Engineering Electromagnetics
전자기학

William H. Hayt 원저, 강형부 외 공역, McGraw Hill Korea
부교재; Schaum's Outline Series ELECTROMAGNETICS

Chap 1. 벡터해석(Vector Analysis)

목적; 전자장 문제를 간단하게 취급
(전기장 및 자기장이 벡터량)

1.1 스칼라와 벡터

•스칼라(scalar); 하나의 실수값(real value)

무게, 속력, 온도, 부피, 저항, 전압, 전류

A, a

•벡터(vector); 크기(+)와 방향

중력, 힘, 속도, 가속도, 전기장, 자기장

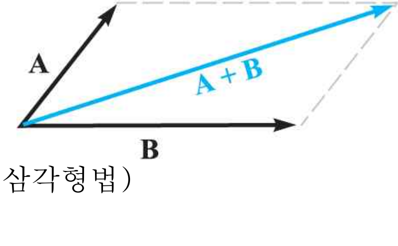
A, $\mathbf{a}$

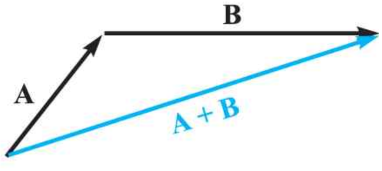
$\overline{\mathrm{A}}$, $\overline{a}$

$\overrightarrow{\mathrm{A}}$, $\overrightarrow{a}$

1.2 벡터 대수

; 계산규칙, 약속





덧셈; $A+B$

(평행사변형법 또는 삼각형법)

뺄셈; $A-B=A+(-B)$

교환법칙(commutative law); $A+B=B+A$

결합법칙(associative law); $A+(B+C)=(A+B)+C$

배분법칙(distributive law); $r(A+B)=rA+rB$

1.3 직각 좌표계
(cartesian or rectangular coordinate system)

•직각 좌표계; x, y, z 세 개의 좌표축이 서로 직각인 오른손 좌표계

(right−handed coordinate system)

그림 1.2 (a)

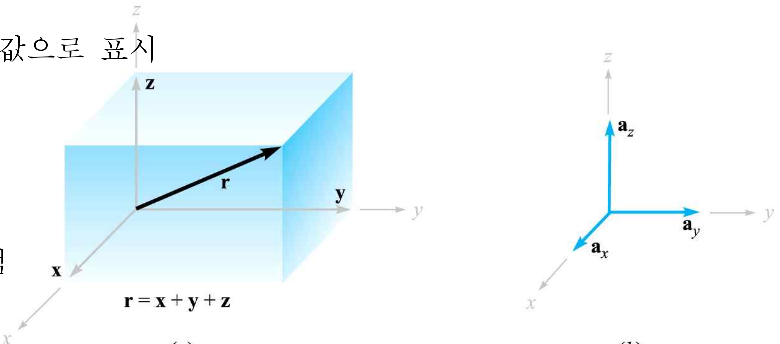
•점의 표시; 한 점의 위치는 x, y 및 z좌표 값으로 표시

$P(1,2,3)$, $Q(2,-2,1)$

그림 1.2 (b)

•점의 의미; $P(1,2,3)$

→ $x=1$, $y=2$, $z=3$인 평면들의 교차점



(a) (b)

•미소체적소의 정의;

점 $P(x,\ y,\ z)$의 각 좌표값이

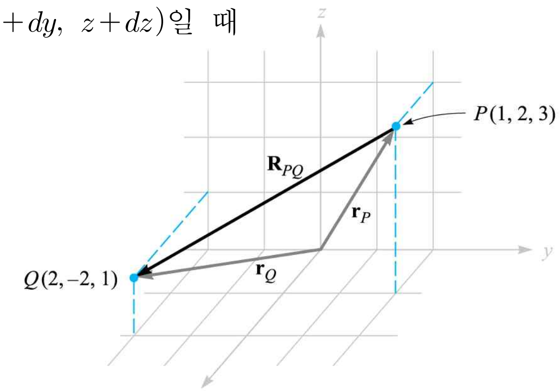
각각 미소량만큼 증가한 점이 $P'(x+dx,\ y+dy,\ z+dz)$일 때

미소체적소; $dv = dx\,dy\,dz$

대각선; $\overline{PP'} = dL = \sqrt{(dx)^2+(dy)^2+(dz)^2}$

면적소(dS); $dx\,dy$, $dy\,dz$, $dz\,dx$

그림 1.2 (c)



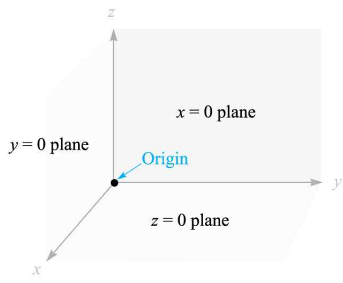
(c)

1.4 벡터성분 및 단위벡터

•성분벡터(component vector); $\boldsymbol{r} = \boldsymbol{x}+\boldsymbol{y}+\boldsymbol{z}$

•단위벡터(unit vector); 크기가 1인 벡터

•기본 단위벡터; 축방향으로의 단위벡터

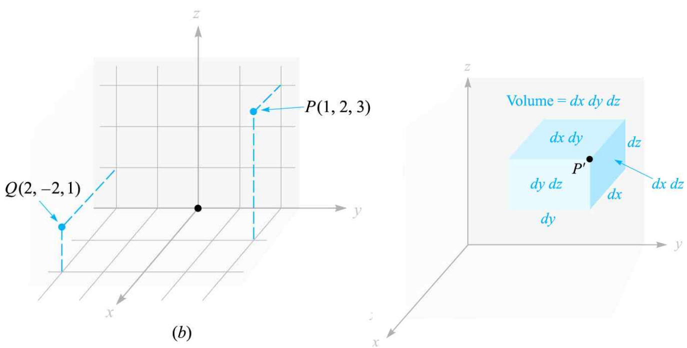


$\boldsymbol{a}_x$	$\boldsymbol{a}_y$	$\boldsymbol{a}_z$
$\boldsymbol{i}$	$\boldsymbol{j}$	$\boldsymbol{k}$
$\boldsymbol{u}_x$	$\boldsymbol{u}_y$	$\boldsymbol{u}_z$

•벡터의 성분표시법; $\boldsymbol{F} = F_x\boldsymbol{a}_x + F_y\boldsymbol{a}_y + F_z\boldsymbol{a}_z$

•벡터의 크기; $|\boldsymbol{F}| = F = \sqrt{F_x^2+F_y^2+F_x^2}$

•$\boldsymbol{F}$ 방향으로의 단위벡터; $\boldsymbol{a}_F = \boldsymbol{F}/|\boldsymbol{F}|$



(b) (c)

(예제 1.1) 원점 O에서 점 $G(2, -2, -1)$로 향하는 단위벡터 a_G

$$G = 2a_x - 2a_y - a_z$$

$$|G| = \sqrt{G_x^2 + G_y^2 + G_z^2} = \sqrt{4+4+1} = 3$$

$$\therefore\ a_G = \frac{G}{|G|} = \frac{2a_x - 2a_y - a_z}{3} = \frac{2}{3}a_x - \frac{2}{3}a_y - \frac{1}{3}a_z$$

(ex 1) 점 $P(1,2,3)$에서 점 $Q(2,-2,1)$까지의 벡터 R_{PQ}

△OPQ로부터 $r_P + R_{PQ} = r_Q$

$$R_{PQ} = r_Q - r_P$$

$$= (2a_x - 2a_y + a_z) - (a_x + 2a_y + 3a_z)$$

$$= a_x - 4a_y - 2a_z$$

(ex 2) $r_A = -a_x - 3a_y - 4a_z$

$r_B = 2a_x + 2a_y + 2a_z$

$C(1, 3, 4)$

(a) $R_{AB} = r_B - r_A =$

(b) $|r_A| = \sqrt{(-1)^2 + (-3)^2 + (-4)^2} =$

(c) $a_A = r_A / |r_A| =$

(d) $a_{AB} = R_{AB} / |R_{AB}| =$

$R_{AB} = r_B - r_A =$

$|R_{AB}| = \sqrt{R_x^{\ 2} + R_y^{\ 2} + R_z^{\ 2}} =$

(e) $a_{CA} = \dfrac{R_{CA}}{|R_{CA}|} =$

$R_{CA} = r_A - r_C =$

$r_C = a_x + 3a_y + 4a_z$

(응용예제 1.1) $M(-1,2,1)$, $N(3,-3,0)$, $P(-2,-3,-4)$

(a) R_{MN}

(b) $R_{MN} + R_{MP}$

(c) $|r_M|$

(d) a_{MP}

(e) $|2r_P - 3r_N|$

1.5 벡터계

(응용예제 1.2)

$$S = \frac{125}{(x-1)^2+(y-2)^2+(z+1)^2}[(x-1)\mathbf{a}_x+(y-2)\mathbf{a}_y+(z+1)\mathbf{a}_z]$$

(a) $S=?$ at $P(2,4,3)$ (b) $\mathbf{a}_S=?$ at $P(2,4,3)$ (c) $|S|=1$이 되는 조건

1.6 내적(스칼라곱, scalar product or dot product)

•정의; $\mathbf{A}\cdot\mathbf{B}=|\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$

•교환법칙; $\mathbf{A}\cdot\mathbf{B}=\mathbf{B}\cdot\mathbf{A}$

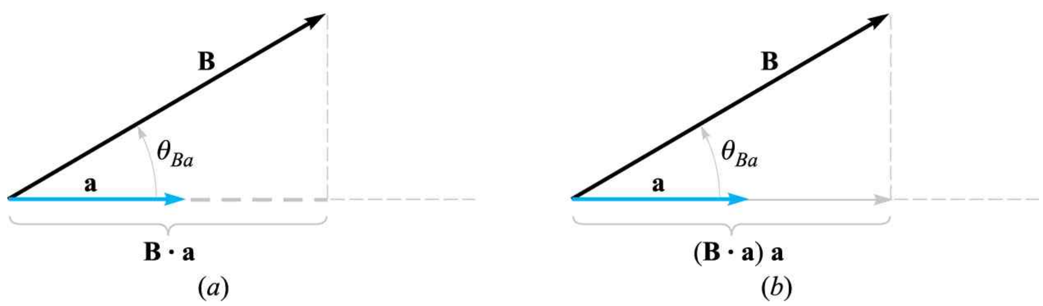
•계산방법; $\mathbf{A}\cdot\mathbf{B}=(A_x\mathbf{a}_x+A_y\mathbf{a}_y+A_z\mathbf{a}_z)\cdot(B_x\mathbf{a}_x+B_y\mathbf{a}_y+B_z\mathbf{a}_z)$

$$\begin{aligned}&=A_x\mathbf{a}_x\cdot(B_x\mathbf{a}_x+B_y\mathbf{a}_y+B_z\mathbf{a}_z)\\&\quad+A_y\mathbf{a}_y\cdot(B_x\mathbf{a}_x+B_y\mathbf{a}_y+B_z\mathbf{a}_z)\\&\quad+A_z\mathbf{a}_z\cdot(B_x\mathbf{a}_x+B_y\mathbf{a}_y+B_z\mathbf{a}_z)\\&=A_xB_x+A_yB_y+A_zB_z\end{aligned}$$

$$\because\ \mathbf{a}_x\cdot\mathbf{a}_x=|\mathbf{a}_x||\mathbf{a}_x|\cos 0^\circ=1\times1\times1=1$$

$$\mathbf{a}_x\cdot\mathbf{a}_y=|\mathbf{a}_x||\mathbf{a}_y|\cos 90^\circ=1\times1\times0=0$$

(ex 3) $\mathbf{A}\cdot\mathbf{A}=|\mathbf{A}||\mathbf{A}|\cos 0^\circ=|\mathbf{A}|^2$ or A^2



•투영(projection); 그림 1.4

(a) $\mathbf{B}$의 $\mathbf{a}$방향의 성분; $\mathbf{B}\cdot\mathbf{a}=|\mathbf{B}||\mathbf{a}|\cos\theta_{Ba}=|\mathbf{B}|\cos\theta_{Ba}$

(b) $\mathbf{B}$의 $\mathbf{a}$방향의 성분벡터; $(\mathbf{B}\cdot\mathbf{a})\mathbf{a}$

(예제 1.2) $\mathbf{G} = y\mathbf{a}_x - 2.5x\mathbf{a}_y + 3\mathbf{a}_z$ at $Q(4,5,2)$, $\mathbf{a}_N = \dfrac{1}{3}(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$

(a) Q에서 $\mathbf{G}$

(b) Q에서 $\mathbf{a}_N$ 방향으로 $\mathbf{G}$의 스칼라 성분

(c) Q에서 $\mathbf{a}_N$ 방향으로 $\mathbf{G}$의 벡터 성분

(d) Q에서 $\mathbf{G}$와 $\mathbf{a}_N$ 사이의 각

(a) $\mathbf{G}_{Q(4,5,2)} = (y\mathbf{a}_x - 2.5x\mathbf{a}_y + 3\mathbf{a}_z)_{Q(4,5,2)}$

$$= 5\mathbf{a}_x - 10\mathbf{a}_y + 3\mathbf{a}_z$$

(b) Q에서 $\mathbf{G}$의 $\mathbf{a}_N$ 방향으로의 성분;

$$\begin{aligned}\mathbf{G}\cdot\mathbf{a}_N &= (5\mathbf{a}_x - 10\mathbf{a}_y + 3\mathbf{a}_z)\cdot(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)/3 \\ &= (10 - 10 - 6)/3 \\ &= -2\end{aligned}$$

(c) Q에서 $\mathbf{G}$의 $\mathbf{a}_N$ 방향으로의 성분벡터;

$$(\mathbf{G}\cdot\mathbf{a}_N)\mathbf{a}_N = -2\mathbf{a}_N = -\frac{2}{3}(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$$

(d) 정의로부터 $\mathbf{G}\cdot\mathbf{a}_N = |\mathbf{G}||\mathbf{a}_N|\cos\theta_{Ga}$

$$\begin{aligned}\cos\theta_{Ga} &= \frac{\mathbf{G}\cdot\mathbf{a}_N}{|\mathbf{G}||\mathbf{a}_N|} \\ &= \frac{-2}{\sqrt{5^2 + (-10)^2 + 3^2}} = \frac{-2}{\sqrt{134}}\end{aligned}$$

$$\therefore\ \theta_{Ga} = \cos^{-1}\frac{-2}{\sqrt{134}} = 99.9^\circ$$

(ex 4) $A(2,5,-1)$, $B(3,-2,4)$, $C(-2,3,1)$

(a) $\mathbf{R}_{AB}\cdot\mathbf{R}_{AC}=(\mathbf{r}_B-\mathbf{r}_A)\cdot(\mathbf{r}_C-\mathbf{r}_A)$

$$=(\mathbf{a}_x-7\mathbf{a}_y+5\mathbf{a}_z)\cdot(-4\mathbf{a}_x-2\mathbf{a}_y+2\mathbf{a}_z)$$

$$=\ -4+14+10$$

$$=\ 20$$

(b) $\mathbf{R}_{AB}$와 $\mathbf{R}_{AC}$ 사이의 각(θ_{BAC})

$$\mathbf{R}_{AB}\cdot\mathbf{R}_{AC}=|\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_{BAC}$$

$$\cos\theta_{BAC}=\frac{\mathbf{R}_{AB}\cdot\mathbf{R}_{AC}}{|\mathbf{R}_{AB}||\mathbf{R}_{AC}|}$$

$$=\frac{20}{\sqrt{1+49+25}\sqrt{16+4+4}}$$

$$=\frac{20}{\sqrt{75}\sqrt{24}}=\frac{2}{\sqrt{18}}=\frac{2}{3\sqrt{2}}$$

$$\therefore\ \theta_{BAC}=\cos^{-1}\frac{2}{3\sqrt{2}}=61.9°$$

(c) $\mathbf{R}_{AB}$의 $\mathbf{R}_{AC}$에 대한 투영의 크기

$$\mathbf{R}_{AB}\cdot\mathbf{a}_{AC}=|\mathbf{R}_{AB}|\cos\theta_{BAC}=5\sqrt{3}\,cos61.9°=4.08$$

(d) $\mathbf{R}_{AB}$의 $\mathbf{R}_{AC}$에 대한 벡터투영

$$(\mathbf{R}_{AB}\cdot\mathbf{a}_{AC})\mathbf{a}_{AC}=4.08\times\frac{-4\mathbf{a}_x-2\mathbf{a}_y+2\mathbf{a}_z}{\sqrt{24}}$$

$$=-3.33\mathbf{a}_x-1.667\mathbf{a}_y+1.667\mathbf{a}_z$$

(응용예제 1.3) $A(6,-1,2)$, $B(-2,3,-4)$, $C(-3,1,5)$

(a) $\mathbf{R}_{AB}$

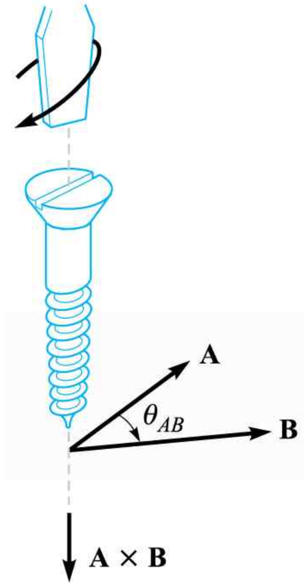
(b) $\mathbf{R}_{AC}$

(c) θ_{BAC}

(d) $\mathbf{R}_{AB}$를 $\mathbf{R}_{AC}$에 (벡터)투영

1.7 외적(벡터곱, vector product or cross product)

•정의; $A \times B = (|A||\mathrm{B}|\sin\theta_{AB})a_N$ → 그림 1.5



•교환법칙; $A \times B = -(B \times A) \ \neq \ B \times A$

•계산방법; $A \times B = (A_x a_x + A_y a_y + A_z a_z) \times (B_x a_x + B_y a_y + B_z a_z)$

$$
\begin{aligned}
&= A_x B_x (a_x \times a_x) + A_x B_y (a_x \times a_y) + A_x B_z (a_x \times a_z) \\
&\quad + A_y B_x (a_y \times a_x) + A_y B_y (a_y \times a_y) + A_y B_z (a_y \times a_z) \\
&\quad + A_z B_x (a_z \times a_x) + A_z B_y (a_z \times a_y) + A_z B_z (a_z \times a_z) \\
&= 0 + A_x B_y a_z + A_x B_z (-a_y) \\
&\quad + A_y B_x (-a_z) + 0 + A_y B_z a_x \\
&\quad + A_z B_x a_y + A_z B_y (-a_x) + 0 \\
&= (A_y B_z - A_z B_y) a_x + (A_z B_x - A_x B_z) a_y + (A_x B_y - A_y B_x) a_z \\
&= \begin{vmatrix} a_x & a_y & a_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}
\end{aligned}
$$

$$\therefore \ |a_x \times a_x| = |a_x||a_x|\sin 0° = 1 \times 1 \times 0 = 0$$

•평행사변형의 면적; $|A \times B| = |A||B|\sin\theta_{AB}$

(ex 5) $A = 2a_x - 3a_y + a_z$, $B = -4a_x - 2a_y + 5a_z$

$$
\begin{aligned}
A \times B &= \begin{vmatrix} a_x & a_y & a_z \\ 2 & -3 & 1 \\ -4 & -2 & 5 \end{vmatrix} \\
&= [(-3)(5) - (-2)(1)]a_x \\
&\quad + [(1)(-4) - (2)(5)]a_y + [(2)(-2) - (-3)(-4)]a_z \\
&= -13a_x - 14a_y - 16a_z
\end{aligned}
$$

(ex 6) $A(2, -5, 1)$, $B(-3, 2, 4)$, $C(0, 3, 1)$

(a) $R_{BC} \times R_{BA} = (r_C - r_B) \times (r_A - r_B)$

$$
\begin{aligned}
&= (3a_x + a_y - 3a_z) \times (5a_x - 7a_y - 3a_z) \\
&= \begin{vmatrix} a_x & a_y & a_z \\ & 3 & 1 \\ -3 & & 5 \\ -7 & -3 & \\ & & \end{vmatrix} \\
&= -24a_x - 6a_y - 26a_z
\end{aligned}
$$

(b) $\text{Area}(\triangle ABC) = \frac{1}{2}\left|\mathbf{R}_{BC}\times\mathbf{R}_{BA}\right|$

$$= \frac{1}{2}\sqrt{(-24)^2+(-6)^2+(-26)^2}$$

$$= 17.94$$

(c) $\triangle ABC$에 수직인 단위벡터

$$\mathbf{a}_N = \pm\frac{\mathbf{R}_{BC}\times\mathbf{R}_{BA}}{\left|\mathbf{R}_{BC}\times\mathbf{R}_{BA}\right|}$$

$$= \pm\frac{-24\mathbf{a}_x - 6\mathbf{a}_y - 26\mathbf{a}_z}{\sqrt{(-24)^2+(-6)^2+(-26)^2}}$$

$$= \pm(0.669\mathbf{a}_x + 0.1672\mathbf{a}_y + 0.724\mathbf{a}_z)$$

(응용예제 1.4) $A(6, -1, 2)$, $B(-2, 3, -4)$, $C(-3, 1, 5)$

(a) $\mathbf{R}_{AB}\times\mathbf{R}_{AC}$

(b) $\triangle ABC$의 면적

(c) $\triangle ABC$에 수직인 단위벡터

1.8 원통좌표계(cylindrical coordinate system)

- 점의 표시; $P(\rho_1,\ \phi_1,\ z_1)$

    그림 1.6 (b)

- $P(\rho_1,\ \phi_1,\ z_1)$의 의미; $\rho=\rho_1,\ \phi=\phi_1,\ z=z_1$인 평면들의 교차점

    그림 1.6 (a)

- 기본 단위벡터; $\boldsymbol{a}_\rho,\ \boldsymbol{a}_\phi,\ \boldsymbol{a}_z$ ← 위치에 대한 함수

    그림 1.6 (b)

- z축만 존재하는 오른손 좌표계; $\boldsymbol{a}_\rho \times \boldsymbol{a}_\phi = \boldsymbol{a}_z$
- 미소체적소의 정의

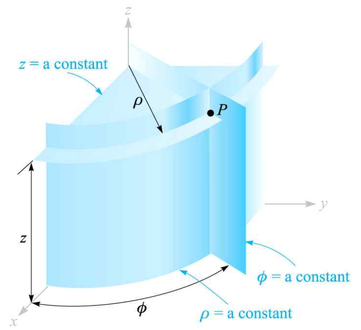
    점 $P(\rho,\ \phi,\ z)$의 각 좌표값이

    각각 미소량만큼 증가한 점이 $P'(\rho+d\rho,\ \phi+d\phi,\ z+dz)$일 때

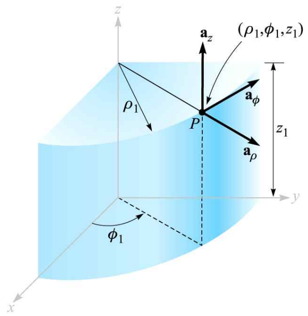
    미소체적소; $dv = d\rho \cdot \rho d\phi \cdot dz = \rho d\rho d\phi dz$

    면적소(dS); $\rho d\rho d\phi,\ \rho d\phi dz,\ dz\,d\rho$

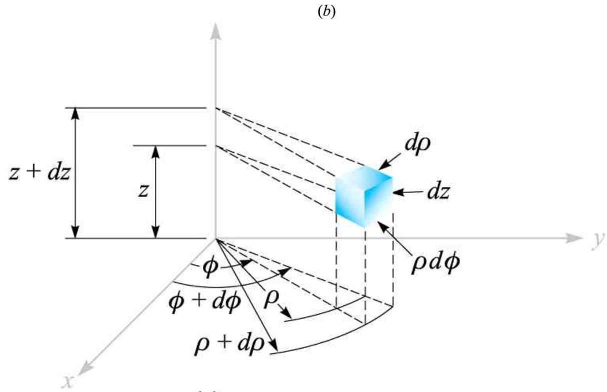
    그림 1.6 (c)



(a)



(b)



(c)

- 원통좌표계 → 직교좌표계의 변환(그림 1.7)

    $x=\rho\cos\phi$

    $y=\rho\sin\phi$

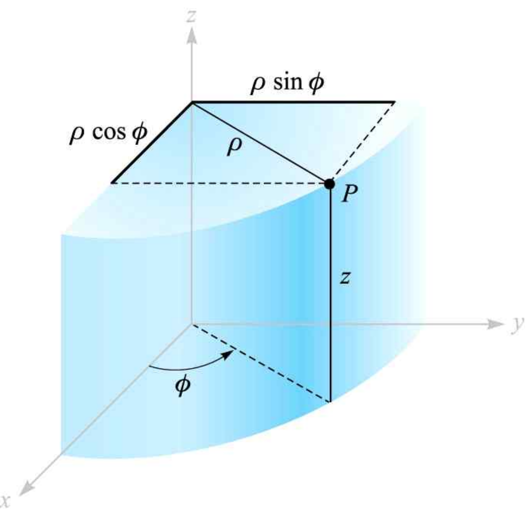
    $z=z$

- 직교좌표계 → 원통좌표계의 변환(그림 1.7)

    $\rho=\sqrt{x^2+y^2}\quad(\rho \geqq 0)$

    $\phi=\tan^{-1}\dfrac{y}{x}$

    $z=z$



- 직교, 원통좌표계 기본 단위벡터의 스칼라곱

	$\boldsymbol{a}_\rho$	$\boldsymbol{a}_\phi$	$\boldsymbol{a}_z$
$\boldsymbol{a}_x\cdot$	$\cos\phi$	$-\sin\phi$	0
$\boldsymbol{a}_y\cdot$	$\sin\phi$	$\cos\phi$	0
$\boldsymbol{a}_z\cdot$	0	0	1

(ex 7) 벡터 $\boldsymbol{A} = A_x\boldsymbol{a_x} + A_y\boldsymbol{a_y} + A_z\boldsymbol{a_z}$를 원통좌표계로 변환하여라.

$$\boldsymbol{A} = A_\rho\boldsymbol{a_\rho} + A_\phi\boldsymbol{a_\phi} + A_z\boldsymbol{a_z}$$

$$\begin{aligned}
A_\rho = \boldsymbol{A}\cdot\boldsymbol{a_\rho} &= (A_x\boldsymbol{a_x} + A_y\boldsymbol{a_y} + A_z\boldsymbol{a_z})\cdot\boldsymbol{a_\rho} \\
&= A_x\boldsymbol{a_x}\cdot\boldsymbol{a_\rho} + A_y\boldsymbol{a_y}\cdot\boldsymbol{a_\rho} + A_z\boldsymbol{a_z}\cdot\boldsymbol{a_\rho} \\
&= A_x\cos\phi + A_y\sin\phi
\end{aligned}$$

$$\begin{aligned}
A_\phi = \boldsymbol{A}\cdot\boldsymbol{a_\phi} &= (A_x\boldsymbol{a_x} + A_y\boldsymbol{a_y} + A_z\boldsymbol{a_z})\cdot\boldsymbol{a_\phi} \\
&= A_x\boldsymbol{a_x}\cdot\boldsymbol{a_\phi} + A_y\boldsymbol{a_y}\cdot\boldsymbol{a_\phi} + A_z\boldsymbol{a_z}\cdot\boldsymbol{a_\phi} \\
&= -A_x\sin\phi + A_y\cos\phi
\end{aligned}$$

$$\begin{aligned}
A_z = \boldsymbol{A}\cdot\boldsymbol{a_z} &= (A_x\boldsymbol{a_x} + A_y\boldsymbol{a_y} + A_z\boldsymbol{a_z})\cdot\boldsymbol{a_z} \\
&= A_x\boldsymbol{a_x}\cdot\boldsymbol{a_z} + A_y\boldsymbol{a_y}\cdot\boldsymbol{a_z} + A_z\boldsymbol{a_z}\cdot\boldsymbol{a_z} \\
&= A_z
\end{aligned}$$

$$\therefore\ \boldsymbol{A} = (A_x\cos\phi + A_y\sin\phi)\boldsymbol{a_\rho} + (-A_x\sin\phi + A_y\cos\phi)\boldsymbol{a_\phi} + A_z\boldsymbol{a_z}$$

(ex 8) $A(x=2,\ y=3,\ z=-1)$, $B(\rho=4,\ \phi=-50°,\ z=2)$

(a) 벡터$\boldsymbol{r_A}$의 크기

(b) 벡터$\boldsymbol{r_B}$의 크기

(c) 벡터$\boldsymbol{R_{AB}}$의 크기

(a) $|\boldsymbol{r_A}| = \sqrt{2^2+3^2+1^2} = \sqrt{14} = 3.74$

(b) $B(x=\rho\cos\phi,\ y=\rho\sin\phi,\ z=z) = B(4\cos(-50°),\ 4\sin(-50°),\ 2)$

$= B(2.57,\ -3.064, 2)$

$\therefore\ |\boldsymbol{r_B}| = \sqrt{2.57^2+(-3.064)^2+2^2} = 4.47$

(c) $|\boldsymbol{R_{AB}}| = |\boldsymbol{r_B} - \boldsymbol{r_A}| = |0.57\boldsymbol{a_x} - 6.064\boldsymbol{a_y} + 3\boldsymbol{a_z}|$

$= \sqrt{0.57^2+(-6.064)^2+3^2}$

$= 6.79$

(ex 9) 직각좌표계 → 원통좌표계

(a) $\boldsymbol{A} = 5\boldsymbol{a}_x$ at $P(\rho = 4,\ \phi = 120°,\ z = 2)$

$$A_\rho = \boldsymbol{A} \cdot \boldsymbol{a}_\rho = 5\boldsymbol{a}_x \cdot \boldsymbol{a}_\rho = 5\cos\phi = 5\cos 120°$$
$$= 5\cos(90° + 30°)$$
$$= 5 \times (-\sin 30°)$$
$$= -2.5$$

$$A_\phi = \boldsymbol{A} \cdot \boldsymbol{a}_\phi = 5\boldsymbol{a}_x \cdot \boldsymbol{a}_\phi = -5\sin\phi = -5\sin 120°$$
$$= -5\sin(90° + 30°)$$
$$= -5\cos 30°$$
$$= -\frac{5\sqrt{3}}{2}$$
$$= -4.33$$

$$\therefore\ \boldsymbol{A} = A_\rho \boldsymbol{a}_\rho + A_\phi \boldsymbol{a}_\phi + A_z \boldsymbol{a}_z$$
$$= -2.5\boldsymbol{a}_\rho - 4.33\boldsymbol{a}_\phi$$

(b) $\boldsymbol{B} = 5\boldsymbol{a}_x$ at $Q(x = 3,\ y = 4,\ z = -1)$

$$Q(x = 3,\ y = 4,\ z = -1) = Q(\rho = \sqrt{x^2 + y^2},\ \phi = \tan^{-1}\frac{y}{x},\ z)$$
$$= Q(\sqrt{3^2 + 4^2},\ \tan^{-1}\frac{4}{3},\ -1)$$
$$= Q(5,\ 53.1°,\ -1)$$

$$B_\rho = \boldsymbol{B} \cdot \boldsymbol{a}_\rho = 5\boldsymbol{a}_x \cdot \boldsymbol{a}_\rho = 5\cos\phi = 5\cos 53.1°$$
$$= 5 \times 0.6$$
$$= 3$$

$$B_\phi = \boldsymbol{B} \cdot \boldsymbol{a}_\phi = 5\boldsymbol{a}_x \cdot \boldsymbol{a}_\phi = -5\sin\phi = -5\sin 53.1°$$
$$= -5 \times 0.8$$
$$= -4$$

$$\therefore\ \boldsymbol{B} = B_\rho \boldsymbol{a}_\rho + B_\phi \boldsymbol{a}_\phi + B_z \boldsymbol{a}_z$$
$$= 3\boldsymbol{a}_\rho - 4\boldsymbol{a}_\phi$$

(c) $\boldsymbol{C} = 4\boldsymbol{a}_x - 2\boldsymbol{a}_y - 4\boldsymbol{a}_z$ at $A(x=2,\ y=3,\ z=5)$

$$A(x=2,\ y=3,\ z=5) = A(\rho = \sqrt{x^2+y^2},\ \phi = \tan^{-1}\frac{y}{x},\ z)$$

$$= A(\sqrt{2^2+3^2},\ \tan^{-1}\frac{3}{2},\ 5)$$

$$= A(\sqrt{13},\ 56.31°,\ 5)$$

$$C_\rho = \boldsymbol{C}\cdot\boldsymbol{a}_\rho = (4\boldsymbol{a}_x - 2\boldsymbol{a}_y - 4\boldsymbol{a}_z)\cdot\boldsymbol{a}_\rho$$

$$= 4\boldsymbol{a}_x\cdot\boldsymbol{a}_\rho - 2\boldsymbol{a}_y\cdot\boldsymbol{a}_\rho - 4\boldsymbol{a}_z\cdot\boldsymbol{a}_\rho$$

$$= 4\cos 56.31° - 2\sin 56.31°$$

$$= 0.555$$

$$C_\phi = \boldsymbol{C}\cdot\boldsymbol{a}_\phi = (4\boldsymbol{a}_x - 2\boldsymbol{a}_y - 4\boldsymbol{a}_z)\cdot\boldsymbol{a}_\phi$$

$$= 4\boldsymbol{a}_x\cdot\boldsymbol{a}_\phi - 2\boldsymbol{a}_y\cdot\boldsymbol{a}_\phi - 4\boldsymbol{a}_z\cdot\boldsymbol{a}_\phi$$

$$= -4\sin 56.31° - 2\cos 56.31°$$

$$= -4.44$$

$$\therefore\ \boldsymbol{C} = C_\rho\boldsymbol{a}_\rho + C_\phi\boldsymbol{a}_\phi + C_z\boldsymbol{a}_z \qquad = 0.555\boldsymbol{a}_\rho - 4.44\boldsymbol{a}_\phi - 4\boldsymbol{a}_z$$

(예제 1.3) $\boldsymbol{B} = y\boldsymbol{a}_x - x\boldsymbol{a}_y + z\boldsymbol{a}_z$ → 원통좌표계

$$B_\rho = \boldsymbol{B}\cdot\boldsymbol{a}_\rho = (B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z)\cdot\boldsymbol{a}_\rho$$

$$= y\boldsymbol{a}_x\cdot\boldsymbol{a}_\rho - x\boldsymbol{a}_y\cdot\boldsymbol{a}_\rho + z\boldsymbol{a}_z\cdot\boldsymbol{a}_\rho$$

$$= y\cos\phi - x\sin\phi$$

$$= \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi$$

$$= 0$$

$$B_\phi = \boldsymbol{B}\cdot\boldsymbol{a}_\phi = (B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z)\cdot\boldsymbol{a}_\phi$$

$$= y\boldsymbol{a}_x\cdot\boldsymbol{a}_\phi - x\boldsymbol{a}_y\cdot\boldsymbol{a}_\phi + z\boldsymbol{a}_z\cdot\boldsymbol{a}_\phi$$

$$= -y\sin\phi - x\cos\phi$$

$$= -\rho\sin^2\phi - \rho\cos^2\phi$$

$$= -\rho$$

$$\therefore\ \boldsymbol{B} = -\rho\boldsymbol{a}_\phi + z\boldsymbol{a}_z$$

(응용예제 1.5)

(a) 점 $C(\rho = 4.4,\ \phi = -115°,\ z = 2)$ → 직각좌표계

$C(x = -1.860,\ y = -3.99,\ z = 2)$

(b) $D(x = -3.1,\ y = 2.6,\ z = -3)$를 원통좌표계로 변환

$D(\rho = 4.05,\ \phi = 140.0°,\ z = -3)$

(c) 점 C에서 점 D에 이르는 거리

8.36

(응용예제 1.6)

(a) 점 $P(10,\ -8,\ 6)$에서 $\boldsymbol{F} = 10\boldsymbol{a}_x - 8\boldsymbol{a}_y + 6\boldsymbol{a}_z$ → 원통좌표계

$\boldsymbol{F} = 12.81\boldsymbol{a}_\rho + 6\boldsymbol{a}_z$

(b) 점 $Q(\rho,\ \phi,\ z)$에서 $\boldsymbol{G} = (2x + y)10\boldsymbol{a}_x - (y - 4x)\boldsymbol{a}_y$ → 원통좌표계

(c) $P(5,\ 2,\ -1)$에서 $\boldsymbol{G} = 20\boldsymbol{a}_\rho - 10\boldsymbol{a}_\phi + 3\boldsymbol{a}_z$ → 직각좌표계

1.9 구좌표계(spherical coordinate system)

• 점의 표시; $P(r_1,\ \theta_1,\ \phi_1)$ (a)

• $P(r_1,\ \theta_1,\ \phi_1)$의 의미; $r=r_1,\ \theta=\theta_1,\ \phi=\phi_1$인 평면들의 교차점

• 기본단위벡터; $\boldsymbol{a}_r,\ \boldsymbol{a}_\theta,\ \boldsymbol{a}_\phi$ ← 위치에 대한 함수

• 오른손 좌표계; $\boldsymbol{a}_r \times \boldsymbol{a}_\theta = \boldsymbol{a}_\phi$

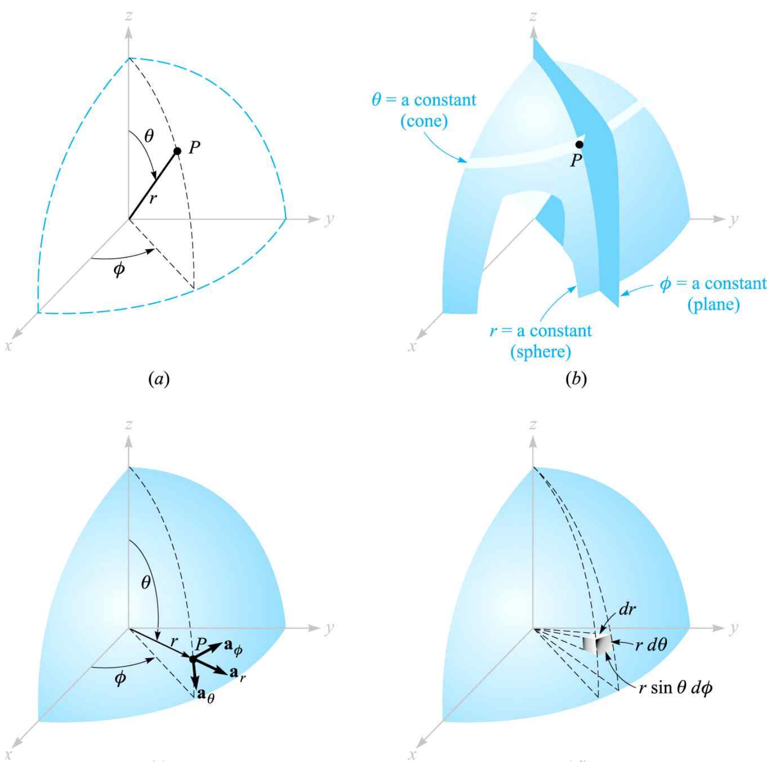
• 미소체적소의 정의;

점 $P(r,\ \theta,\ \phi)$의 각 좌표값이 각각 미소량만큼

증가한 점이 $P'(r+dr,\ \theta+d\theta,\ \phi+d\phi)$일 때

미소체적소; $dv = dr \cdot r\sin\theta d\phi \cdot rd\theta = r^2\sin\theta drd\theta d\phi$

면적소(dS); $r\sin\theta drd\theta,\ r^2\sin\theta d\theta d\phi,\ r\sin\theta drd\theta$



•직교좌표계 → 구좌표계의 변환

$$r = \sqrt{x^2+y^2+z^2} \quad (r \geqq 0)$$

$$\theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\frac{r}{\sqrt{x^2+y^2+z^2}} \quad (0^o \leqq \theta \leqq 180^o)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

•구좌표계 → 직교좌표계의 변환

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

•직교, 구좌표계 기본단위벡터의 스칼라곱

	$\boldsymbol{a}_r$	$\boldsymbol{a}_\theta$	$\boldsymbol{a}_\phi$
$\boldsymbol{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\boldsymbol{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\boldsymbol{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

(ex 10) $A(x=2,\ y=3,\ z=-1)$, $B(r=4,\ \theta=25°,\ \phi=120°)$

(a) 점 A → 구좌표계

(b) 점 B → 직각좌표계

(c) 점 A와 점 B 사이의 거리

(a) $$r = \sqrt{x^2+y^2+z^2} = \sqrt{2^2+3^2+(-1)^2} = \sqrt{14} = 3.74$$

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos^{-1}\frac{-1}{\sqrt{14}} = 105.5°$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{3}{2} = 56.3°$$

$$\therefore\ A(r=3.74,\ \theta=105.5°,\ \phi=56.3°)$$

(b) $$x = r\sin\theta\cos\phi = 4\sin25°\cos120° = -0.845$$

$$y = r\sin\theta\sin\phi = 4\sin25°\sin120° = 1.464$$

$$z = r\cos\theta = 4\cos25° = 3.63$$

$$\therefore\ B(x=-0.845,\ y=1.464,\ z=3.63)$$

(c) $$|\boldsymbol{R}_{AB}| = |\boldsymbol{r}_B - \boldsymbol{r}_A| = |-2.845\boldsymbol{a}_x - 1.536\boldsymbol{a}_y + 4.63\boldsymbol{a}_z|$$

$$= \sqrt{(-2.845^2+(-1.536)^2+4.63^2} = 5.64$$

(ex 11) 직각좌표계 → 구좌표계

(a) $\boldsymbol{A} = 5\boldsymbol{a}_x$ at $B(r=4,\ \theta=25^o,\ \phi=120^o)$

$$A_r = \boldsymbol{A}\cdot\boldsymbol{a}_r = 5\boldsymbol{a}_x\cdot\boldsymbol{a}_r = 5\sin\theta\cos\phi$$
$$= 5\sin25°\cos120°$$
$$= -1.057$$

$$A_\theta = \boldsymbol{A}\cdot\boldsymbol{a}_\theta = 5\boldsymbol{a}_x\cdot\boldsymbol{a}_\theta = 5\cos\theta\cos\phi$$
$$= 5\cos25°\cos120°$$
$$= -2.27$$

$$A_\phi = \boldsymbol{A}\cdot\boldsymbol{a}_\phi = 5\boldsymbol{a}_x\cdot\boldsymbol{a}_\phi = 5(-\sin\phi)$$
$$= -5\sin120°$$
$$= -4.33$$

$$\therefore\ \boldsymbol{A} = A_r\boldsymbol{a}_r + A_\theta\boldsymbol{a}_\theta + A_\phi\boldsymbol{a}_\phi$$
$$= -1.057\boldsymbol{a}_r - 2.27\boldsymbol{a}_\theta - 4.33\boldsymbol{a}_\phi$$

(b) $\boldsymbol{B} = 5\boldsymbol{a}_x$ at $A(x=2,\ y=3,\ z=-1)$

$$A(x=2,\ y=3,\ z=-1)$$
$$\rightarrow A\left(r=\sqrt{x^2+y^2+z^2},\ \theta=\cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}},\ \phi=\tan^{-1}\frac{y}{x}\right)$$
$$\rightarrow A\left(r=\sqrt{2^2+3^2+(-1)^2},\ \theta=\cos^{-1}\frac{-1}{\sqrt{2^2+3^2+(-1)^2}},\ \phi=\tan^{-1}\frac{3}{2}\right)$$
$$\rightarrow A\left(r=\sqrt{14},\ \theta=\cos^{-1}\frac{-1}{\sqrt{14}},\ \phi=\tan^{-1}\frac{3}{2}\right)$$

$$B_r = \boldsymbol{B}\cdot\boldsymbol{a}_r = 5\boldsymbol{a}_x\cdot\boldsymbol{a}_r = 5\sin\theta\cos\phi$$
$$= 5\sin\left(\cos^{-1}\frac{-1}{\sqrt{14}}\right)\cos\left(\tan^{-1}\frac{3}{2}\right) = 2.67$$

$$B_\theta = \boldsymbol{B}\cdot\boldsymbol{a}_\theta = 5\boldsymbol{a}_x\cdot\boldsymbol{a}_\theta = 5\cos\theta\cos\phi$$
$$= 5\cos\left(\cos^{-1}\frac{-1}{\sqrt{14}}\right)\cos\left(\tan^{-1}\frac{3}{2}\right) = -0.741$$

$$B_\phi = \boldsymbol{B}\cdot\boldsymbol{a}_\phi = 5\boldsymbol{a}_x\cdot\boldsymbol{a}_\phi = 5(-\sin\phi)$$
$$= -5\sin\left(\tan^{-1}\frac{3}{2}\right) = -4.16$$

$$\therefore\ \boldsymbol{B} = B_r\boldsymbol{a}_r + B_\theta\boldsymbol{a}_\theta + B_\phi\boldsymbol{a}_\phi = 2.67\boldsymbol{a}_r - 0.741\boldsymbol{a}_\theta - 4.16\boldsymbol{a}_\phi$$

(c) $\boldsymbol{C}=4\boldsymbol{a}_x-2\boldsymbol{a}_y-4\boldsymbol{a}_z$ at $P(x=-2,\ y=-3,\ z=4)$

$P(x=-2,\ y=-3,\ z=4)\ \rightarrow\ P(r=\qquad,\ \theta=\qquad,\ \phi=\qquad)$

$$
\begin{aligned}
C_r &= \boldsymbol{C}\cdot\boldsymbol{a}_r = (4\boldsymbol{a}_x-2\boldsymbol{a}_y+4\boldsymbol{a}_z)\cdot\boldsymbol{a}_r \\
&= 5\boldsymbol{a}_x\cdot\boldsymbol{a}_r-2\boldsymbol{a}_y\cdot\boldsymbol{a}_r+4\boldsymbol{a}_z\cdot\boldsymbol{a}_r \\
&= 5\sin\theta\cos\phi-2\sin\theta\sin\phi+4\cos\theta \\
&= -3.34
\end{aligned}
$$

$$C_\theta = \boldsymbol{C}\cdot\boldsymbol{a}_\theta = (4\boldsymbol{a}_x-2\boldsymbol{a}_y+4\boldsymbol{a}_z)\cdot\boldsymbol{a}_\theta =$$

$$C_\phi = \boldsymbol{C}\cdot\boldsymbol{a}_\phi = (4\boldsymbol{a}_x-2\boldsymbol{a}_y+4\boldsymbol{a}_z)\cdot\boldsymbol{a}_\phi =$$

$$
\begin{aligned}
\therefore\ \boldsymbol{C} &= C_r\boldsymbol{a}_r+C_\theta\boldsymbol{a}_\theta+C_\phi\boldsymbol{a}_\phi \\
&= -3.34\boldsymbol{a}_r+2.27\boldsymbol{a}_\theta+4.44\boldsymbol{a}_\phi
\end{aligned}
$$

(예제 1.4) G$=\dfrac{xz}{y}\boldsymbol{a}_x$ → 구좌표계

$$
\begin{aligned}
G_r &= \boldsymbol{G}\cdot\boldsymbol{a}_r = \frac{xz}{y}\boldsymbol{a}_x\cdot\boldsymbol{a}_r = \frac{xz}{y}sin\theta\cos\phi = \frac{r\sin\theta\cos\phi\cdot r\cos\theta}{r\sin\theta\sin\phi}sin\theta\cos\phi \\
&= r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}
\end{aligned}
$$

$$
\begin{aligned}
G_\theta &= \boldsymbol{G}\cdot\boldsymbol{a}_\theta = \frac{xz}{y}\boldsymbol{a}_x\cdot\boldsymbol{a}_\theta = \frac{xz}{y}cos\theta\cos\phi = \frac{r\sin\theta\cos\phi\cdot r\cos\theta}{r\sin\theta\sin\phi}cos\theta\cos\phi \\
&= r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}
\end{aligned}
$$

$$
\begin{aligned}
G_\phi &= \boldsymbol{G}\cdot\boldsymbol{a}_\phi = \frac{xz}{y}\boldsymbol{a}_x\cdot\boldsymbol{a}_\phi = \frac{xz}{y}(-\sin\phi) = \frac{r\sin\theta\cos\phi\cdot r\cos\theta}{r\sin\theta\sin\phi}(-\sin\phi) \\
&= -r\cos\theta\cos\phi
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ \boldsymbol{G} &= r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}\boldsymbol{a}_r+r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}\boldsymbol{a}_\theta-r\cos\theta\cos\phi\boldsymbol{a}_\phi \\
&= r\cos\theta\cos\phi\left(\sin\theta\frac{\cos\phi}{\sin\phi}\boldsymbol{a}_r+\cos\theta\frac{\cos\phi}{\sin\phi}\boldsymbol{a}_\theta-\boldsymbol{a}_\phi\right) \\
&= r\cos\theta\cos\phi(\sin\theta\cot\phi\boldsymbol{a}_r+\cos\theta\cot\phi\boldsymbol{a}_\theta-\boldsymbol{a}_\phi)
\end{aligned}
$$

(응용예제 1.7) $C(-3,\ 2,\ 1)$, $D(r=5,\ \theta=20°,\ \phi=-70°)$

(a) C의 구좌표

(b) D의 직각좌표

(c) C와 D 사이의 거리

(응용예제 1.8) 직각좌표계 → 구좌표계

(a) $10\boldsymbol{a}_x$ at $P(x=-3,\ y=2,\ z=4)$

(b) $10\boldsymbol{a}_y$ at $Q(\rho=5,\ \phi=30°,\ z=4)$

(c) $10\boldsymbol{a}_z$ at $M(r=4,\ \theta=110°,\ \phi=120°)$