4 moving from left to right along the top of the circuit.

At the left-most mesh, $i_x = -5\, i_1$ [1]
For the supermesh, we can write $4i_1 - 2i_x + 2 + 2i_3 = 0$ [2]
and the corresponding KCL equation: $i_3 - i_1 = 1$ [3]

Substituting Eq. [1] into Eq. [2] and simplifying,

$14i_1 + 2i_3 = -2$
$-i_1 + i_3 = 1$

Solving, $i_1 = -250$ mA and $i_3 = 750$ mA.
Then, $i_x = -5\, i_1 = 1.35$ A and $i_2 = i_1 - 2 = -2.25$ A

Nodal voltages are straightfoward to find, then:

$V_4 = 2i_3 = 1.5$ V
$V_3 = 2 + V_4 = 3.5$ V
$V_2 - V_3 = 2\, i_1$ or $V_2 = 2\, i_1 + V_3 = 3$ V
$V_1 - V_2 = 2\, i_x$ or $V_1 = 2\, i_x + V_2 = 5.5$ V

61. **Hand analysis:**

Define three clockwise mesh currents: i_1 in the bottom left mesh, i_2 in the top mesh, and i_3 in the bottom right mesh.

MESH 1: $i_1 = 5$ mA *by inspection* [1]

SUPERMESH:
$$i_1 - i_2 = 0.4\, i_{10}$$
$$i_1 - i_2 = 0.4(i_3 - i_2)$$
$$i_1 - 0.6\, i_2 - 0.4\, i_3 = 0 \quad [2]$$

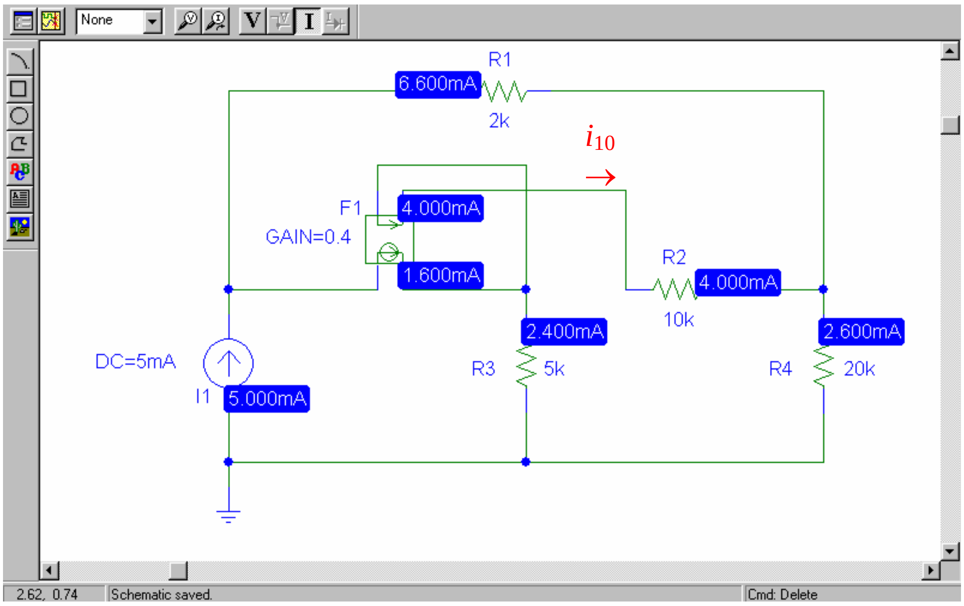
MESH 3: $-5000\, i_1 - 10000\, i_2 + 35000\, i_3 = 0$ [3]

Simplify:
$$0.6\, i_2 + 0.4\, i_3 = 5\times10^{-3} \quad [2]$$
$$-10000\, i_2 + 35000\, i_3 = 25 \quad [3]$$

Solving, we find $i_2 = 6.6$ mA and $i_3 = 2.6$ mA. Since $i_{10} = i_3 - i_2$, we find that

$$\boxed{i_{10} = -4 \text{ mA.}}$$

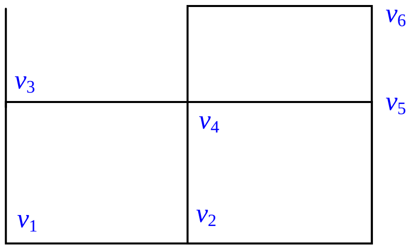
PSpice simulation results:



Summary: The current entering the right-hand node of the 10-kΩ resistor R2 is equal to 4.000 mA. Since this current is $-i_{10}$, $i_{10} = -4.000$ mA as found by hand.

62. **Hand analysis:**

Define the nodes as:



NODE 1: $-2\times10^{-3} = (v_1 - 1.3)/\ 1.8\times10^{3} \quad \rightarrow \quad v_1 = -2.84$ V.

2, 4 Supernode:

$2.3\times10^{-3} = (v_2 - v_5)/\ 1\text{x}10^{3} + (v_4 - 1.3)/\ 7.3\times10^{3} + (v_4 - v_5)/\ 1.3\times10^{3} + v_4/\ 1.5\times10^{3}$

KVL equation: $-v_2 + v_4 = 5.2$

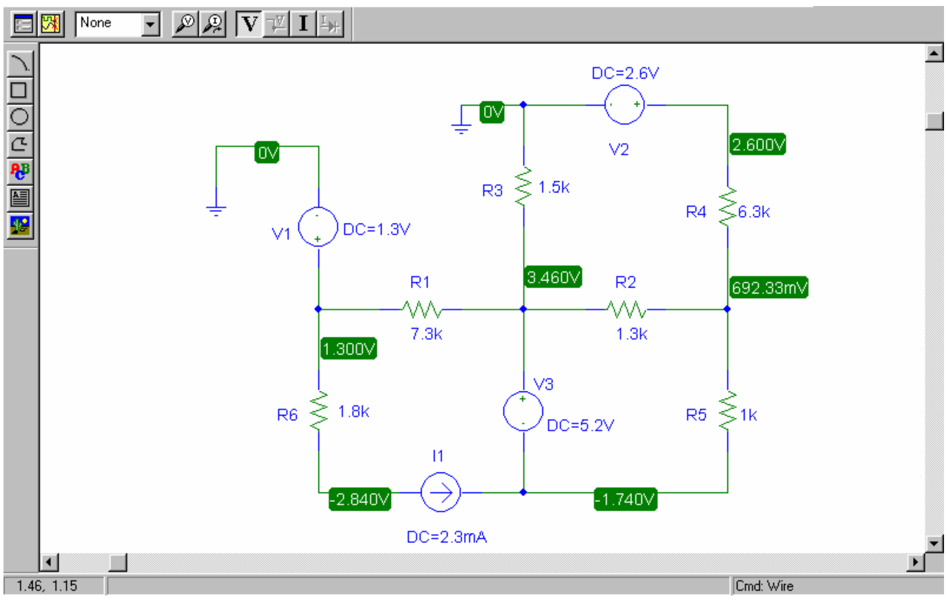
Node 5: $0 = (v_5 - v_2)/\ 1\text{x}10^{3} + (v_5 - v_4)/\ 1.3\text{x}10^{3} + (v_5 - 2.6)/\ 6.3\text{x}10^{3}$

Simplifying and collecting terms,

$$
\begin{aligned}
14.235\ v_2 + 22.39\ v_4 - 25.185\ v_5 &= 35.275 \quad [1]\\
-v_2 + \qquad v_4 \qquad\qquad &= 5.2 \qquad [2]\\
-8.19\ v_2 \quad - 6.3\ v_4 + 15.79\ v_5 &= 3.38 \qquad [3]
\end{aligned}
$$

Solving, we find the voltage at the central node is $\boxed{v_4 = 3.460 \text{ V.}}$

PSpice simulation results:



Summary: The voltage at the center node is found to be 3.460 V, which is in agreement with our hand calculation.

63. **Hand analysis:**

At the 1,2 supermesh: $2000\ i_1 + 6000\ i_2 - 3 + 5000\ i_2 = 0$ [1]

and $i_1 - i_2 = 2\times10^{-3}$ [2]

by inspection, $i_4 = -1$ mA. However, this as well as any equation for mesh four are unnecessary: we already have two equations in two unknowns and i_1 and i_2 are sufficient to enable us to find the voltage across the current source.

Simplifying, we obtain $2000\ i_1 + 11000\ i_2 = 3$ [1]

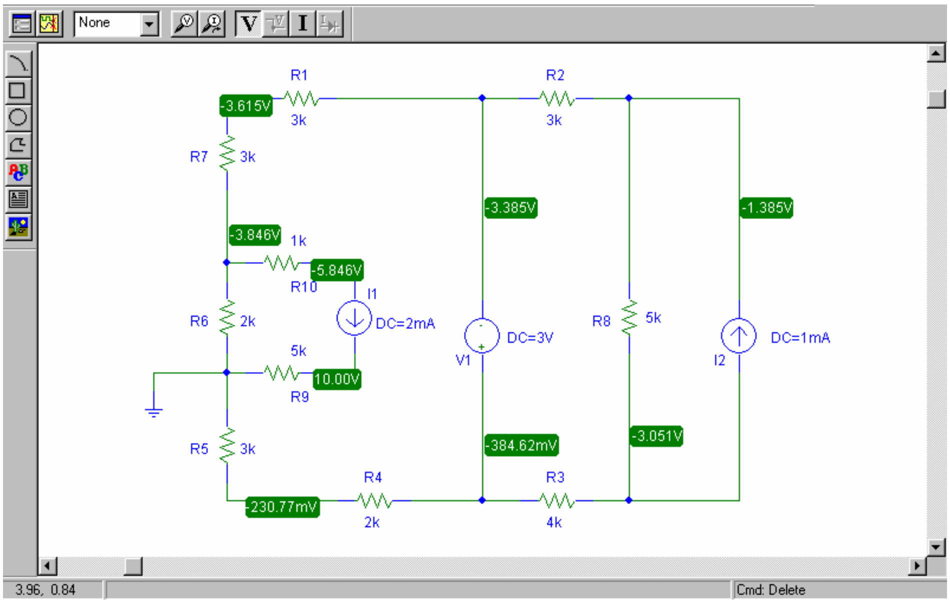
$1000\ i_1 - 1000\ i_2 = 2$ [2]

Solving, $i_1 = 1.923$ mA and $i_2 = -76.92$ μA.

Thus, the voltage across the 2-mA source ("+" reference at the top of the source) is

$$v = -2000\ i_1 - 6000\ (i_1 - i_2) = \boxed{-15.85\text{ V.}}$$

PSpice simulation results:



Summary: Again arbitrarily selecting the "+" reference as the top node of the 2-mA current source, we find the voltage across it is –5.846 – 10 = -15.846 V, in agreement with our hand calculation.

64. **Hand analysis:**

Define a voltage v_x at the top node of the current source I_2, and a clockwise mesh current i_b in the right-most mesh.

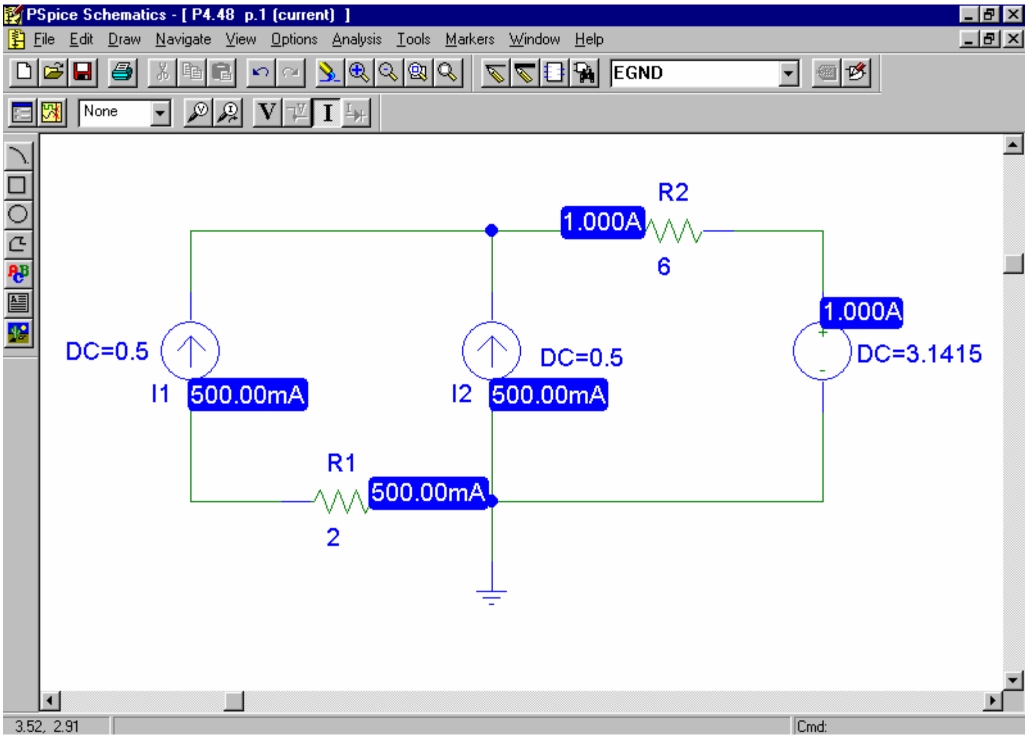
We want 6 W dissipated in the 6-Ω resistor, which leads to the requirement i_b = 1 A. Applying nodal analysis to the circuit,

$$I_1 + I_2 \ = \ (v_x - v_1)/\,6 \ = \ 1$$

so our requirement is $I_1 + I_2 = 1$. There is no constraint on the value of v_1 other than we are told to select a nonzero value.

Thus, $\boxed{\text{we choose } I_1 = I_2 = 500 \text{ mA and } v_1 = 3.1415 \text{ V.}}$

PSpice simulation results:



Summary: We see from the labeled schematic above that our choice for I_1, I_2 and V_1 lead to 1 A through the 6-Ω resistor, or 6 W dissipated in that resistor, as desired.

65. **Hand analysis:**

Define node 1 as the top left node, and node 2 as the node joining the three 2-Ω resistors. Place the "+" reference terminal of the 2-V source at the right. The right-most 2-Ω resistor has therefore been shorted out. Applying nodal analysis then,

Node 1: $-5\, i_1 = (v_1 - v_2)/ 2$ [1]

Node 2: $0 = (v_2 - v_1)/ 2 + v_2/ 2 + (v_2 - 2)/ 2$ [2]

and, $i_1 = (v_2 - 2)/ 2$ [3]

Simplifying and collecting terms,

$v_1 + v_2 = 10$ [1]

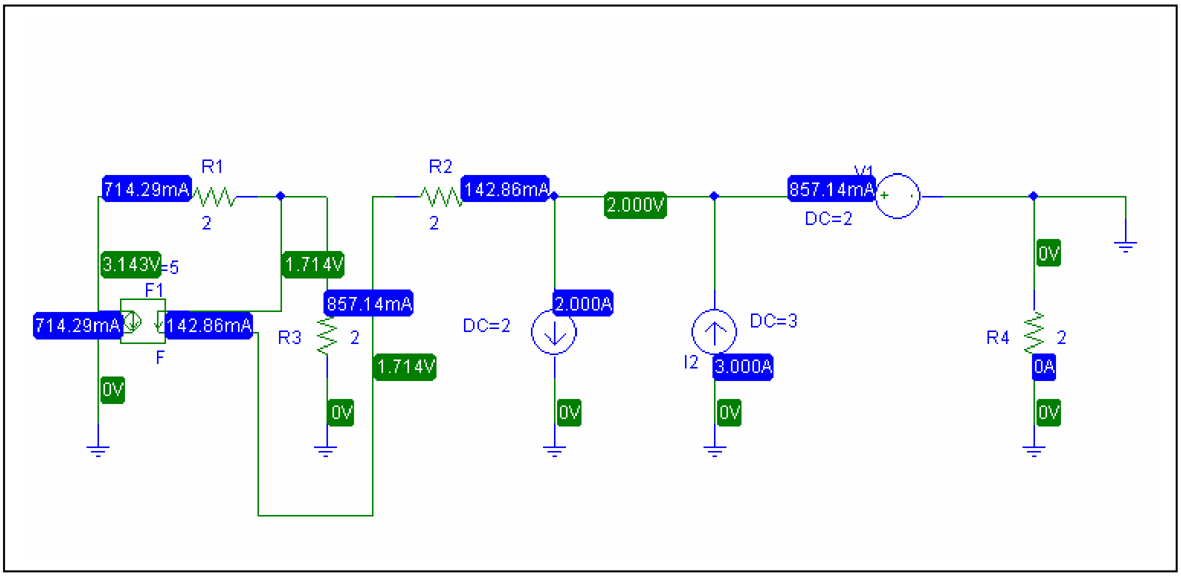
$-v_1 + 3\, v_2 = 2$ [2]

Solving, we find that $\boxed{v_1 = 3.143 \text{ V} \quad \text{and} \quad v_2 = 1.714 \text{ V.}}$

Defining clockwise mesh currents i_a, i_b, i_c, i_d starting with the left-most mesh and proceeding right, we may easily determine that

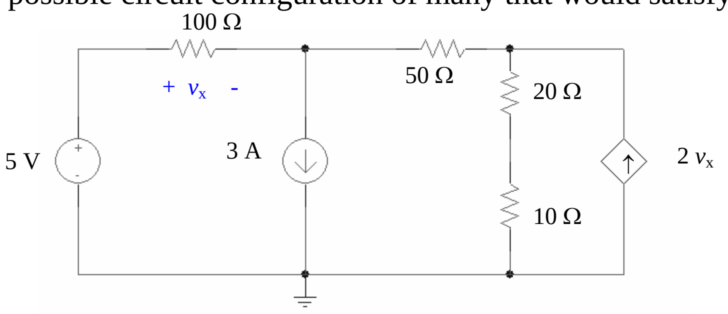
$$\boxed{\begin{aligned} i_a &= -5\, i_1 = 714.3 \text{ mA} \\ i_b &= -142.9 \text{ mA} \\ i_c &= i_1 - 2 = -2.143 \text{ A} \\ i_d &= 3 + i_c = 857.1 \text{ mA} \end{aligned}}$$

PSpice simulation results:



Summary: The simulation results agree with the hand calculations.

66. (a) One possible circuit configuration of many that would satisfy the requirements:



At node 1: $-3 = (v_1 - 5)/ 100 + (v_1 - v2)/ 50$ [1]

At node 2: $2\ v_x = (v2 - v_1)/ 50 + v_2/ 30$ [2]

and, $v_x = 5 - v_1$ [3]

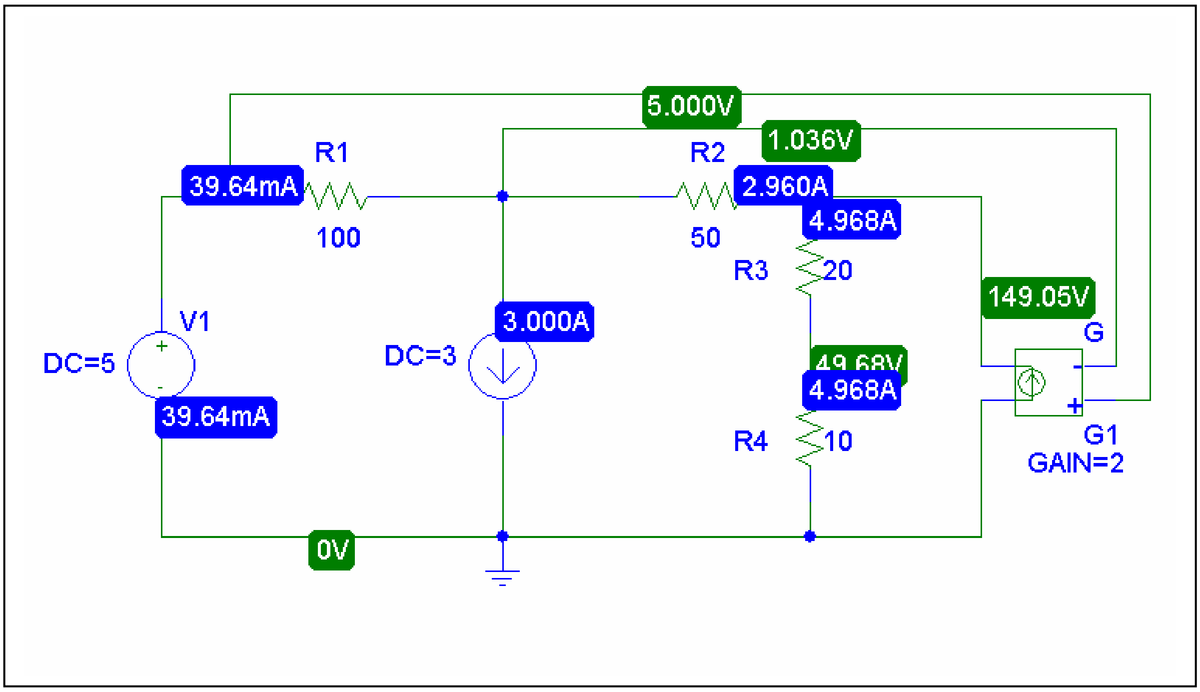
Simplifying and collecting terms,

$150\ v_1 - 100\ v_2 = -14750$ [1]

$2970\ v_1 + 80\ v_2 = 15000$ [2]

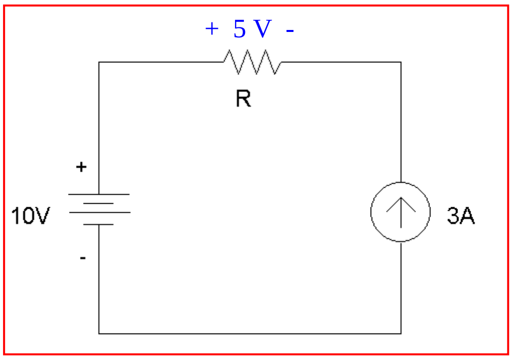
Solving, we find that $\boxed{v_1 = 1.036 \text{ V}}$ and $\boxed{v_2 = 149.1 \text{ V.}}$
The current through the 100-Ω resistor is simply $(5 - v_1)/100 = \boxed{39.64 \text{ mA}}$
The current through the 50-Ω resistor is $(v_1 - v_2)/ 50 = \boxed{-2.961 \text{ A,}}$
and the current through the 20-Ω and 10-Ω series combination is $v_2/ 30 = \boxed{4.97 \text{ A.}}$
Finally, the dependent source generates a current of $2\ v_x = \boxed{7.928 \text{ A.}}$

(b) PSpice simulation results



Summary: The simulated results agree with the hand calculations.

67. One possible solution of many:



Choose R so that 3R = 5; then the voltage across the current source will be 5 V, and so will the voltage across the resistor R.

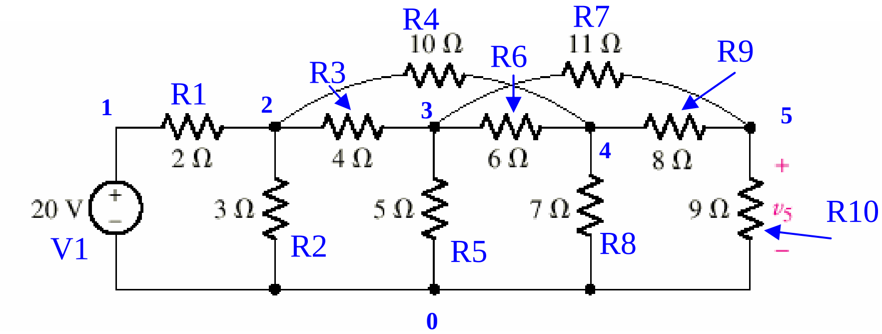
R = 5/3 Ω. To construct this from 1-Ω resistors, note that

$$5/3\ \Omega = 1\ \Omega + 2/3\ \Omega = 1\ \Omega + 1\ \Omega \| 1\Omega \| 1\Omega + 1\Omega \| 1\Omega \| 1\Omega$$

```
* Solution to Problem 4.57

.OP

V1 1 0 DC 10
I1 0 4 DC 3
R1 1 2 1
R2 2 3 1
R3 2 3 1
R4 2 3 1
R5 3 4 1
R6 3 4 1
R7 3 4 1

.END
```

```
****    SMALL SIGNAL BIAS SOLUTION      TEMPERATURE =   27.000 DEG C
******************************************************************************
 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(   1)  10.0000  (   2)   7.0000  (   3)   6.0000  (   4)   5.0000

   VOLTAGE SOURCE CURRENTS
   NAME         CURRENT

   V1          -3.000E+00
```

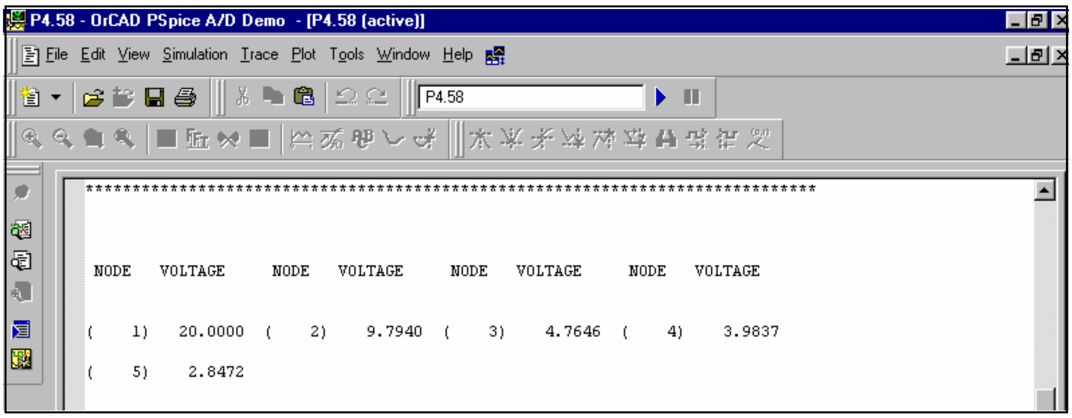
68. We first name each node, resistor and voltage source:



We next write an appropriate input deck for SPICE:

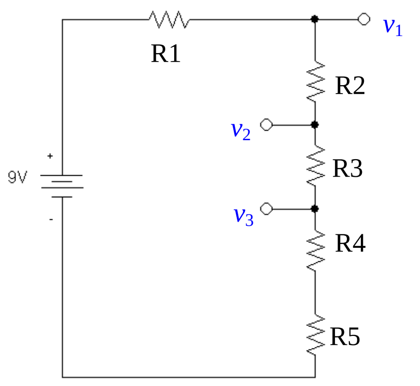
```
*  Solution to Problem 4.58

.OP

V1 1 0 DC 20
R1 1 2 2
R2 2 0 3
R3 2 3 4
R4 2 4 10
R5 3 0 5
R6 3 4 6
R7 3 5 11
R8 4 0 7
R9 4 5 8
R10 5 0 9

.END
```

And obtain the following output:



We see from this simulation result that the voltage v_5 = 2.847 V.

69. One possible solution of many:



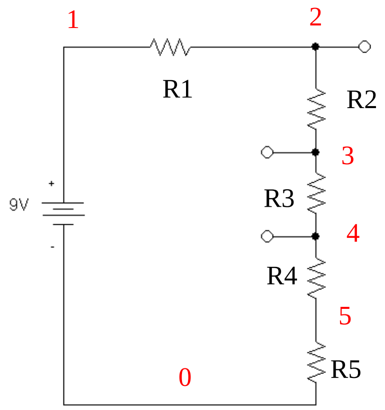
All resistors are 1 Ω, except R1, which represents 5 1-Ω resistors in series.

Verify: $v_1 = 9(4/9) = 4$ V
$v_2 = 9(3/9) = 3$ V
$v_3 = 9(2/9) = 2$ V

SPICE INPUT DECK:

```
* Solution to Problem 4.59

.OP

V1 1 0 DC 9
R1 1 2 5
R2 2 3 1
R3 3 4 1
R4 4 5 1
R5 5 0 1

.END
```

```
**** 07/29/01 21:36:26 *********** Evaluation PSpice (Nov 1999) **************

 * Solution to Problem 4.59
 ****     SMALL SIGNAL BIAS SOLUTION      TEMPERATURE =   27.000 DEG C
**********************************************************************
*******

 NODE    VOLTAGE      NODE    VOLTAGE      NODE    VOLTAGE      NODE
VOLTAGE

(   1)    9.0000  (   2)    4.0000  (   3)    3.0000  (   4)    2.0000

(   5)    1.0000
```



70. (a) If only two bulbs are not lit (and thinking of each bulb as a resistor), the bulbs must be in parallel- otherwise, the burned out bulbs, acting as short circuits, would prevent current from flowing to the "good" bulbs.

(b) In a parallel connected circuit, each bulb "sees" 115 VAC. Therefore, the individual bulb current is 1 W/ 115 V = 8.696 mA. The resistance of each "good" bulb is V/I = 13.22 kΩ. A simplified, electrically-equivalent model for this circuit would be a 115 VAC source connected in parallel to a resistor R_{eq} such that

$1/R_{eq} = 1/13.22\times10^3 + 1/13.22\times10^3 + \ldots + 1/13.22\times10^3$ (400 – 2 = 398 terms)
or $R_{eq} = 33.22\ \Omega$. We expect the source to provide 398 W.

```
* Solution to Problem 4.60

.OP

V1 1 0 AC 115 60
R1 1 0 33.22

.AC LIN 1 60 60
.PRINT AC VM(1)IM(V1)

.END
```

```
**** 07/29/01 21:09:32 *********** Evaluation PSpice (Nov 1999) **************

 * Solution to Problem 4.60

 ****     SMALL SIGNAL BIAS SOLUTION       TEMPERATURE =   27.000 DEG C
******************************************************************************

 NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE     NODE   VOLTAGE
(    1)    0.0000

   VOLTAGE SOURCE CURRENTS
   NAME         CURRENT

   V1          0.000E+00

   TOTAL POWER DISSIPATION   0.00E+00  WATTS

**** 07/29/01 21:09:32 *********** Evaluation PSpice (Nov 1999)
 * Solution to Problem 4.60


 ****     AC ANALYSIS                    TEMPERATURE =   27.000 DEG C
******************************************************************************

  FREQ        VM(1)       IM(V1)
  6.000E+01   1.150E+02   3.462E+00
```

This calculated power is not the value sought. It is an artifact of the use of an ac source, which requires that we perform an ac analysis. The supplied power is then separately computed as $(1.15\times10^2)(3.462) = 398.1$ W.

(c) The inherent series resistance of the wire connections leads to a voltage drop which increases the further one is from the voltage source. Thus, the furthest bulbs actually have less than 115 VAC across them, so they draw slightly less current and glow more dimly.