7장 정상자계

전계; 실험법칙 → 가우스 법칙

자계; 실험법칙 → 암페어의 주회법칙

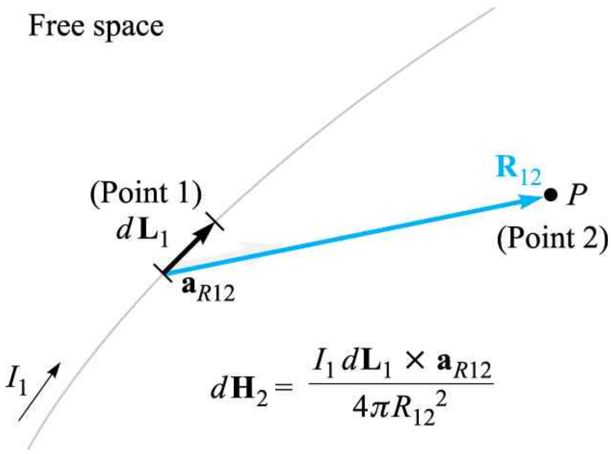
7.1 Biot-Savart의 법칙

- 정상자계(직류자계)의 근원

    영구자석

    시간에 대한 변화율이 일정한 전계

    직류전류



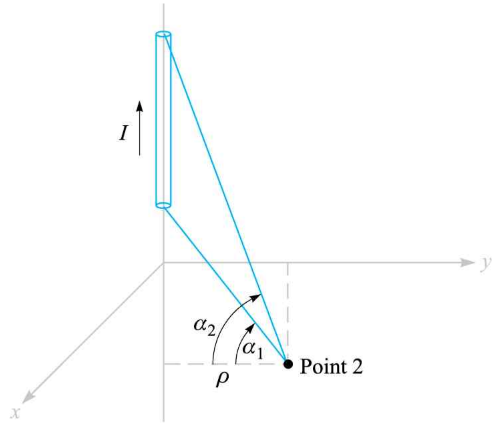
- 직류전류에 의한 자계의 세기

    → Biot-Savart의 실험법칙(그림 7.1)

$$H_2 = \oint \frac{I_1\, dL_1 \times a_{R_{12}}}{4\pi R_{12}^2} = \oint \frac{I_1\ dL_1 \times R_{12}}{4\pi R_{12}^3} \ \text{A/m}$$

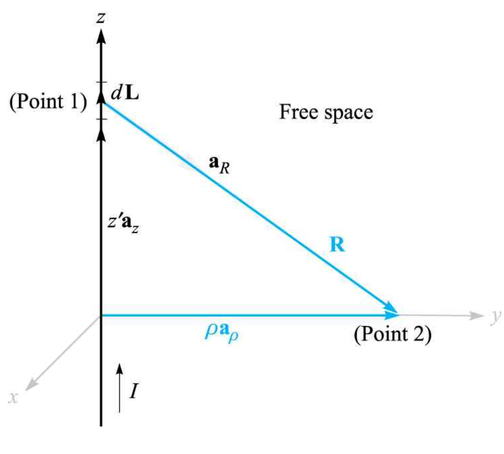
- 유한길이의 선전류에 의한 자계의 세기(그림 7.5)

$$H = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)a_\phi \ \text{A/m}$$



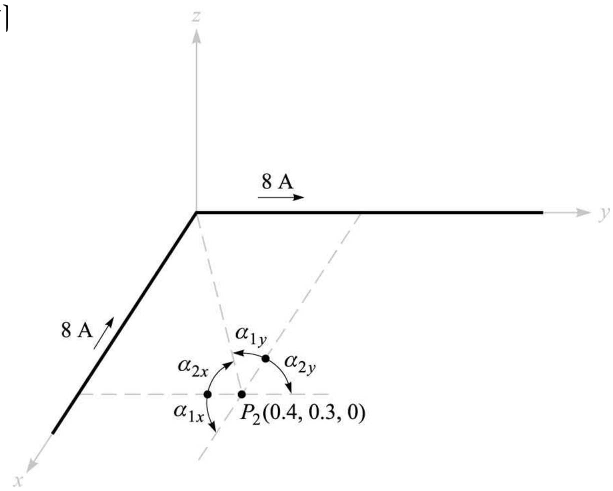
- 무한길이의 선전류에 의한 자계의 세기(그림 7.3)

$$H = \frac{I}{4\pi\rho}\left[\sin 90^o - \sin(-90^o)\right]a_\phi$$
$$= \frac{I}{2\pi\rho}a_\phi \quad [\text{A/m}]$$



(예제 7.1) 두 개의 반무한 선전류에 의한 자계의 세기

$I = 8\mathrm{A}$, $P_2(0.4,\ 0.3,\ 0) \rightarrow \boldsymbol{H_2} = ?$



$$\boldsymbol{H_2} = \boldsymbol{H_{2(x)}} + \boldsymbol{H_{2(y)}}$$

$$\alpha_{1x} = -90^o,\ \alpha_{2x} = \tan^{-1}(0.4/0.3) = 53.1^o$$

$$\rho_x = 0.3$$

$$\boldsymbol{H_{2(x)}} = \frac{I_x}{4\pi\rho_x}(\sin\alpha_{2x} - \sin\alpha_{1x})\boldsymbol{a_\phi}$$

$$= \frac{8}{4\pi \times 0.3}(\sin 53.1^0 - \sin(-90^0))\boldsymbol{a_\phi}$$

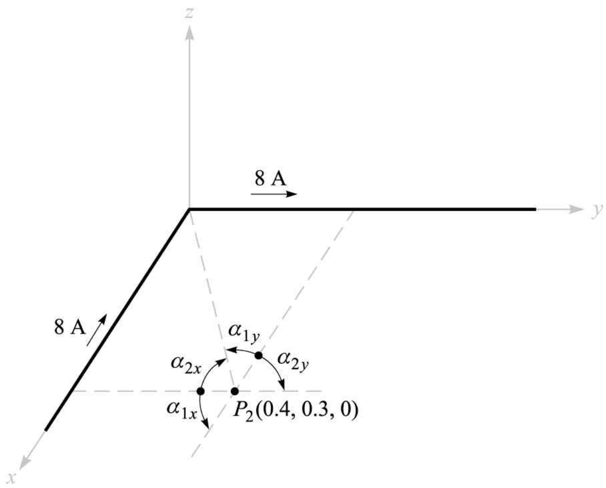
$$= \frac{2}{0.3\pi}(1.8)\boldsymbol{a_\phi}$$

$$= \frac{12}{\pi}\boldsymbol{a_\phi}$$

$$= -\frac{12}{\pi}\boldsymbol{a_z}\ \ \mathrm{A/m}$$

$$\alpha_{1y} = -\tan^{-1}(0.3/0.4) = -36.9^o,\ \alpha_{2y} = 90^o$$

$$\rho_y = 0.4$$



$$\boldsymbol{H_{2(y)}} = \frac{I_y}{4\pi\rho_y}(\sin\alpha_{2y} - \sin\alpha_{1y})\boldsymbol{a_\phi}$$

$$= \frac{8}{4\pi \times 0.4}(\sin 90^o - \sin(-36.9^0))\boldsymbol{a_\phi}$$

$$= \frac{2}{0.4\pi}(1.6)\boldsymbol{a_\phi}$$

$$= \frac{8}{\pi}\boldsymbol{a_\phi}$$

$$= -\frac{8}{\pi}\boldsymbol{a_z}\ \ \mathrm{A/m}$$

$$\therefore\ \boldsymbol{H_2} = \left(\frac{-12}{\pi}\right)\boldsymbol{a_z} + \left(\frac{-8}{\pi}\right)\boldsymbol{a_z}$$

$$= -\frac{20}{\pi}\boldsymbol{a_z}$$

$$= -6.37\boldsymbol{a_z}\ \ \mathrm{A/m}$$

(ex 1) $I = 24\text{A}$, $P_2(1.5,\ 2,\ 3) \rightarrow \boldsymbol{H}_2 = ?$

$$\rho = \sqrt{1.5^2 + 2^2} = 2.5$$

$$\boldsymbol{H}_2 = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\boldsymbol{a}_\phi$$

(a) from $z=0$ to $z=6$

$$\alpha_1 = -\tan^{-1}(3/\rho) = -50.2°,\quad \alpha_2 = 50.2°$$

$$\boldsymbol{H}_2 = \frac{24}{4\pi \times 2.5}(\sin 50.2° + \sin 50.2°)\boldsymbol{a}_\phi$$

$$= \frac{12}{2.5\pi}\sin 50.2° \mathbf{a}_\phi$$

$$= 1.174\boldsymbol{a}_\phi$$

$$= H_x\boldsymbol{a}_x + H_y\boldsymbol{a}_y$$

$$H_x = \boldsymbol{H}_2 \cdot \boldsymbol{a}_x$$

$$= 1.174\boldsymbol{a}_\phi \cdot \boldsymbol{a}_x$$

$$= 1.174(-\sin\phi)$$

$$= 1.174 \times (-2/2.5)$$

$$= -0.9392$$

$$H_y = \boldsymbol{H}_2 \cdot \boldsymbol{a}_y$$

$$= 1.174\boldsymbol{a}_\phi \cdot \boldsymbol{a}_y$$

$$= 1.174\cos\phi$$

$$= 1.174 \times (1.5/2.5)$$

$$= 0.7044$$

$$\therefore\ \boldsymbol{H}_2 = H_x\boldsymbol{a}_x + H_y\boldsymbol{a}_y$$

$$= -0.9392\boldsymbol{a}_x + 0.7044\boldsymbol{a}_y \ \text{A/m}$$

(b) from $z=6$ to $z=\infty$

(c) from $z=-\infty$ to $z=\infty$

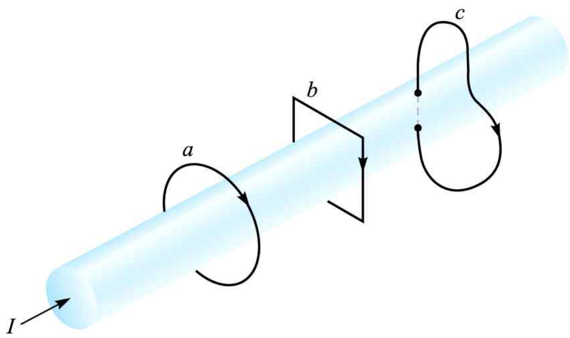
응용예제 7.1)

응용예제 7.2)

7.2 Ampere의 주회법칙(Circuital Law)

- Ampere의 주회법칙 (그림 7.7)

$$\oint H \cdot dL = I$$



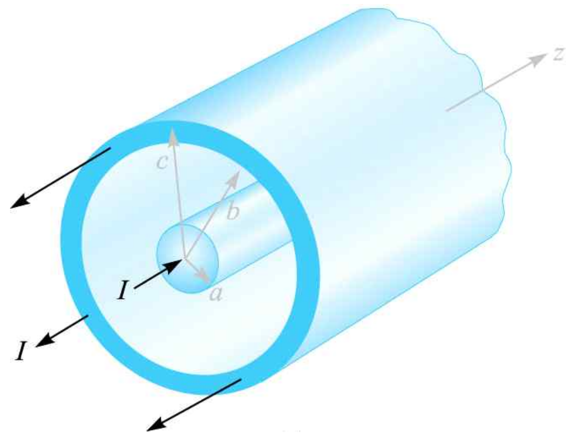
폐곡로를 따라 H 를 선적분한 결과는
폐곡로로 둘러싸인 직류전류와 같다.

- 무한길이의 선전류에 의한 자계의 세기(그림 7.3)

$$\oint H \cdot dL = \int_0^{2\pi} H_\phi \ \rho \, d\phi = H_\phi \, \rho \int_0^{2\pi} d\phi = H_\phi \, 2\pi\rho = I$$

$$\therefore H_\phi = \frac{I}{2\pi\rho} \text{A/m}$$

- 동축케이블에서의 자계의 세기(그림 7.8)



i) $\rho < a$

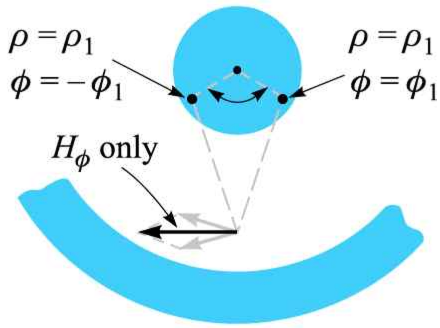
$$H_\phi = \frac{I'}{2\pi\rho} \qquad \leftarrow \ \frac{I'}{I} = \frac{\pi\rho^2}{\pi a^2}$$

$$= \frac{I}{2\pi\rho}\frac{\rho^2}{a^2} = \frac{I}{2\pi a^2}\rho \ \text{A/m}$$

ii) $a \leqq \rho < b$

$$H_\phi = \frac{I}{2\pi\rho} = \frac{I}{2\pi}\frac{1}{\rho} \ \text{A/m}$$

iii) $b \leqq \rho < c$



$$H_\phi = \frac{I - I''}{2\pi\rho} \qquad \leftarrow \frac{I''}{I} = \frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}$$

$$= \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

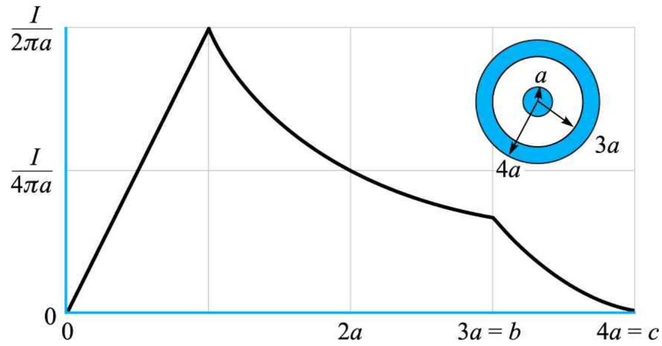
$$= \frac{I}{2\pi\rho}\left(\frac{c^2 - b^2 - \rho^2 + b^2}{c^2 - b^2}\right)$$

$$= \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right) \ \text{A/m}$$

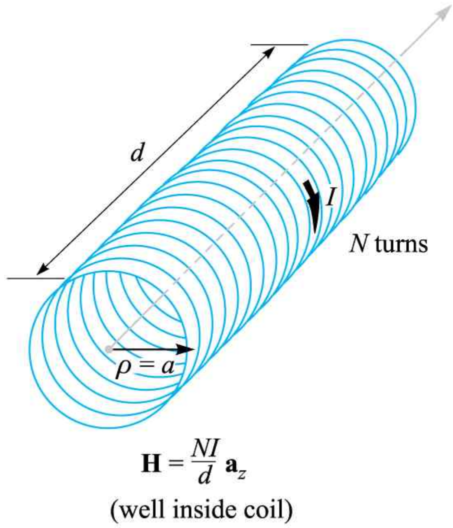
iv) $\rho \geqq d$

$$H_\phi = \frac{0}{2\pi\rho} = 0$$

⇨ 그림 7.9



• 솔레노이드(solenoid)에서의 자계의 세기(그림 7.11b)



$$\oint \boldsymbol{H} \cdot d\boldsymbol{L} = NI$$

$$\boldsymbol{H}_1 \cdot d\boldsymbol{L}_1 + \boldsymbol{H}_2 \cdot d\boldsymbol{L}_2 + \boldsymbol{H}_3 \cdot d\boldsymbol{L}_3 + \boldsymbol{H}_4 \cdot d\boldsymbol{L}_4 = NI$$

$$\leftarrow \boldsymbol{H}_2 \perp d\boldsymbol{L}_2,\ \boldsymbol{H}_4 \perp d\boldsymbol{L}_4$$

$$\boldsymbol{H}_1 \cdot d\boldsymbol{L}_1 + \boldsymbol{H}_3 \cdot d\boldsymbol{L}_3 = NI \qquad \leftarrow \boldsymbol{H}_3 = 0$$

$$\boldsymbol{H}_1 \cdot d\boldsymbol{L}_1 = NI \qquad \leftarrow \boldsymbol{H}_1 \parallel d\boldsymbol{L}_1$$

$$H_1\, dL_1 = NI$$

$$H_1 = \frac{NI}{dL_1} \qquad \leftarrow H_1 = H_z,\ dL_1 = d$$

$$\therefore\ H_z = \frac{NI}{d} = nI \text{ A/m} \qquad \leftarrow \frac{N}{d} = n \text{ (권선밀도)}$$

- 토로이드(toroid)에서의 자계의 세기(그림 7.12b)

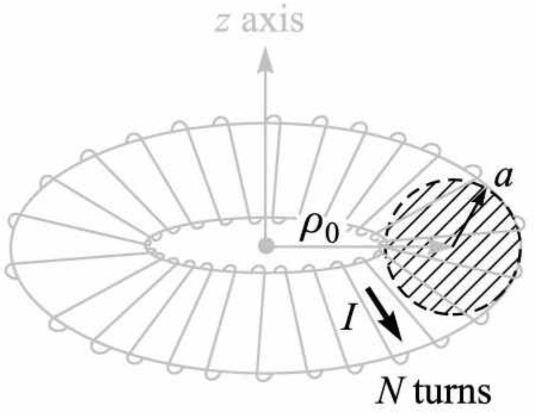
  i) 토로이드 외부 : $\oint \boldsymbol{H} \cdot d\boldsymbol{L} = 0$

  ii) 토로이드 내부 : $\oint \boldsymbol{H} \cdot d\boldsymbol{L} = NI$

$$\int_0^{2\pi} H_\phi \, \rho \, d\phi = NI$$

$$H_\phi \, 2\pi\rho = NI$$

$$\therefore \; H_\phi = \frac{NI}{2\pi\rho} \text{ A/m}$$



(ex 2) $\boldsymbol{H}$=? at $P(0.01, 0, 0)$

(a) 0.08A at z축 & −0.08A at $x = 0.015,\ y = 0$

$$\boldsymbol{H} = \boldsymbol{H}_1 + \boldsymbol{H}_2$$

$$= \frac{0.08}{2\pi \times 0.01}\boldsymbol{a}_y + \frac{0.08}{2\pi \times 0.005}\boldsymbol{a}_y$$

$$= \frac{0.08}{2\pi}\left(\frac{1}{0.01} + \frac{1}{0.005}\right)\boldsymbol{a}_y = 3.82\boldsymbol{a}_y \text{ A/m}$$

(b) $a = 6$㎜, $b = 9$㎜, $c = 12$㎜인 동축케이블의 내부도체에 $I = 0.8\boldsymbol{a}_z$ A

$b < \rho = 0.01 < c$ 이므로

$$\boldsymbol{H} = \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)\boldsymbol{a}_\phi$$

$$= \frac{0.8}{2\pi \times 0.01}\frac{0.012^2 - 0.01^2}{0.012^2 - 0.009^2}\boldsymbol{a}_y$$

$$= 8.8925\boldsymbol{a}_y \text{ A/m}$$

(c)

(d) 원점을 중심으로 하고

중심선이 y축인 토로이드($\rho_0 = 12$㎜, $a = 3$㎜, $N = 250$회)에서,

밖에서 $I = 2\boldsymbol{a}_y$ ㎃

$\rho_0 - a < \rho = 0.01 < \rho_0 + a$ (토로이드 내부)이므로

$$\boldsymbol{H} = \frac{NI}{2\pi\rho}\boldsymbol{a}_\phi$$

$$= \frac{250 \times 0.002}{2\pi \times 0.01}\boldsymbol{a}_z$$

$$= 7.958\boldsymbol{a}_z \text{ A/m}$$

7.3 벡터의 회전(Curl)

- 벡터의 회전(curl)

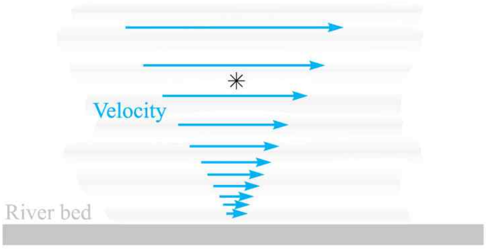
  Gauss의 법칙 → 벡터의 발산(divergence)

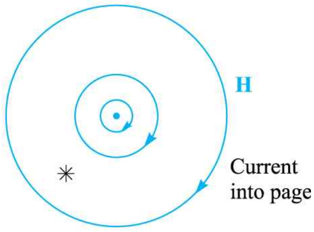
  Ampere의 주회법칙 → 벡터의 회전(curl)

$$\text{curl } \boldsymbol{H} = \nabla \times \boldsymbol{H} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\boldsymbol{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\boldsymbol{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\boldsymbol{a}_z$$

$$= \mathrm{J}$$





- Ampere의 주회법칙 미분형

$$\oint \mathrm{H} \cdot d\mathrm{L} = I \text{ (적분형)} \quad \leftrightarrow \quad \nabla \times \mathrm{H} = \mathrm{J} \text{ (미분형)}$$

응용예제 7.5) $\nabla \times \mathrm{G} = ?$

a) $\mathrm{G} = xyz(\boldsymbol{a}_x + \boldsymbol{a}_y)$ at P(3, 2, 1)

$$\nabla \times \mathrm{G} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ G_x & G_y & G_z \end{vmatrix}$$

$$= \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right)\boldsymbol{a}_x + \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right)\boldsymbol{a}_y + \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)\boldsymbol{a}_z$$

$$= (0 - xy)\boldsymbol{a}_x + (xy - 0)\boldsymbol{a}_y + (yz - xz)\boldsymbol{a}_z$$

$$= -6\boldsymbol{a}_x + 6\boldsymbol{a}_y - \boldsymbol{a}_z$$

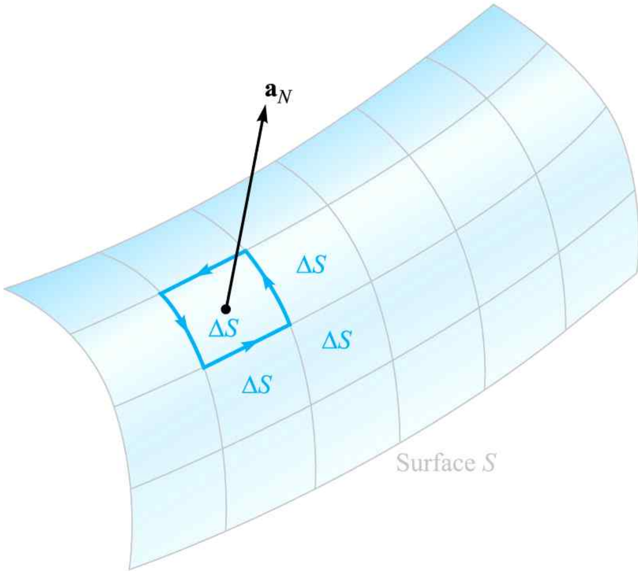
7.4 Stokes의 정리

- $\oint \mathrm{H} \cdot d\mathrm{L} \equiv \int_S (\nabla \times \mathrm{H}) \cdot d\mathrm{S}$

$$\because \oint \mathrm{H} \cdot d\mathrm{L} = I$$

$$= \int_S \mathrm{J} \cdot d\mathrm{S}$$

$$= \int_S (\nabla \times \mathrm{H}) \cdot d\mathrm{S}$$



- 전류의 연속성; 임의의 폐곡면을 통하여 나가는 전류의 합은 항상 0

$$\nabla \cdot \mathrm{J} = \nabla \cdot (\nabla \times \mathrm{H}) \equiv 0$$

7.5 자속과 자속밀도

- 자속밀도(magnetic flux density)의 정의

$$\mathrm{B} = \mu_0 \mathrm{H} \ [\mathrm{Wb/m^2} = \mathrm{T} = 10000\,\mathrm{G}]$$ (자유공간에서)

where, $\mu_0 = 4\pi \times 10^{-7}$ [H/m]

- 자속

전계의 세기 E $\rightarrow$ 전속밀도 $\mathrm{D} = \epsilon \mathrm{E}$ $\rightarrow$ 전속 $\Psi = \oint_S \mathrm{D} \cdot d\mathrm{S} = Q$

자계의 세기 H $\rightarrow$ 자속밀도 $\mathrm{B} = \mu \mathrm{H}$ $\rightarrow$ 자속 $\Phi = \oint_S \mathrm{B} \cdot d\mathrm{S} = 0$