11. AC Circuit Power Analysis

11.1 Instantaneous power

◆ Resistance

$$\begin{aligned} p(t) &= v(t)\cdot i(t) \\ &= i^2(t)\cdot R \\ &= v^2(t)/R \end{aligned}$$

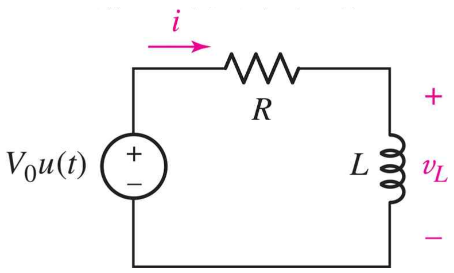
◆ Inductance

$$\begin{aligned} p(t) &= v(t)\cdot i(t) \\ &= L\ i(t)\frac{d\,i(t)}{dt} \end{aligned}$$

◆ Capacitance

$$\begin{aligned} p(t) &= v(t)\cdot i(t) \\ &= C\ v(t)\frac{d\,v(t)}{dt} \end{aligned}$$

Example)



$$i(t) = \frac{V_0}{R}(1 - e^{\frac{-R}{L}t})u(t)$$

total power delivered by the source

$$\begin{aligned} p(t) &= v(t)\cdot i(t) \\ &= \frac{V_0^2}{R}(1 - e^{\frac{-R}{L}t})u(t) \end{aligned}$$

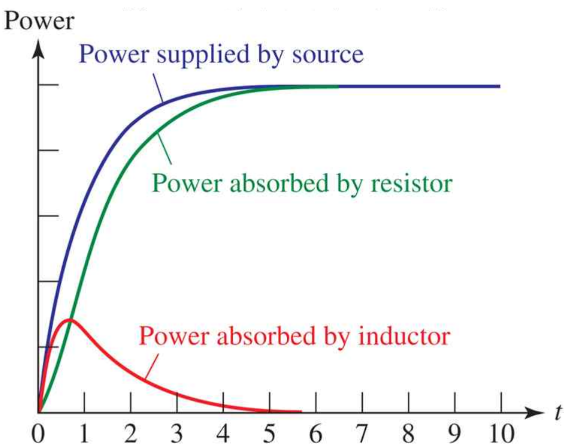
power delivered to (dissipated by) the R

$$\begin{aligned} p_R(t) &= i^2(t)R \\ &= \frac{V_0^2}{R}(1 - e^{\frac{-R}{L}t})^2\,u(t) \end{aligned}$$

power absorbed by inductor L

$$\begin{aligned} p_L(t) &= v_L(t)\cdot i(t) \\ &= \frac{V_0^2}{R}e^{\frac{-R}{L}t}(1 - e^{\frac{-R}{L}t})\,u(t) \end{aligned}$$

where $v_L(t) = L\dfrac{di(t)}{dt} = V_0 e^{\frac{-R}{L}t}u(t)$



11.2 Average power.

over the interval $[t_1,\ t_2]$.

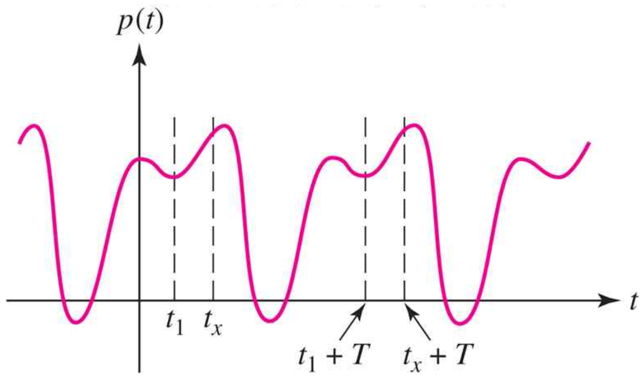
$$P=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}p(t)dt$$

for a periodic fn.

$$f(t)=f(t+T)$$

average power over one period.

$$P=\frac{1}{T}\int_{t_1}^{t_1+T}p(t)dt$$



◆ Average power in the sinusoidal steady state.

$$v(t)=V_m\cos(wt+\theta) \text{ and } i(t)=I_m\cos(wt+\phi)$$

instantaneous power.

$$\begin{aligned}p(t)&=V_mI_m\cos(wt+\theta)\cos(wt+\phi)\\&=\frac{1}{2}V_mI_m\cos(2wt+\theta+\phi)+\frac{1}{2}V_mI_m\cos(\theta-\phi)\\&=[\text{sinusoids with twice the applied freg.}]+[\text{constant term}]\end{aligned}$$

Hence, average power.

$$P=\frac{1}{2}V_mI_m\cos(\theta-\phi)$$

Example 11.2) $v(t)=4\cos(\pi t/6)$,

$V=4(V)$ and $Z=2\angle 60^\circ$

Average power and instantaneous power?

sol)

Phasor current

$$I=\frac{V}{Z}=2\angle-60^\circ \qquad \Rightarrow\ i(t)=2\cos(\pi t/6-60^\circ)$$

Average power

$$P=\frac{1}{2}\cdot 4\cdot 2\cos(0^\circ+60^\circ)=4\cos(60^\circ)=2(W)$$

instantaneous power.

$$\begin{aligned}p(t)&=8\cos(\pi t/6)\cos(\pi t/6-60^\circ)\\&=2+4\cos(\pi t/3-60^\circ)\end{aligned}$$

◆ Average power absorbed by Resistor

same phase (in phase)

$$v(t) = V_m \cos(wt+\theta) \text{ and } i(t) = I_m \cos(wt+\theta)$$

Average power

$$P = \frac{1}{2} V_m I_m \cos(\theta - \theta)$$

$$= \frac{1}{2} V_m I_m \quad = \frac{1}{2} \frac{V_m^2}{R} \quad = \frac{1}{2} I_m^2 R$$

◆ Average power absorbed by Reactive elements (L and C)

phase difference : $\pm 90^\circ$

Average power

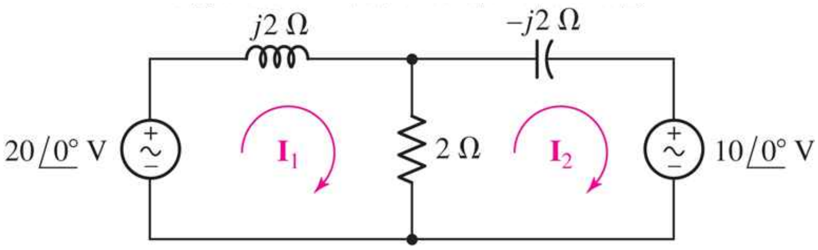
$$P = \frac{1}{2} V_m I_m \cos(\pm 90^\circ) = 0(W)$$

Example 11.3) $Z_L = 8 - j11(\Omega)$ by a current $I = 5\angle 20^\circ$ (A)

sol) $Z_L = 8 - j11(\Omega) = R + jX$ (저항 + 리액턴스)

$$\Rightarrow P = \frac{1}{2} I_m^2 R = 100(W)$$

Example 11.4) Average power of each elements?



sol)

$P_L = P_C = 0(W)$

$$I_1 = 5 - j10 = 11.18\angle -63.43^\circ \qquad \Rightarrow I_1 - I_2 = -j5 = 5\angle -90^\circ$$
$$I_2 = 5 - j5 \ = 7.07\angle -45^\circ$$

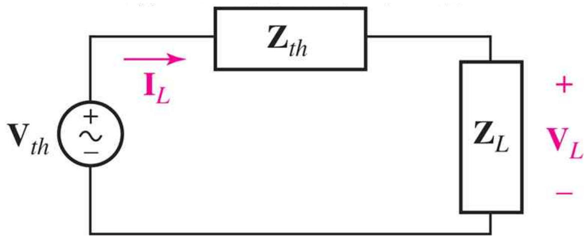
$$P_{2\Omega} = \frac{1}{2} I_m^2 R = 25(W)$$

$$P_{l_{source}} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} \times 20 \times (-11.18) \times \cos(63.43^\circ) = -50(W)$$

$$P_{r_{source}} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} \times 20 \times 11.18 \times \cos(45^\circ) = 25(W)$$

◆ Maximum power transfer.

The average power delivered to the load is maximum when $Z_L = Z_{th}$. (impedance matching)



11.3 Effective values of current and voltage

※ ac 110v or 220v having a freq. of 60Hz

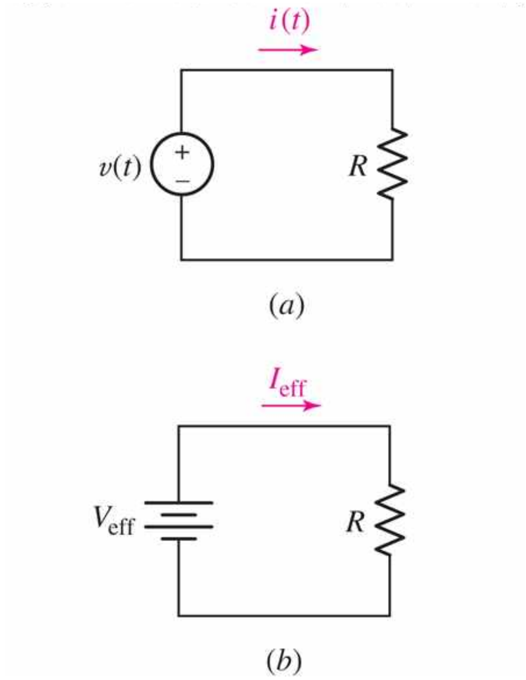
⇒ What is meant by "110" volts?

"Effective value" or "rms(root mean square) value"

(dc V_{eff}와 동일한 효과의 전압을 저항에 공급)

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} = V_{eff}$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} = I_{eff}$$



◆ rms value of Sinusoids

$$\begin{aligned}
I_{rms} &= \sqrt{\frac{1}{T}\int_0^T i^2(t)\,dt} \qquad \leftarrow i(t) = I_m \cos(wt+\phi) \\
&= I_m \sqrt{\frac{1}{T}\int_0^T \cos^2(wt+\phi)\,dt} \qquad \leftarrow \cos^2(A) = [1+\cos(2A)]/2 \\
&= I_m \sqrt{\frac{1}{2T}\int_0^T [1+\cos(2wt+2\phi)]\,dt} \\
&= I_m \sqrt{\frac{1}{2T}[T+0]} \\
&= \frac{I_m}{\sqrt{2}}
\end{aligned}$$

11.4 Apparent Power and Power Factor

Impedance Z_L applied by $v(t) = V_m \cos(wt + \theta)$

$\Rightarrow$ the current $i(t) = I_m \cos(wt + \phi)$

Average power

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$
$$= V_{eff} I_{eff} \cos(\theta - \phi)$$

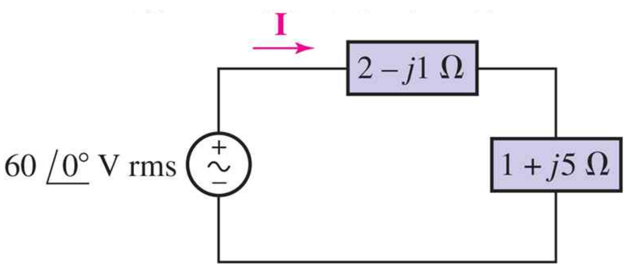
where apparent power is

$$P_{apparent} = V_{eff} I_{eff}$$

Power factor

$$PF = \frac{Average\ Power}{Apparent\ Power} = \cos(\theta - \phi)$$

Example 11.8) Average power, Apparent power, and Power factor?



sol)

total impedance

$$Z = 3 + j4 = 5\angle 53.13^\circ$$

current

$$I = \frac{V}{Z} = \frac{60\angle 0^\circ}{5\angle 53.13} = 12\angle -53.13^\circ$$

Average power

$$P = \frac{1}{2} V_m I_m \cos(\theta - \phi)$$
$$= V_{eff} I_{eff} \cos(\theta - \phi)$$
$$= I_{eff}^2 R = 12^2 \times 3 = 432(W)$$

Apparent power

$$P = V_{eff} I_{eff} = 60 \times 12 = 720(W)$$

Power factor

$$PF = \cos(\theta - \phi) = \cos(53.13^\circ) = 0.6$$
$$= \frac{432}{720} = 0.6$$