4장 전계의 에너지 및 전위

4.1 전계내에서 점전하를 이동시키는데 소요되는 에너지

- 전계의 세기($\boldsymbol{E}$; electric field intensity); 전계 내의 단위시험전하에 작용하는 힘
- 전계 $\boldsymbol{E}$ 내에서 전하 Q를 한 점에서 다른 점까지 이동시킬 때 필요한 에너지;

  전계에 의해서 Q에 작용하는 힘 : $\boldsymbol{F}_E = Q\boldsymbol{E}$

  → 이 힘의 $d\boldsymbol{L}$ 방향의 성분 : $F_{EL} = \boldsymbol{F}_E \cdot \boldsymbol{a}_L = Q\boldsymbol{E}\cdot\boldsymbol{a}_L$

  → 전하를 이동시키기 위하여 전하에 가해야 할 힘 $F_{appl} = -F_{EL} = -Q\boldsymbol{E}\cdot\boldsymbol{a}_L$

  → 전하를 이동시키는데 필요한 에너지(일=힘×거리) $dW = -Q\boldsymbol{E}\cdot d\boldsymbol{L}$

  여기서, $d\boldsymbol{L} = \boldsymbol{a}_L dL$

  $= dx\boldsymbol{a}_x + dy\boldsymbol{a}_y + dz\boldsymbol{a}_z$ (직각좌표계)

  $= d\rho\boldsymbol{a}_r + \rho d\phi\boldsymbol{a}_\phi + dz\boldsymbol{a}_z$ (원통좌표계)

  $= dr\boldsymbol{a}_r + rd\theta\boldsymbol{a}_\theta + r\sin\theta\boldsymbol{a}_\phi$ (구좌표계)

  → $\therefore\ W = \int_{init}^{final} dW = -Q\int_{init}^{final}\boldsymbol{E}\cdot d\boldsymbol{L}$

(ex 1) $\boldsymbol{E} = \left(\frac{x}{2}+2y\right)\boldsymbol{a}_x + 2x\boldsymbol{a}_y$ V/m에 점전하 $Q = -20\mu\text{C}$가 위치

전하 Q를 점 $O(0,\ 0,\ 0)$에서 점 $P(4,\ 0,\ 0)$까지 이동시키는데 필요한 일 W = ?

$$W = -Q\int_{init}^{final}\boldsymbol{E}\cdot d\boldsymbol{L}$$

$$= 20\times10^{-6}\int_O^P\left[\left(\frac{x}{2}+2y\right)\boldsymbol{a}_x + 2x\boldsymbol{a}_y\right]\cdot[dx\boldsymbol{a}_x]$$

$$= 20\times10^{-6}\int_0^4\left(\frac{x}{2}+2y\right)dx\Big|_{y=0}$$

$$= 20\times10^{-6}\cdot\left[\frac{x^2}{4}+0\right]_{x=0}^{x=4}$$

$$= 80\ \mu J$$

(응용예제 4.1) $\boldsymbol{E}=\frac{1}{z^2}\left(8xyz\boldsymbol{a}_x+4x^2z\boldsymbol{a}_y-4x^2y\boldsymbol{a}_z\right)$ V/m 내에서

6nC의 전하를 점 $P(2,\ -2,\ 3)$에서 다음의 $\boldsymbol{a}_L$ 방향으로 2μm 움직일 때 소요되는 일

(a) $\boldsymbol{a}_L=-\frac{6}{7}\boldsymbol{a}_x+\frac{3}{7}\boldsymbol{a}_y+\frac{2}{7}\boldsymbol{a}_z$

-149.3 fJ

(b) $\boldsymbol{a}_L=\frac{6}{7}\boldsymbol{a}_x-\frac{3}{7}\boldsymbol{a}_y-\frac{2}{7}\boldsymbol{a}_z$

149.3 fJ

(c) $\boldsymbol{a}_L=\frac{3}{7}\boldsymbol{a}_x+\frac{6}{7}\boldsymbol{a}_y$

0

4.2 선적분

(예제 4.1) $\boldsymbol{E} = y\boldsymbol{a}_x + x\boldsymbol{a}_y + 2\boldsymbol{a}_z$ V/m에 점전하 $Q = 2$C가 위치, 전하 Q를 점 $B(1,\ 0,\ 1)$에서 점 $A(0.8,\ 0.6,\ 1)$까지 $x^2 + y^2 = 1$, $z = 1$인 원의 원호를 따라 이동시키는데 필요한 일 W = ?

$$W = -Q\int_B^A \boldsymbol{E}\cdot d\boldsymbol{L}$$

$$= -2\int_B^A (y\boldsymbol{a}_x + x\boldsymbol{a}_y + 2\boldsymbol{a}_z)\cdot(dx\boldsymbol{a}_x + dy\boldsymbol{a}_y + dz\boldsymbol{a}_z)$$

$$= -2\int_B^A (ydx + xdy + 2dz)$$

$$= -2\left(\int_1^{0.8} ydx + \int_0^{0.6} xdy + \int_1^1 2dz\right)$$

$$\leftarrow\ y = \sqrt{1-x^2},\ \ x = \sqrt{1-y^2}$$

$$= -2\left(\int_1^{0.8}\sqrt{1-x^2}\,dx + \int_0^{0.6}\sqrt{1-y^2}\,dy + 0\right)$$

$$\leftarrow\ x = \cos\theta \ \rightarrow\ dx = -\sin\theta\, d\theta,\ \ \sqrt{1-x^2} = \sin\theta$$

$$\leftarrow\ y = \cos\phi \ \rightarrow\ dy = -\sin\phi\, d\phi,\ \ \sqrt{1-y^2} = \sin\phi$$

$$= -2\left(\int_{\theta_1}^{\theta_2}(-\sin^2\theta)d\theta + \int_{\phi_1}^{\phi_2}(-\sin^2\phi)d\phi\right)$$

$$\leftarrow\ \int udv = uv - \int vdu \qquad\qquad u = \sin\theta \ \rightarrow\ du = \cos\theta\, d\theta$$

$$dv = -\sin\theta\, d\theta \ \rightarrow\ v = \cos\theta$$

$$\leftarrow\ \int(-\sin^2\theta)\,d\theta = \sin\theta\cos\theta - \int\cos^2\theta\, d\theta$$

$$= \sin\theta\cos\theta - \int(1-\sin^2\theta)d\theta = \sin\theta\cos\theta - \int d\theta + \int\sin^2\theta d\theta$$

$$\leftarrow\ 2\int(-\sin^2\theta)d\theta = \sin\theta\cos\theta - \theta$$

$$\therefore\ \int(-\sin^2\theta)d\theta = \frac{1}{2}(\sin\theta\cos\theta - \theta)$$

$$= -\left[\sin\theta\cos\theta - \theta\right]_{\theta_1}^{\theta_2} - \left[\sin\phi\cos\phi - \phi\right]_{\phi_1}^{\phi_2}$$

$$= -\left[x\sqrt{1-x^2} - \cos^{-1}x\right]_1^{0.8} - \left[x\sqrt{1-x^2} - \cos^{-1}x\right]_0^{0.6}$$

$$= -\left[(0.8\times0.6 - \cos^{-1}0.8) - (-\cos^{-1}1)\right] - \left[(0.6\times0.8 - \cos^{-1}0.6) - (-\cos^{-1}0)\right]$$

$$= -(-0.1635 + 0) - (-0.4473 + 1.571) = -0.96\text{J}$$

(예제 4.2) $\boldsymbol{E} = y\boldsymbol{a}_x + x\boldsymbol{a}_y + 2\boldsymbol{a}_z$ V/m에 점전하 $Q = 2$C가 위치, 전하 Q를 점 $B(1,\ 0,\ 1)$에서 점 $A(0.8,\ 0.6,\ 1)$까지 직선을 따라 이동시키는데 필요한 일 W = ?

점 $A(0.8,\ 0.6)$와 점 $B(1,\ 0)$를 잇는 직선의 방정식

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$y - 0 = \frac{0.6 - 0}{0.8 - 1}(x - 1)$$

$$y = -3(x - 1)$$

$$W = -Q\int_B^A \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$= -2\int_B^A (y\boldsymbol{a}_x + x\boldsymbol{a}_y + 2\boldsymbol{a}_z) \cdot (dx\boldsymbol{a}_x + dy\boldsymbol{a}_y + dz\boldsymbol{a}_z) = -2\int_B^A (ydx + xdy + 2dz)$$

$$= -2\left(\int_1^{0.8} ydx + \int_0^{0.6} xdy + \int_1^1 2dz\right)$$

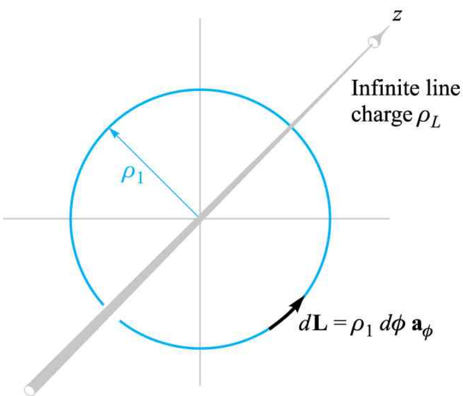
$$\leftarrow\ y = -3x + 3,\ x = -\frac{1}{3}y + 1$$

$$= -2\left(\int_1^{0.8}(-3x+3)dx + \int_0^{0.6}(-\frac{1}{3}y+1)dy + 0\right)$$

$$= -2\left(\left[-\frac{3}{2}x^2 + 3x\right]_1^{0.8} + \left[-\frac{1}{6}y^2 + y\right]_0^{0.6}\right)$$

$$= -2([1.44 - 1.5] + [0.54 - 0])$$

$$= -0.96\ [J]$$



(ex 2) z축을 따라 선전하 ρ_L

전하 Q; $B(\rho_1,\ 0,\ 0) \rightarrow \rho = \rho_1,\ z = 0$인 원 $\rightarrow B(\rho_1,\ 2\pi,\ 0)$, W = ?

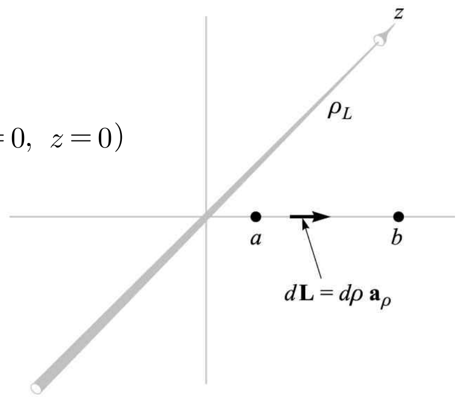
선전하 ρ_L에 의한 전계; $\boldsymbol{E} = E_\rho \boldsymbol{a}_\rho = \dfrac{\rho_L}{2\pi\epsilon_0\rho}\boldsymbol{a}_\rho$

$$W = -Q\int_{init}^{final} \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$= -Q\int_B^B \frac{\rho_L}{2\pi\epsilon_0\rho}\boldsymbol{a}_\rho \cdot (d\rho\boldsymbol{a}_\rho + \rho d\phi\boldsymbol{a}_\phi + dz\boldsymbol{a}_z)$$

$$= -Q\int_0^{2\pi} \frac{\rho_L}{2\pi\epsilon_0} d\phi \boldsymbol{a}_\rho \cdot \boldsymbol{a}_\phi = 0$$

(ex 3) z축을 따라 선전하 ρ_L



전하 Q; $B(\rho=a,\ \phi=0,\ z=0) \rightarrow A(\rho=b,\ \phi=0,\ z=0)$

$W = ?$

$$W = -Q\int_{init}^{final} \boldsymbol{E}\cdot d\boldsymbol{L}$$

$$= -Q\int_{B}^{A} \frac{\rho_L}{2\pi\epsilon_0\rho}\boldsymbol{a}_\rho \cdot (d\rho\boldsymbol{a}_\rho + \rho d\phi\boldsymbol{a}_\phi + dz\boldsymbol{a}_z)$$

$$= -Q\int_{a}^{b} \frac{\rho_L}{2\pi\epsilon_0}\frac{1}{\rho}d\rho$$

$$= -Q\left[\frac{\rho_L}{2\pi\epsilon_0}ln\rho\right]_a^b \quad = -\frac{Q\rho_L}{2\pi\epsilon_0}[\ln b - \ln a] \quad = -\frac{Q\rho_L}{2\pi\epsilon_0}ln\frac{b}{a}\mathrm{J}$$

(ex 4) z축을 따라 선전하 ρ_L, 전하Q; $B(\rho=b,\ \phi=0,\ z=0) \rightarrow A(\rho=a,\ \phi=0,\ z=0)$, $W = ?$

선전하 ρ_L에 의한 전계

$$\boldsymbol{E} = E_\rho\boldsymbol{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0\rho}\boldsymbol{a}_\rho$$

$$W = -Q\int_{init}^{final} \boldsymbol{E}\cdot d\boldsymbol{L}$$

$$= -Q\int_{B}^{A} \frac{\rho_L}{2\pi\epsilon_0\rho}\boldsymbol{a}_\rho \cdot (d\rho\boldsymbol{a}_\rho + \rho d\phi\boldsymbol{a}_\phi + dz\boldsymbol{a}_z)$$

$$= -Q\int_{b}^{a} \frac{\rho_L}{2\pi\epsilon_0}\frac{1}{\rho}d\rho$$

$$= -Q\left[\frac{\rho_L}{2\pi\epsilon_0}ln\rho\right]_b^a = -\frac{Q\rho_L}{2\pi\epsilon_0}[\ln a - \ln b]$$

$$= -\frac{Q\rho_L}{2\pi\epsilon_0}ln\frac{a}{b} \ \text{or}\ \frac{Q\rho_L}{2\pi\epsilon_0}ln\frac{b}{a}\mathrm{J}$$

(응용예제 4.2) 다음의 전계 내에서

4C의 전하를 점 $B(1,\ 0,\ 0)$에서 점 $A(0,\ 2,\ 0)$까지

$y=2-2x$, $z=0$의 경로를 따라 이동할 때 필요한 에너지 $W = ?$

(a) $\boldsymbol{E} = 5\boldsymbol{a}_x \mathrm{V/m}$

(b) $\boldsymbol{E} = 5x\boldsymbol{a}_x \mathrm{V/m}$

(c) $\boldsymbol{E} = 5x\boldsymbol{a}_x + 5y\boldsymbol{a}_y \mathrm{V/m}$

4.3 전위차 및 전위의 정의

- 전계 $\boldsymbol{E}$ 내에서 전하 Q를 한 점에서 다른 한점까지 이동시킬 때 필요한 일;

$$W = -Q\int_{init}^{final} \boldsymbol{E} \cdot d\boldsymbol{L}$$

- 전위차의 정의

  단위시험전하에 작용하는 힘

  → 전계의 세기 $\boldsymbol{E}$

  단위시험전하를 한 점에서 다른 한 점까지 이동시키는데 필요한 일

  → 전위차(potential difference) V

- 전위차의 계산

  단위시험전하를 점B에서 점A까지 이동시키는데 필요한 일

$$\text{전위차} = V_{AB} = \frac{W}{Q}$$

$$= -\int_{init}^{final} \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$= -\int_{B}^{A} \boldsymbol{E} \cdot d\boldsymbol{L} \ \mathrm{V}$$

- 전위(potential) 또는 절대전위; 전위가 영인 기준점에 대한 특정 점의 전위차
- 점B에 대한 점A의 전위차; $V_{AB} = V_A - V_B$

(ex 5) z축을 따라 선전하 ρ_L, 점$B(\rho=b,\ 0,\ 0)$에 대한 점$A(\rho=a,\ 0,\ 0)$의 전위차 $V_{ab} = ?$

선전하 ρ_L에 의한 전계

$$\boldsymbol{E} = E_\rho \boldsymbol{a}_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho}\boldsymbol{a}_\rho$$

$$V_{ab} = -\int_{init}^{final} \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$= -\int_{B}^{A} \frac{\rho_L}{2\pi\epsilon_0\rho}\boldsymbol{a}_\rho \cdot (dr\boldsymbol{a}_\rho + \rho d\phi \boldsymbol{a}_\phi + dz\boldsymbol{a}_z)$$

$$= -\int_{b}^{a} \frac{\rho_L}{2\pi\epsilon_0}\frac{1}{\rho}d\rho$$

$$= -\left[\frac{\rho_L}{2\pi\epsilon_0} ln\rho\right]_b^a = -\frac{\rho_L}{2\pi\epsilon_0}[\ln a - \ln b]$$

$$= -\frac{\rho_L}{2\pi\epsilon_0} ln\frac{a}{b} \ \text{or} \ \frac{\rho_L}{2\pi\epsilon_0} ln\frac{b}{a} \ \ \mathrm{J}$$

(ex 6) 원점에 점전하 Q

점$B(r=r_B,\ 0,\ 0)$와 점$A(r=r_A,\ 0,\ 0)$의 전위차 V_{AB} = ?

점전하 Q에 의한 전계

$$\boldsymbol{E}=E_r\boldsymbol{a_r}=\frac{Q}{4\pi\epsilon_0 r}\boldsymbol{a_r}$$

$$V_{AB}=-\int_{init}^{final}\boldsymbol{E}\cdot d\boldsymbol{L}$$

$$=-\int_B^A\frac{Q}{4\pi\epsilon_0 r^2}\boldsymbol{a_r}\cdot(dr\boldsymbol{a_r}+rd\theta\boldsymbol{a_\theta}+r\sin\theta d\phi\boldsymbol{a_\phi})$$

$$=-\int_{r_B}^{r_A}\frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}dr$$

$$=-\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_B}^{r_A}$$

$$=\frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A}-\frac{1}{r_B}\right]=\frac{Q}{4\pi\epsilon_0 r_A}-\frac{Q}{4\pi\epsilon_0 r_B}$$

$$=V_A-V_B$$

(ex 7) $\boldsymbol{E}=-\frac{6y}{x^2}\boldsymbol{a_x}+\frac{6}{x}\boldsymbol{a_y}+5\boldsymbol{a_z}$ V/m

(a) 점 $P(-7,\ 2,\ 1)$와 점$Q(4,\ 1,\ 2)$ 사이의 전위차 $V_{PQ}=?$

경로; $Q(4,\ 1,\ 2)\rightarrow R_1(-7,\ 1,\ 2)\rightarrow R_2(-7,\ 2,\ 2)\rightarrow P(-7,\ 2,\ 1)$

$$V_{PQ}=-\int_{init}^{final}\boldsymbol{E}\cdot d\boldsymbol{L}$$

$$=-\int_Q^P\left(-\frac{6y}{x^2}\boldsymbol{a_x}+\frac{6}{x}\boldsymbol{a_y}+5\boldsymbol{a_z}\right)\cdot\left(dx\boldsymbol{a_x}+dy\boldsymbol{a_y}+dz\boldsymbol{a_z}\right)$$

$$=-\left(\int_Q^{R_1}\boldsymbol{E}\cdot d\boldsymbol{L}+\int_{R_1}^{R_2}\boldsymbol{E}\cdot d\boldsymbol{L}+\int_{R_2}^{P}\boldsymbol{E}\cdot d\boldsymbol{L}\right)$$

$$=-\int_4^{-7}\left(-\frac{6y}{x^2}\right)dx\bigg|_{y=1}-\int_1^2\frac{6}{x}dy\bigg|_{x=-7}-\int_2^1 5dz$$

$$=\int_4^{-7}\frac{6}{x^2}dx+\int_1^2\frac{6}{7}dy-\int_2^1 5dz$$

$$=6\left[-\frac{1}{x}\right]_4^{-7}+\frac{6}{7}[y]_1^2-5[z]_2^1$$

$$=6\left[\frac{1}{7}+\frac{1}{4}\right]+\frac{6}{7}[2-1]-5[1-2]=8.214\text{V}$$

(b) $V_Q = 0 \rightarrow V_P = ?$

$$V_{PQ} = V_P - V_Q$$

$$\therefore V_P = V_Q + V_{PQ}$$

$$= 0 + 8.214$$

$$= 8.214\text{V}$$

(c) $V_R = 0$ where $R(2,\ 0,\ -1) \rightarrow V_P = ?$ where $P(-7,\ 2,\ 1)$

경로; $R(2,\ 0,\ -1) \rightarrow T_1(-7,\ 0,\ -1)$

$\rightarrow T_2(-7,\ 2,\ -1)$

$\rightarrow P(-7,\ 2,\ 1)$

$$V_P = V_R + V_{PR}$$

$$= V_{PR}$$

$$= -\int_{init}^{final} \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$= -\int_R^P \left(-\frac{6y}{x^2}\boldsymbol{a}_x + \frac{6}{x}\boldsymbol{a}_y + 5\boldsymbol{a}_z\right) \cdot \left(dx\boldsymbol{a}_x + dy\boldsymbol{a}_y + dz\boldsymbol{a}_z\right)$$

$$= -\left(\int_R^{T_1} \boldsymbol{E} \cdot d\boldsymbol{L} + \int_{T_1}^{T_2} \boldsymbol{E} \cdot d\boldsymbol{L} + \int_{T_2}^{P} \boldsymbol{E} \cdot d\boldsymbol{L}\right)$$

$$= -\int_2^{-7} \left(-\frac{6y}{x^2}\right) dx \Big|_{y=0} - \int_0^2 \frac{6}{x} dy \Big|_{x=-7} - \int_{-1}^{1} 5dz$$

$$= \int_0^2 \frac{6}{7} dy - \int_{-1}^{1} 5dz$$

$$= \frac{6}{7}[y]_0^2 - 5[z]_{-1}^1$$

$$= \frac{6}{7}[2-0] - 5[1+1]$$

$$= -8.286\text{V}$$

(응용예제 4.4) $\boldsymbol{E} = 6x^2\boldsymbol{a}_x + 6y\boldsymbol{a}_y + 5\boldsymbol{a}_z$ V/m

(a) 점 $M(2,\ 6,\ -1)$와 점 $N(-3,\ -3,\ 2)$ 사이의 전위차 $V_{MN} = ?$

(b) 점 $Q(4,\ -2,\ -35)$에서 $V_Q = 0$일 때 $V_M = ?$

(c) 점 $P(1,\ 2,\ -4)$에서 $V_P = 2$일 때 $V_N = ?$

4.4 점전하에 의한 전위

- 점전하 Q에 의한 전계

$$\boldsymbol{E} = E_r \boldsymbol{a_r} = \frac{Q}{4\pi\epsilon_0 r}\boldsymbol{a_r} \ \mathrm{V/m}$$

- 원점에 점전하 Q가 위치할 때,
  $r = r_A$인 점과 $r = r_B$인 점 사이의 전위차 V_{AB}

$$V_{AB} = -\int_{init}^{final} \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$= -\int_B^A \frac{Q}{4\pi\epsilon_0 r^2}\boldsymbol{a_r} \cdot (d\rho \boldsymbol{a_r} + r d\theta \boldsymbol{a_\theta} + r\sin\theta d\phi \boldsymbol{a_\phi})$$

$$= -\int_{r_B}^{r_A} \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2}dr$$

$$= -\frac{Q}{4\pi\epsilon_0}\left[-\frac{1}{r}\right]_{r_B}^{r_A}$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right] \ \mathrm{V}$$

- 점전하 Q가 원점에 위치할 때,
  $r = \infty$인 곳을 영전위 기준점으로 정할 경우 점A에서의 전위 V_A

$$V_A = V_{A\infty} - V_\infty$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_A} - \frac{1}{\infty}\right) + 0$$

$$= \frac{Q}{4\pi\epsilon_0 r_A} \ \mathrm{V}$$

- 점전하 Q가 원점에 위치할 때,
  원점에서의 거리가 r인 곳에서의 전위 V

$$V = \frac{Q}{4\pi\epsilon_0 r} \ \mathrm{V}$$

(ex 8) $Q = 6\text{nC}$ at $O(0,\ 0,\ 0)$

$V_P\ =\ ?$, where $P(0.2,\ -0.4,\ 0.4)$

(a) $V_\infty = 0$,

$$V_P = \frac{Q}{4\pi\epsilon_0 r}$$

$$= \frac{Q}{4\pi\epsilon_0\sqrt{x^2+y^2+z^2}}$$

$$= \frac{6\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}\times\sqrt{0.2^2+(-0.4)^2+0.4^2}} = \frac{54}{\sqrt{0.36}} = 90\text{V}$$

(b) $V_Q = 0$, where $Q(1,\ 0,\ 0)$

$$V_P = V_{QP} + V_Q$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_Q}\right) + 0$$

$$= \frac{6\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}}\left(\frac{1}{0.36} - 1\right) = 36\text{V}$$

(c) $V_R = 20$, where $R(-0.5,\ 1,\ -1)$

$$V_P = V_{PR} + V_R$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r_P} - \frac{1}{r_R}\right) + 20$$

$$= \frac{6\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}}\left(\frac{1}{0.6} - \frac{1}{\sqrt{(-0.5)^2+1^2+(-1)^2}}\right) + 20$$

$$= 54\left(\frac{1}{0.6} - \frac{1}{1.5}\right)$$

$$= 74\text{V}$$

(응용예제 4.5) $Q = 15\text{nC}$ at $O(0,\ 0,\ 0)$

$V_1\ =\ ?$, where $P_1(-2,\ 3,\ -1)$

(a) $V_2 = 0$, where $P_2(6,\ 5,\ 4)$

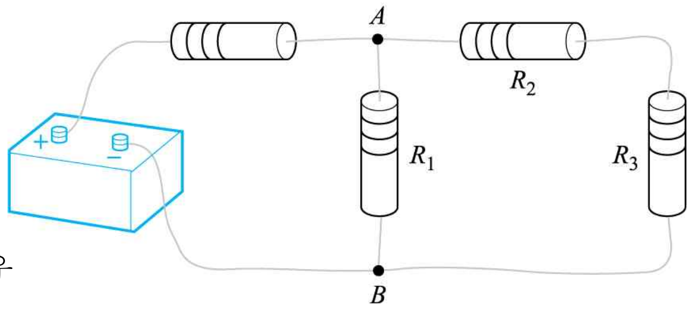
(b) $V_\infty = 0$

(c) $V_3 = 5\text{V}$, where $P_3(2,\ 0,\ 4)$

4.5 전하 시스템의 전위계: 보전적 성질

- 전위; 단위전하를 영전위 기준점으로부터 어떤 점까지 이동시키는데 필요한 일

$$V(\boldsymbol{r}) = \int_{vol} \frac{\rho_v(\boldsymbol{r}')dv'}{4\pi\epsilon_0|\boldsymbol{r}-\boldsymbol{r}'|}$$



- 전위차;

$$V_{AB} = V_A - V_B = -\int_B^A \boldsymbol{E} \cdot d\boldsymbol{L}$$

무한원점을 영전위 기준점으로 할 경우

$$V_A = -\int_\infty^A \boldsymbol{E} \cdot d\boldsymbol{L}$$

- 보존계(conservative field); 폐곡선에 대한 선적분이 0

$\oint \boldsymbol{E} \cdot d\boldsymbol{L} = 0$ → 이동경로와는 무관 → Kirchhoff의 전압법칙

(ex 9) $V_{\infty} = 0$

$V_P = ?$, where $P(0,\ 0,\ 10)$

(a) $Q = 20\text{nC}$ at $O(0,\ 0,\ 0)$

$$V_P = V_{P\infty} + V_{\infty}$$

$$= \frac{Q}{4\pi\epsilon_0 r} = \frac{20\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}\times10} = 18\text{V}$$

(b) $\rho_L = 10\text{nC/m}$ at $x=0,\ z=0,\ -1<y<1$

$$V = \int_{vol}\frac{\rho_v(\boldsymbol{r}')dv'}{4\pi\epsilon_0|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$= \int_l \frac{\rho_L(\boldsymbol{r}')dl'}{4\pi\epsilon_0|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$= \int_{-1}^{+1}\frac{\rho_L dy'}{4\pi\epsilon_0|10\boldsymbol{a}_z - y'\boldsymbol{a}_y|}$$

$$= \int_{-1}^{+1}\frac{\rho_L dy'}{4\pi\epsilon_0\sqrt{10^2+y'^2}} \quad \leftarrow \begin{cases} y' = 10\tan\theta \\ dy' = 10\sec^2\theta d\theta \end{cases} \quad \text{and} \quad \sqrt{10^2+y'^2} = 10\sec\theta$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\int_{\theta_1}^{\theta_2}\frac{10\sec^2\theta\, d\theta}{10\sec\theta}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\int_{\theta_1}^{\theta_2}\sec\theta\, d\theta$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\left[\ln(\sec\theta+\tan\theta)\right]_{\theta_1}^{\theta_2}$$

$$\leftarrow \sec\theta = \frac{\sqrt{10^2+y'^2}}{10} \quad \text{and} \quad \tan\theta = \frac{y'}{10}$$

$$= \frac{\rho_L}{4\pi\epsilon_0}\left[\ln\left(\frac{\sqrt{10^2+y'^2}}{10}+\frac{y'}{10}\right)\right]_{-1}^{1}$$

$$= \frac{10\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}}\left[\ln\left(\frac{\sqrt{101}+1}{10}\right)-\ln\left(\frac{\sqrt{101}-1}{10}\right)\right]$$

$$= 10\times9\times\ln\frac{\sqrt{101}+1}{\sqrt{101}-1}$$

$$= 17.97\text{V}$$

(c) $\rho_L = 10\text{nC/m}$ at $x=0,\ y=0,\ -1<z<1$

$$V = \int_{vol} \frac{\rho_v(\boldsymbol{r}')dv'}{4\pi\epsilon_0|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$= \int_l \frac{\rho_L(\boldsymbol{r}')dl'}{4\pi\epsilon_0|\boldsymbol{r}-\boldsymbol{r}'|}$$

$$= \int_{-1}^{+1} \frac{\rho_L dz'}{4\pi\epsilon_0|10\boldsymbol{a}_z - z'\boldsymbol{a}_z|}$$

$$= \int_{-1}^{+1} \frac{\rho_L dz'}{4\pi\epsilon_0(10-z')} \qquad \leftarrow \begin{cases} 10-z' = u \\ -dz' = du \end{cases}$$

$$= -\frac{\rho_L}{4\pi\epsilon_0}\int_{u_1}^{u_2}\frac{du}{u}$$

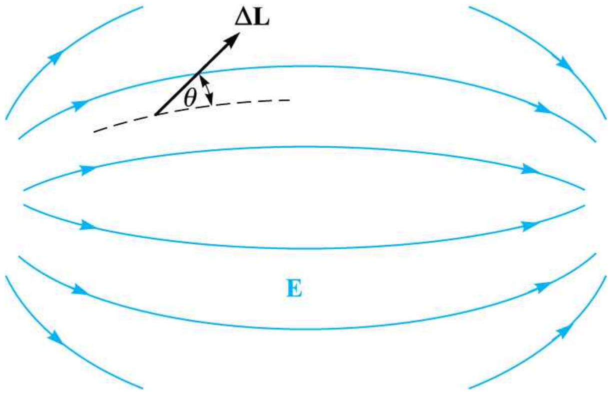
$$= -\frac{\rho_L}{4\pi\epsilon_0}[\ln u]_{u_1}^{u_2} \qquad \leftarrow\ u = 10-z'$$

$$= -\frac{\rho_L}{4\pi\epsilon_0}[\ln(10-z')]_{-1}^{1}$$

$$= -\frac{10\times10^{-9}}{4\pi\times\frac{1}{36\pi}\times10^{-9}}[\ln 9 - \ln 11] = -10\times9\times\ln\frac{9}{11}$$

$$= 18.06\text{V}$$

4.6 전위 경도



지금까지; 전계의 세기$\boldsymbol{E}$ 또는 전하밀도ρ_v $\rightarrow$ V

일반적으로; $V \rightarrow \boldsymbol{E}$

$$V = -\int \boldsymbol{E} \cdot d\boldsymbol{L}$$

$$\Delta V \doteq -\boldsymbol{E} \cdot \Delta \boldsymbol{L}$$

$$\Delta V \doteq -E \Delta L \cos\theta$$

$$\frac{dV}{dL} = -E\cos\theta$$

$\cos\theta = -1$, 즉 $\Delta \boldsymbol{L}$의 방향이 $\boldsymbol{E}$와 반대방향일 때

$$\left.\frac{dV}{dL}\right|_{\max} = E$$

(1) 전계의 세기의 크기는 거리에 대한 전위의 변화율의 최대값과 같다.

(2) 전위의 변화율의 최대값은 $\boldsymbol{E}$의 방향이 전위가 증가하는 방향과 반대일 때 얻어진다.

$$\rightarrow \quad \boldsymbol{E} = -\left.\frac{dV}{dL}\right|_{\max} \boldsymbol{a}_N$$

$$= -\frac{dV}{dN}\boldsymbol{a}_N$$

일반적으로 스칼라 T로부터 벡터를 얻는 연산을 경도 또는 구배(gradient)라 부르고, 다음과 같이 정의한다.

$$T\text{의 구배} = \mathrm{grad}\,T = \frac{dT}{dN}\boldsymbol{a}_N$$

$$\therefore \ \boldsymbol{E} = -\mathrm{grad}\,V$$

한편 $$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

$$dV = -\boldsymbol{E} \cdot d\boldsymbol{L} = -E_x dx - E_y dy - E_z dz$$

$$\rightarrow \quad E_x = -\frac{\partial V}{\partial x}, \ E_y = -\frac{\partial V}{\partial y}, \ E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \ \boldsymbol{E} = -\left(\frac{\partial V}{\partial x}\boldsymbol{a}_x + \frac{\partial V}{\partial y}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right)$$

$$= -\left(\frac{\partial}{\partial x}\boldsymbol{a}_x + \frac{\partial}{\partial y}\boldsymbol{a}_y + \frac{\partial}{\partial z}\boldsymbol{a}_z\right)V$$

$$= -\nabla V = -\mathrm{grad}\,V$$

• 전위경도의 계산

$$\nabla V = \frac{\partial V}{\partial x}\boldsymbol{a}_x + \frac{\partial V}{\partial y}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z \qquad \text{(직각좌표계)}$$

$$\nabla V = \frac{\partial V}{\partial \rho}\boldsymbol{a}_x + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z \qquad \text{(원통좌표계)}$$

$$\nabla V = \frac{\partial V}{\partial r}\boldsymbol{a}_x + \frac{1}{r}\frac{\partial V}{\partial \theta}\boldsymbol{a}_y + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\boldsymbol{a}_z \quad \text{(구좌표계)}$$

• $\begin{cases} \text{발산(divergence); 벡터} \rightarrow \text{스칼라} \\ \text{경도(gradient); 스칼라} \rightarrow \text{벡터} \end{cases}$

(ex 7) $V = 2x^2y - 5z$

(a) $V=?$ at $P(-4,\ 3,\ 6)$

$$V_P = 2(-4)^2(3) - 5(6) = 66\ [\mathrm{V}]$$

(b) 전계의 세기$\boldsymbol{E}$의 크기와 방향 at $P(-4,\ 3,\ 6)$

$$\begin{aligned} \boldsymbol{E} = -\nabla V &= -\left(\frac{\partial V}{\partial x}\boldsymbol{a}_x + \frac{\partial V}{\partial y}\boldsymbol{a}_y + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right) \\ &= -4xy\boldsymbol{a}_x - 2x^2\boldsymbol{a}_y + 5\boldsymbol{a}_z\ [\mathrm{V/m}] \end{aligned}$$

$$\boldsymbol{E}_P = 48\boldsymbol{a}_x - 32\boldsymbol{a}_y + 5\boldsymbol{a}_z\ [\mathrm{V/m}]$$

$$|\boldsymbol{E}_P| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{48^2 + (-32)^2 + 5^2} = 57.9\ [\mathrm{V/m}]$$

$$\boldsymbol{a}_{E,P} = \frac{\boldsymbol{E}}{|\boldsymbol{E}|} = \frac{48\boldsymbol{a}_x - 32\boldsymbol{a}_y + 5\boldsymbol{a}_z}{57.9} = 0.829\boldsymbol{a}_x - 0.553\boldsymbol{a}_y + 0.086\boldsymbol{a}_z$$

(c) 자유공간에서의 전속밀도$\boldsymbol{D}$

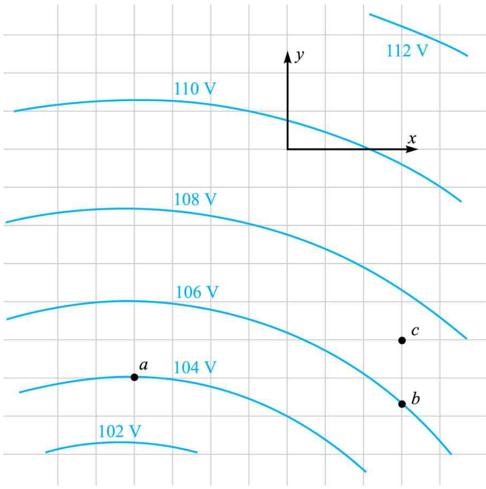
$$\begin{aligned} \boldsymbol{D} = \epsilon_0\boldsymbol{E} &= \left(8.854\times10^{-12}\right)\cdot\left(-4xy\boldsymbol{a}_x - 2x^2\boldsymbol{a}_y + 5\boldsymbol{a}_z\right) \\ &= -35.4xy\boldsymbol{a}_x - 17.71x^2\boldsymbol{a}_y + 44.3\boldsymbol{a}_z\ [\mathrm{pC/m^2}] \end{aligned}$$

(d) 전계를 만들어 낸 원인(체적전하밀도)

$$\rho_v = \nabla\cdot\boldsymbol{D} = \frac{\partial Dx}{\partial x} + \frac{\partial Dy}{\partial y} + \frac{\partial Dz}{\partial z} = -35.4y\ [\mathrm{pC/m^3}]$$

(ex 14) 101쪽 응용예제 4.7

→ 그림 4.8



(ex 15) 101쪽 예제 4.8

$$V=\frac{(60\sin\theta)}{r^2}\ [\mathrm{V}],\qquad P(r=3m,\ \theta=60^o,\ \phi=25^o)$$

(a) $V_P = \dfrac{60\sin60^o}{3^2} = 5.773\ [\mathrm{V}]$

(b)
$$\begin{aligned} \boldsymbol{E_P} &= -\nabla V\big|_P = -\left(\frac{\partial V}{\partial r}\boldsymbol{a_x}+\frac{1}{r}\frac{\partial V}{\partial\theta}\boldsymbol{a_y}+\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\boldsymbol{a_z}\right)\Bigg|_P \\ &= -\left((-2)\cdot\frac{60\sin\theta}{r^3}\boldsymbol{a_r}+\frac{1}{r}\cdot\frac{60\cos\theta}{r^2}\boldsymbol{a_\theta}\right)\Bigg|_P \\ &= \frac{2\times60\sin60^o}{3^3}\boldsymbol{a_r}-\frac{60\cos60^o}{3^3}\boldsymbol{a_\theta} \\ &= 3.85\boldsymbol{a_r}-1.11\boldsymbol{a_\theta}\ [\mathrm{V/m}] \end{aligned}$$

(c) $\dfrac{dV}{dN}$ at P

$$\frac{dV}{dN}\bigg|_P = |\boldsymbol{E_P}| = \sqrt{3.85^2+(-1.11)^2} = 4.006\ [\mathrm{V/m}]$$

(d) 점 P에서 $\boldsymbol{a_N}$

$$\begin{aligned} \boldsymbol{a_{N,P}} = -\boldsymbol{a_{E,P}} &= -\frac{\boldsymbol{E}}{|\boldsymbol{E}|}\bigg|_P \\ &= -\frac{3.85\boldsymbol{a_r}-1.11\boldsymbol{a_\theta}}{4.01} \\ &= -0.96\boldsymbol{a_r}+0.277\boldsymbol{a_\theta} \end{aligned}$$

(e) 점 P에서 ρ_v

$$\boldsymbol{D} = \epsilon_0\boldsymbol{E} = 120\epsilon_0\frac{\sin\theta}{r^3}\boldsymbol{a_r}-60\epsilon_0\frac{\cos\theta}{r^3}\boldsymbol{a_\theta}$$

$$\begin{aligned} \rho_v\big|_P = \nabla\cdot\boldsymbol{D}\big|_P &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2D_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta D_\theta)+\frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial\phi}\bigg|_P \\ &= \frac{1}{r^2}\frac{\partial}{\partial r}\frac{120\epsilon_0\sin\theta}{r}+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\frac{-60\epsilon_0\sin\theta\cos\theta}{r^3}\bigg|_P \\ &= -\frac{120\epsilon_0\sin\theta}{r^4}-\frac{60\epsilon_0}{r^4\sin\theta}(\cos^2\theta-\sin^2\theta)\bigg|_P \\ &= -\frac{60\epsilon_0}{r^4}\left[2\sin\theta+\frac{\cos^2\theta-\sin^2\theta}{\sin\theta}\right]\bigg|_P \\ &= -\frac{60\times8.854\times10^{-12}}{3^4}\left[2\times\sin60^o-\frac{\cos^260^o-\sin^260^o}{\sin60^o}\right] \\ &= -7.57\ [\mathrm{pC/m^3}] \end{aligned}$$

4.7 전기쌍극자

전기쌍극자(electric dipole); 크기가 같고 부호가 반대인 두 점전하가 가까운 거리에 존재하여 하나의 입자처럼 행동

4.8 정전계 내의 에너지밀도

$$
\begin{aligned}
W_E &= \frac{1}{2}\int_{vol} \rho_v V dv \\
&= \frac{1}{2}\int_{vol} \boldsymbol{D}\cdot\boldsymbol{E} dv \\
&= \frac{1}{2}\int_{vol} \epsilon_0 E^2 dv
\end{aligned}
$$

(ex 13) 113쪽 예제 4.11

$W_E = ?$ in $0<\rho<a,\ 0<\phi<\pi,\ 0<z<2$

$(a)\ \ V = V_0\dfrac{\rho}{a}$

$$
\begin{aligned}
\boldsymbol{E} \ = \ -\nabla V &= -\left(\frac{\partial V}{\partial \rho}\boldsymbol{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\boldsymbol{a}_\phi + \frac{\partial V}{\partial z}\boldsymbol{a}_z\right) \\
&= -\frac{V_0}{a}\boldsymbol{a}_\rho
\end{aligned}
$$

$$
\begin{aligned}
\therefore\ W_E &= \frac{1}{2}\int_{vol}\epsilon_0 E^2 dv \\
&= \frac{1}{2}\int_0^2\int_0^\pi\int_0^a \epsilon_0\frac{V_0^2}{a^2}\rho d\rho d\phi dz \\
&= \frac{\epsilon_0}{2}\frac{V_0^2}{a^2}\times 2\times\pi\times\frac{1}{2}\rho^2\Big]_0^a \\
&= \frac{\pi}{2}\epsilon_0 V_0^2\ \text{[J]}
\end{aligned}
$$