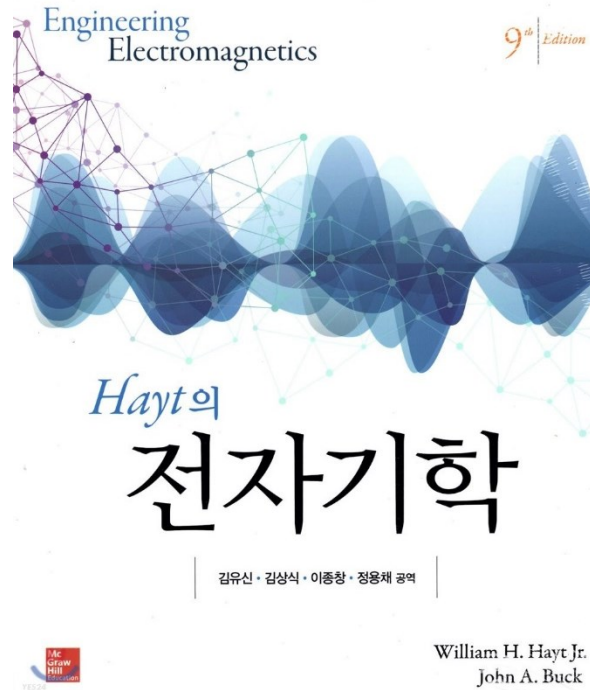


전기자기학 I

(강의자료 #2)



교과목명 : 전기자기학 I

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교재명 : Hayt의 전자기학



Ch. 2. 쿨롱의 법칙과 전계세기

*Hayt*의

전자기학

CH. 2 : 쿨롱의 법칙과 전계세기

- 1) 쿨롱의 실험 법칙
- 2) 전계의 세기
- 3) 연속적인 체적전하분포에 의한 전계
- 4) 선전하에 의한 전계
- 5) 면전하에 의한 전계
- 6) 전계의 유선(streamline)과 스케치

2.1 쿨롱의 실험 법칙 - 정의

- 쿨롱의 법칙 (Coulomb's Law)

진공 또는 자유공간 내에서 두 개의 하전 입자 사이의 힘은 각각의 입자가 가지고 있는 전하량에 비례하고 그들 사이의 거리의 제곱에 반비례한다.

$$F = k \frac{Q_1 Q_2}{R^2}$$

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$$

뉴턴(newton, N)

Q_1, Q_2 : 물체들이 갖는 전하량, 쿨롱 (coulomb, C)

R : 물체 사이의 거리, 미터(meter, m)

k : 비례상수

$k = \frac{1}{4\pi\epsilon_0}$ 자유 공간의 유전율(permittivity), (farad/meter, F/m)

$$\epsilon_0 = 8.854 \times 10^{-12} \approx \frac{1}{36\pi} \times 10^{-9} \text{ [F/m]}$$

- 전하의 단위; C(쿨롱, Coulomb)

- 전자의 전하량 $e = -1.602 \times 10^{-19} \text{ C}$

- $1 \text{ C} = -6 \times 10^{18} eC = (6 \times 10^{18})(1.602 \times 10^{-19}) \text{ C}$

- $F = \frac{1}{4\pi\epsilon_0} \frac{1C \times 1C}{(1m)^2} = 9 \times 10^9 N \approx 100\text{만톤}$

2.1 쿨롱의 실험 법칙 - 정의

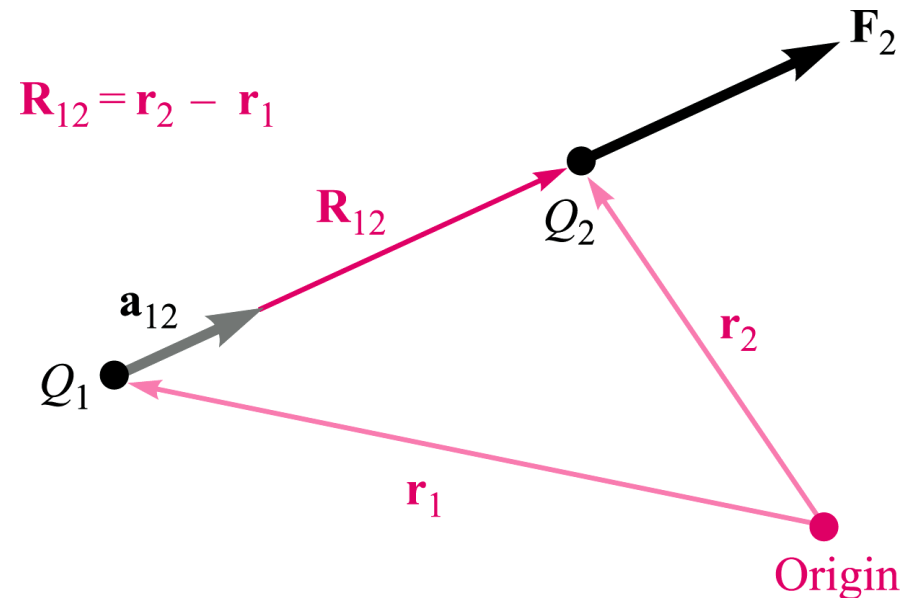
- 전기력의 벡터 표시

➤ Q_1, Q_2 의 부호가 같으면 Q_2 에 작용하는 힘 $\mathbf{F}_2$ 의 방향은 벡터 $\mathbf{R}_{12}$ 의 방향과 같다.

❖ $\mathbf{F}_2 = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{12}^2} \mathbf{a}_{12}$

✓ 여기서, $\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|}$

❖ $\mathbf{F}_1 = \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{21}^2} \mathbf{a}_{21}$
 $= \frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{R_{12}^2} (-\mathbf{a}_{12}) = -\mathbf{F}_2$



2.1 쿨롱의 실험 법칙 - 예제

- (예제 2.1) 점 $M(1,2,3)$ 에 $Q_1 = 3 \times 10^{-4}[\text{C}]$, 점 $N(2,0,5)$ 에 $Q_2 = -10^{-4}[\text{C}]$ 인 전하가 자유공간에 놓여져 있다. Q_1 에 의해서 Q_2 에 가해지는 힘을 구하라.

$$\text{➤ } \mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = (2 - 1)\mathbf{a}_x + (0 - 2)\mathbf{a}_y + (5 - 3)\mathbf{a}_z = \mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$$

$$\text{➤ } \mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3}$$

$$\begin{aligned}\therefore \mathbf{F}_2 &= \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{12} \\ &= \frac{3 \times 10^{-4} (-10^{-4})}{4\pi(1/36\pi)10^{-9} \times 9} \left(\frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3} \right) \\ &= -30 \left(\frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3} \right) = -10\mathbf{a}_x + 20\mathbf{a}_y - 20\mathbf{a}_z [\text{N}]\end{aligned}$$

$$\text{➤ } \mathbf{F}_1 = -\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2} \mathbf{a}_{21} = -\frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{12}$$

2.1 쿨롱의 실험 법칙 - 예제

- (ex 1) $Q_1 = 2mC$ at $P_1(3, -2, -4)$, $Q_2 = -5\mu C$ at $P_2(1, -4, 2)$

➤ (a) $\mathbf{R}_{12}$ (b) $|\mathbf{F}_1|$

➤ (a) $\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = -2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{(-2)^2 + (-2)^2 + 6^2}} = \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{44}}$$

$$\therefore \mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \mathbf{a}_{12} \text{ [N]}$$

$$= \frac{2 \times 10^{-3} \times (-5) \times 10^{-6}}{4\pi(1/36\pi)10^{-9} \times 44} \cdot \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{44}} \text{ [N]}$$

$$= 2.045 \frac{2\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z}{\sqrt{44}} \text{ [N]}$$

$$= 0.616\mathbf{a}_x + 0.616\mathbf{a}_y - 1.848\mathbf{a}_z \text{ [N]}$$

➤ (b) $|\mathbf{F}_1| = |\mathbf{F}_2| = \sqrt{0.616^2 + 0.616^2 + (-1.848)^2} = 2.04 \text{ [N]}$

2.1 쿨롱의 실험 법칙 - 예제

- (응용예제 2.1) $Q_A = 20\mu C$ at $A(-6,4,7)$, $Q_B = 50\mu C$ at $B(5,8,-2)$
 - (a) $\mathbf{R}_{AB}$ (b) R_{AB} (c) $\mathbf{F}_A$

2.2 전계의 세기

- 전하 Q_1 의 위치를 고정하면 제2의 전하 Q_2 는 어떠한 위치에서도 힘을 받는다.

→ Q_1 에 의한 힘의 장(계)(force field)이 존재

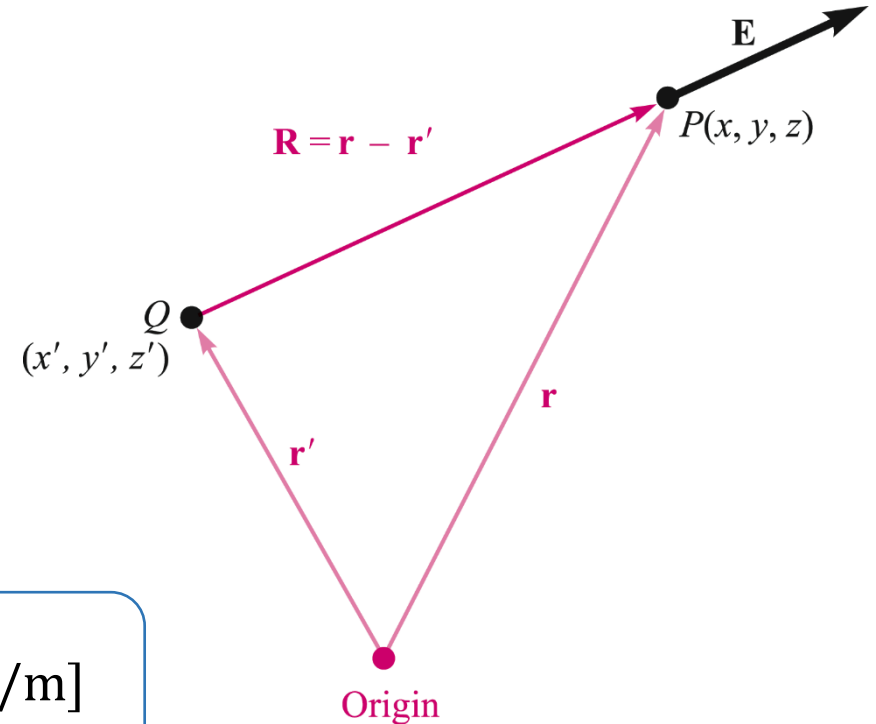
→ Q_2 를 Q_t 로 대체하면 $\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\epsilon_0 R_{1t}^2} \mathbf{a}_{1t}$

- 시험전하의 단위전하당 작용하는 힘

➤ $\frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \mathbf{a}_{1t} \rightarrow$ 전계세기(electric field intensity) $\mathbf{E}$

➤ 전계의 세기 (the electric field intensity: 전하 → 전기장)

$$\therefore \mathbf{E} = \frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \mathbf{a}_{1t} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R \text{ [V/m]}$$



2.2 전계의 세기

- (ex 2) Q_1 at $O(r = 0, \theta = 0, \phi = 0) \rightarrow r = r_1$ 인 곳에서의 $\mathbf{E}$?

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R = \frac{Q_1}{4\pi\epsilon_0 r_1^2} \mathbf{a}_r$$

- (ex 3) Q at $O(x = 0, y = 0, z = 0) \rightarrow P(x = x_1, y = y_1, z = z_1)$ 에서의 $\mathbf{E}$?

$$\mathbf{R} = x_1 \mathbf{a}_x + y_1 \mathbf{a}_y + z_1 \mathbf{a}_z$$

$$\mathbf{a}_R = \frac{x_1 \mathbf{a}_x + y_1 \mathbf{a}_y + z_1 \mathbf{a}_z}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$$

$$\therefore \mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R = \frac{Q}{4\pi\epsilon_0 (x_1^2 + y_1^2 + z_1^2)} \cdot \frac{x_1 \mathbf{a}_x + y_1 \mathbf{a}_y + z_1 \mathbf{a}_z}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$$

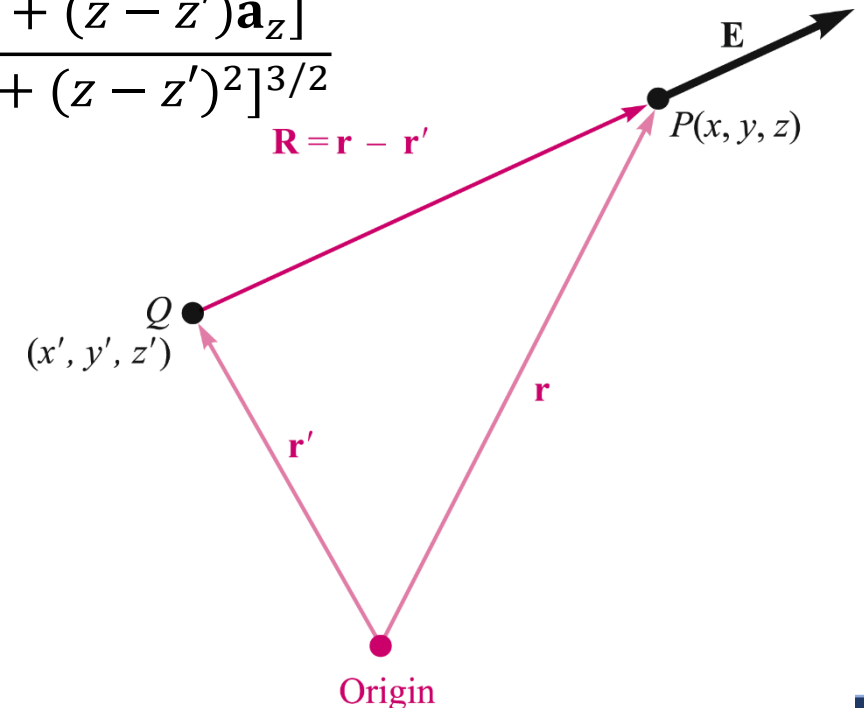
2.2 전계의 세기

- (ex 4) Q at $O(x', y', z') \rightarrow P(x, y, z)$ 에서의 $\mathbf{E}$?

$$\mathbf{R} = \mathbf{r}_P - \mathbf{r}_O = (x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z$$

$$\mathbf{a}_R = \frac{\mathbf{R}}{R} = \frac{(x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$

$$\therefore \mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_R = \frac{Q}{4\pi\epsilon_0 R^3} \mathbf{R} = \frac{Q[(x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z]}{4\pi\epsilon_0 [(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}}$$

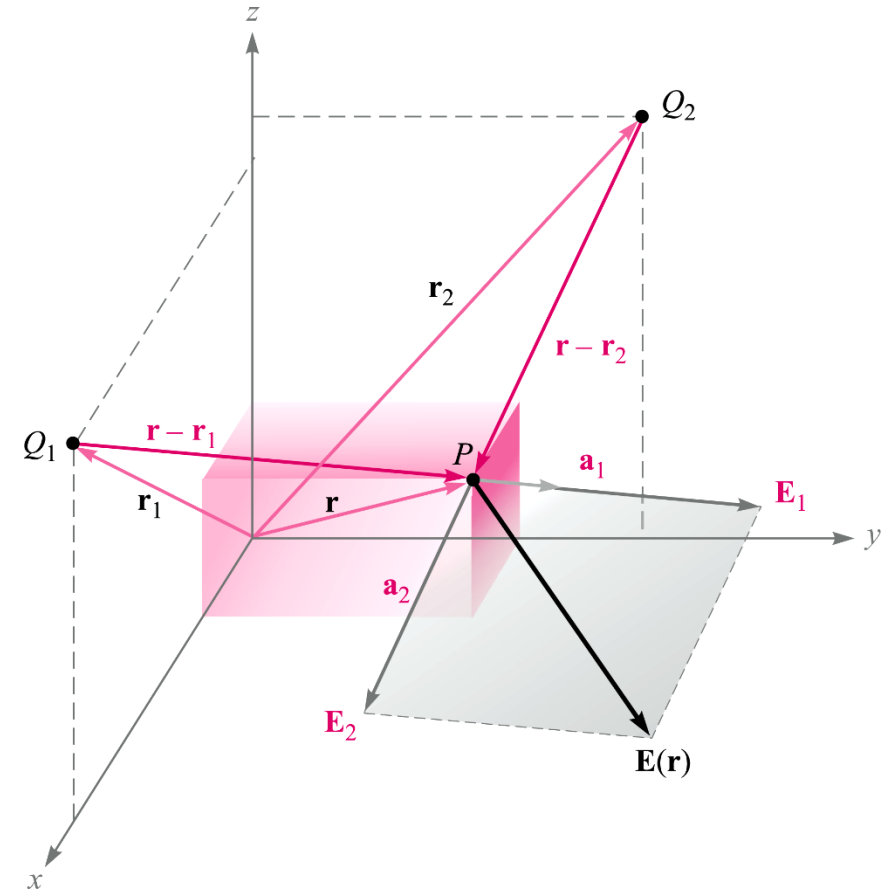


2.2 전계의 세기

- 중첩의 원리 (Superposition)

➤ 두 점전하에 의한 전계의 세기는 각각 단독으로 있을 때 작용하는 힘을 더한 것과 같다.

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{Q_1}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_1|} \mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_2|} \mathbf{a}_2$$



2.2 전계의 세기

- Q_1 at r_1, Q_2 at $r_2, \dots, Q_n$ at $r_n \rightarrow r$ 인 곳에서의 $\mathbf{E}$?

➤ $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \dots + \mathbf{E}_n$

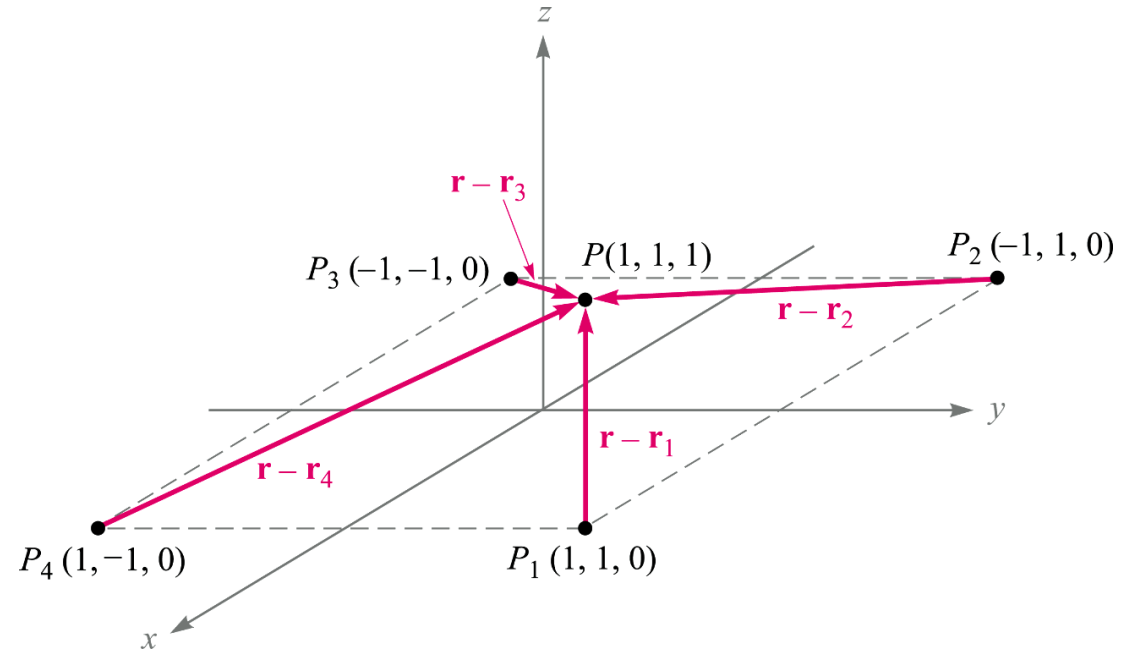
$$= \frac{Q_1}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_1|} \mathbf{a}_1 + \frac{Q_2}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_2|} \mathbf{a}_2 + \dots + \frac{Q_n}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_n|} \mathbf{a}_n$$

$$= \sum_{m=1}^n \frac{Q_m}{4\pi\epsilon_0|\mathbf{r} - \mathbf{r}_m|} \mathbf{a}_m$$

2.2 전계의 세기

- (예제 2.2) 3 nC at $P_1(1,1,0)$, 3 nC at $P_2(-1,1,0)$, 3 nC at $P_3(-1,-1,0)$, 3 nC at $P_4(1,-1,0)$, $\rightarrow \mathbf{E} = ?$ at $P(1,1,1)$

$$\begin{aligned}\mathbf{E} &= \sum_{m=1}^4 \mathbf{E}_m \\ &= \sum_{m=1}^4 \frac{Q_m}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_m|} \mathbf{a}_m \\ &= \sum_{m=1}^4 \frac{Q_m}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}_m|} \frac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|} = \sum_{m=1}^4 \frac{Q_m}{4\pi\epsilon_0} \frac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|^2} \\ &= \frac{Q}{4\pi\epsilon_0} \left[\frac{\mathbf{r} - \mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|^2} + \frac{\mathbf{r} - \mathbf{r}_2}{|\mathbf{r} - \mathbf{r}_2|^2} + \frac{\mathbf{r} - \mathbf{r}_3}{|\mathbf{r} - \mathbf{r}_3|^2} + \frac{\mathbf{r} - \mathbf{r}_4}{|\mathbf{r} - \mathbf{r}_4|^2} \right]\end{aligned}$$



2.2 전계의 세기

$$\triangleright \mathbf{E} = \sum_{m=1}^4 \mathbf{E}_m = \frac{Q}{4\pi\epsilon_0} \left[\frac{\mathbf{r}-\mathbf{r}_1}{|\mathbf{r}-\mathbf{r}_1|} + \frac{\mathbf{r}-\mathbf{r}_2}{|\mathbf{r}-\mathbf{r}_2|} + \frac{\mathbf{r}-\mathbf{r}_3}{|\mathbf{r}-\mathbf{r}_3|} + \frac{\mathbf{r}-\mathbf{r}_4}{|\mathbf{r}-\mathbf{r}_4|} \right]$$

$$\text{where, } \mathbf{r} - \mathbf{r}_1 = \mathbf{a}_z \quad |\mathbf{r} - \mathbf{r}_1| = 1$$

$$\mathbf{r} - \mathbf{r}_2 = 2\mathbf{a}_x + \mathbf{a}_z \quad |\mathbf{r} - \mathbf{r}_2| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\mathbf{r} - \mathbf{r}_3 = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z \quad |\mathbf{r} - \mathbf{r}_3| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

$$\mathbf{r} - \mathbf{r}_4 = 2\mathbf{a}_y + \mathbf{a}_z \quad |\mathbf{r} - \mathbf{r}_4| = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\begin{aligned} \therefore \mathbf{E} &= \frac{3 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12}} \left[\frac{\mathbf{a}_z}{1^3} + \frac{2\mathbf{a}_x + \mathbf{a}_z}{\sqrt{5}} + \frac{2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z}{3^3} + \frac{2\mathbf{a}_y + \mathbf{a}_z}{\sqrt{5}} \right] \\ &= 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z \text{ [V/m]} \end{aligned}$$

2.2 전계의 세기

- (ex 7) $\mathbf{E}$ at $M(3, -4, 2)$?

➤ (a) $Q_1 = 2\mu\text{C}$ at $P_1(0, 0, 0)$

$$\mathbf{r}_M - \mathbf{r}_1 = 3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z$$

$$|\mathbf{r}_M - \mathbf{r}_1| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$$

$$\begin{aligned}\therefore \mathbf{E}_{1M} &= \frac{Q}{4\pi\epsilon_0 R_{1M}^2} \mathbf{a}_R = \frac{Q_1}{4\pi\epsilon_0 |\mathbf{r}_M - \mathbf{r}_1|} \frac{\mathbf{r}_M - \mathbf{r}_1}{|\mathbf{r}_M - \mathbf{r}_1|} \\ &= \frac{2 \times 10^{-6}}{4\pi \cdot 8.854 \times 10^{-12} \cdot 29} \frac{3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{29}} \\ &= 620 \frac{3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{29}} \text{ [V/m]} \\ &= 345\mathbf{a}_x - 460\mathbf{a}_y + 230\mathbf{a}_z \text{ [V/m]}\end{aligned}$$

2.2 전계의 세기

➤ (b) $Q_2 = 3\mu\text{C}$ at $P_2(-1,2,3)$

$$\mathbf{r}_M - \mathbf{r}_2 = 4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{r}_M - \mathbf{r}_2| = \sqrt{4^2 + (-6)^2 + (-1)^2} = \sqrt{53}$$

$$\begin{aligned}\therefore \mathbf{E}_{2M} &= \frac{Q}{4\pi\epsilon_0 R_{2M}^2} \mathbf{a}_R = \frac{Q_1}{4\pi\epsilon_0 |\mathbf{r}_M - \mathbf{r}_2|} \frac{\mathbf{r}_M - \mathbf{r}_2}{|\mathbf{r}_M - \mathbf{r}_2|} \\ &= \frac{3 \times 10^{-6}}{4\pi \cdot 8.854 \times 10^{-12} \cdot 53} \frac{4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z}{\sqrt{53}} \\ &= 509 \frac{4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z}{\sqrt{53}} \text{ [V/m]} \\ &= 280\mathbf{a}_x - 419\mathbf{a}_y - 69.9\mathbf{a}_z \text{ [V/m]}\end{aligned}$$

2.2 전계의 세기

➤ (c) $Q_1 = 2\mu C$ at $P_1(0,0,0)$ and $Q_2 = 3\mu C$ at $P_2(-1,2,3)$

$$\mathbf{E}_M = \mathbf{E}_{1M} + \mathbf{E}_{2M}$$

$$= (345\mathbf{a}_x - 460\mathbf{a}_y + 230\mathbf{a}_z) + (280\mathbf{a}_x - 419\mathbf{a}_y - 69.9\mathbf{a}_z)$$

$$= 625\mathbf{a}_x - 879\mathbf{a}_y + 160.1\mathbf{a}_z \text{ [V/m]}$$

2.2 전계의 세기

- (응용예제 2.2) $Q_A = -0.3\mu C$ at $A(25, -30, 15)$ cm,
 $Q_B = 0.5\mu C$ at $B(-10, 8, 12)$ cm,
➤ $\mathbf{E} = ?$ at (a) 원점 (b) $P(15, 20, 50)$ cm

- (응용예제 2.3)

➤ (a) $\sum_{m=0}^5 \frac{1+(-1)^m}{m^2+1}$

(b) $\sum_{m=1}^4 \frac{(0.1)^m + 1}{(4+m^2)^{1.5}}$

2.3 연속적인 체적전하분포에 의한 전기

- 전하가 점이 아닌 부피를 가지고 분포 $\rightarrow$ 체적전하밀도(ρ_v [C/m³])로 표시

$$\rho_v = \frac{Q}{v} \rightarrow \rho_v = \lim_{\Delta v \rightarrow 0} \frac{\Delta Q}{\Delta v} \rightarrow \rho_v = \frac{\Delta Q}{\Delta v} = \frac{dQ}{dv}$$

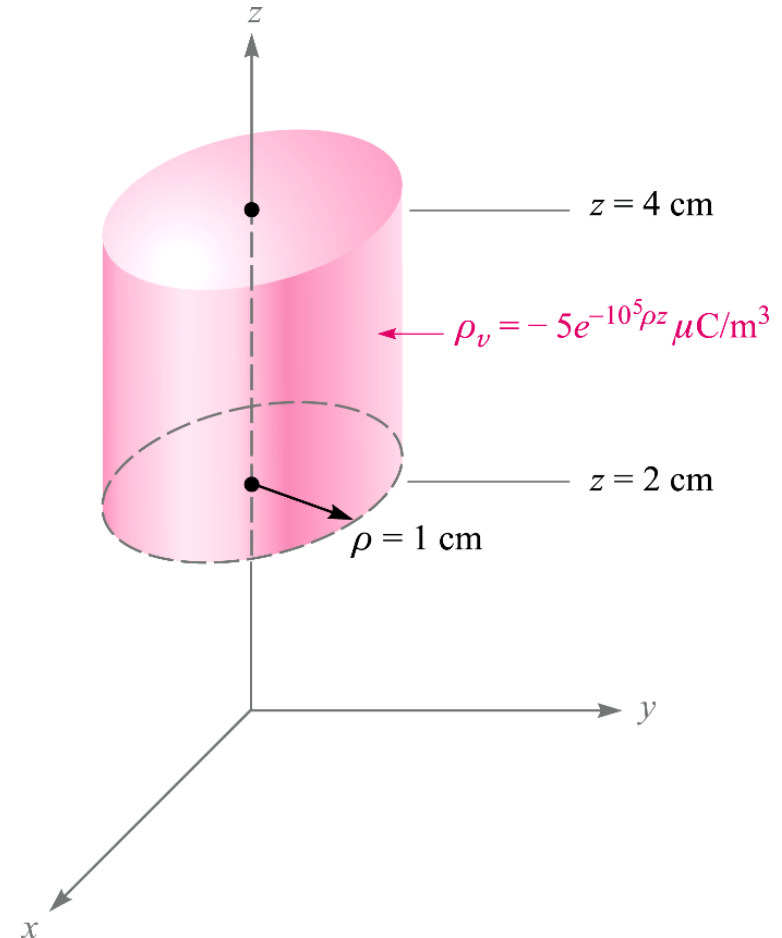
$$\therefore Q = \int_{vol} dQ = \int_{vol} \rho_v dv \text{ [C]}$$

$$\begin{aligned} \rightarrow \mathbf{E} &= \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_r \\ &= \frac{Q}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \\ &= \frac{\int_{vol} \rho_v(\mathbf{r}') dv'}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|} \\ &= \int_{vol} \frac{\rho_v(\mathbf{r}') dv' (\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3} \end{aligned}$$

2.3 연속적인 체적전하분포에 의한 전기

- (예제 2.3) 그림 2.5와 같이 체적전하밀도가 $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z} [\text{C/m}^3]$ 이고, 길이 2 cm, 반지름 1 cm인 전자빔의 총전하량 Q ?

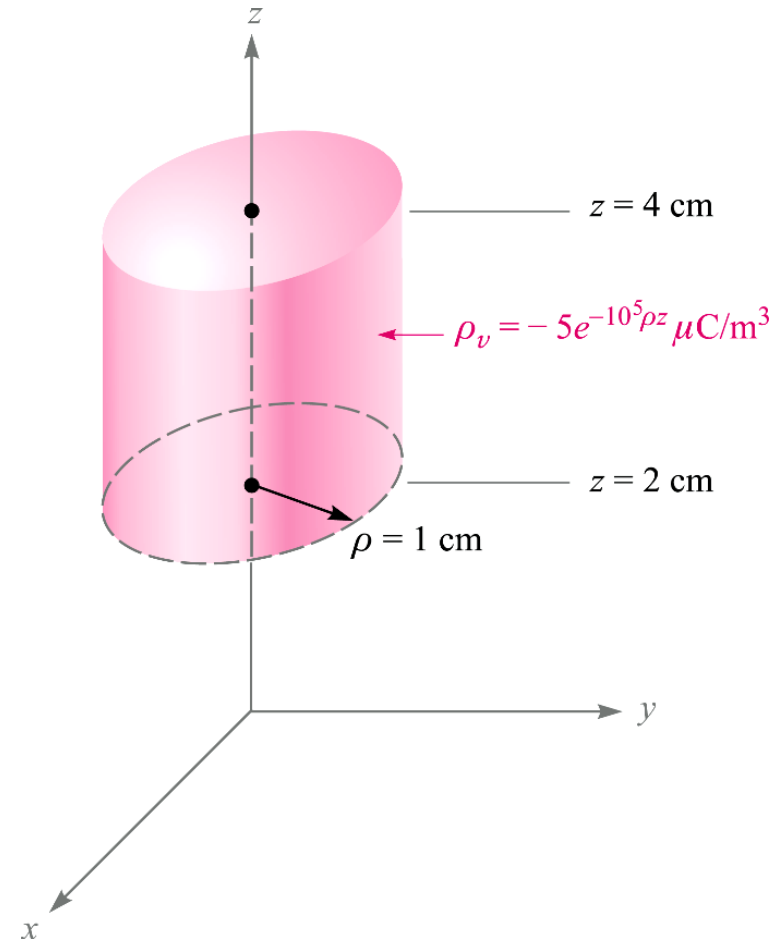
$$\begin{aligned}
 Q &= \int_{vol} dQ = \int_{vol} \rho_v dv \\
 &= \int_{vol} -5 \times 10^{-6} e^{-10^5 \rho z} \rho d\rho d\phi dz \\
 &= \int_{z=0.02}^{z=0.04} \int_{\phi=0}^{\phi=2\pi} \int_{\rho=0}^{\rho=0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho d\rho d\phi dz \\
 &= -5 \times 10^{-6} \int_{z=0.02}^{z=0.04} \int_{\rho=0}^{\rho=0.01} e^{-10^5 \rho z} \rho d\rho \left(\int_{\phi=0}^{\phi=2\pi} d\phi \right) dz \\
 &= -5 \times 10^{-6} \int_{z=0.02}^{z=0.04} \int_{\rho=0}^{\rho=0.01} e^{-10^5 \rho z} \cdot 2\pi \rho d\rho dz \\
 &= -10^{-5} \pi \int_{\rho=0}^{\rho=0.01} \rho \left(\int_{z=0.02}^{z=0.04} e^{-10^5 \rho z} \right) dz d\rho \\
 &= -10^{-5} \pi \int_{\rho=0}^{\rho=0.01} \rho \left[\frac{1}{-10^5 \rho} e^{-10^5 \rho z} \right] d\rho
 \end{aligned}$$



2.3 연속적인 체적전하분포에 의한 전기

- (예제 2.3) 그림 2.5와 같이 체적전하밀도가 $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z} \text{ [C/m}^3\text{]}$ 이고, 길이 2 cm, 반지름 1 cm인 전자빔의 총전하량 Q ?

$$\begin{aligned} Q &= \int_{vol} dQ = \int_{vol} \rho_v dv \\ &= -10^{-5} \pi \int_{\rho=0}^{\rho=0.01} \rho \left[\frac{1}{-10^5 \rho} e^{-10^5 \rho z} \right] d\rho \\ &= 10^{-10} \pi \int_{\rho=0}^{\rho=0.01} (e^{-4000 \rho} - e^{-2000 \rho}) d\rho \\ &= 10^{-10} \pi \left[\frac{e^{-4000 \rho}}{-4000} - \frac{e^{-2000 \rho}}{-2000} \right] \\ &= 10^{-10} \pi \left\{ \left(\frac{e^{-40}}{-4000} - \frac{e^{-20}}{-2000} \right) - \left(\frac{1}{-4000} - \frac{1}{-2000} \right) \right\} \\ &\approx 10^{-10} \pi \left(\frac{1}{4000} - \frac{1}{2000} \right) = 10^{-10} \pi \cdot \left(-\frac{1}{4000} \right) \\ &= -\frac{\pi}{40} 10^{-12} = -\frac{\pi}{40} \text{ pC} = -0.0785 \text{ pC} \end{aligned}$$



2.3 연속적인 체적전하분포에 의한 전기

- (응용예제 2.4) $Q = ?$

- (a) $0.1 \leq |x|, |y|, |z| \leq 0.2; \rho_v = \frac{1}{x^3 y^3 z^3}$

- (b) $0 \leq \rho \leq 0.1, 0 \leq \phi \leq \pi, 2 \leq \rho \leq 4; \rho_v = \rho^2 z^2 \sin 0.6\phi$

- (c) 우주; $\rho_v = \frac{e^{-2r}}{r^2}$

2.4 선전하에 의한 전기

- 전하가 점이 아닌 라인 위에 분포 → 선전하밀도(ρ_L [C/m])로 표시

$$\rho_L = \frac{Q}{l} \rightarrow \rho_L = \frac{\Delta Q}{\Delta l} \rightarrow \rho_L = \frac{dQ}{dl}$$

$$\therefore Q = \int_l dQ = \int_l \rho_L dl$$

$$\rightarrow \mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2} \mathbf{a}_r = \frac{Q}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^2} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

$$= \frac{\int_l \rho_L(\mathbf{r}') dl'}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^2} \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

$$= \int_l \frac{\rho_L(\mathbf{r}') dl' (\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

2.4 선전하에 의한 전기

- z 축의 $-\infty$ 에서 $+\infty$ 까지 선전하밀도 ρ_L 의 전하가 균일하게 분포되어 있을 때, z 축에서 ρ 만큼 떨어진 곳에서의 전기세기?

$$d\mathbf{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \mathbf{a}_R$$

$$= \frac{dQ(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

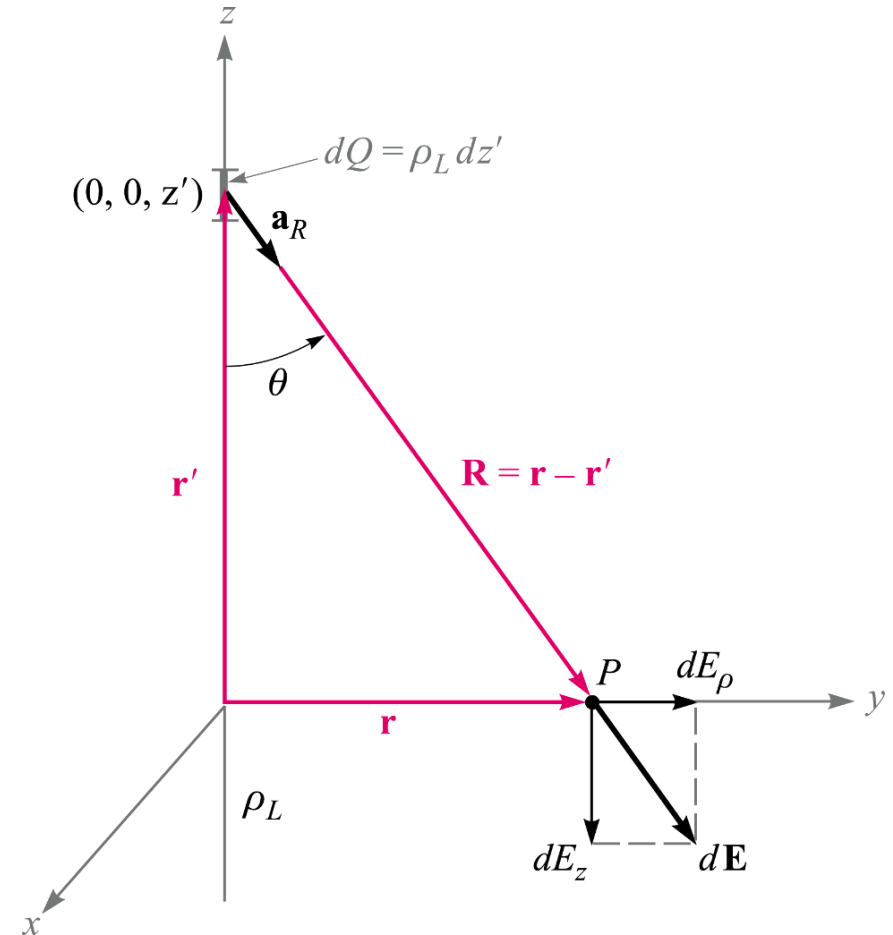
$$= \frac{\rho_L dz'(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

$$dQ = \rho_L dz'$$

$$\mathbf{r} = \rho \mathbf{a}_\rho = y \mathbf{a}_y$$

$$\mathbf{r}' = z' \mathbf{a}_z$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \rho \mathbf{a}_\rho - z' \mathbf{a}_z$$



2.4 선전하에 의한 전기

$$d\mathbf{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \mathbf{a}_R = \frac{dQ(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3} = \frac{\rho_L dz'(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

$$d\mathbf{E} = \frac{\rho_L dz'(\rho \mathbf{a}_\rho - z' \mathbf{a}_z)}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$dE_\rho = \frac{\rho_L \rho dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$= \frac{\rho_L \rho}{4\pi\epsilon_0 \rho^2} \left(\frac{z'}{\sqrt{\rho^2 + z'^2}} \right)_{-\infty}^{\infty}$$

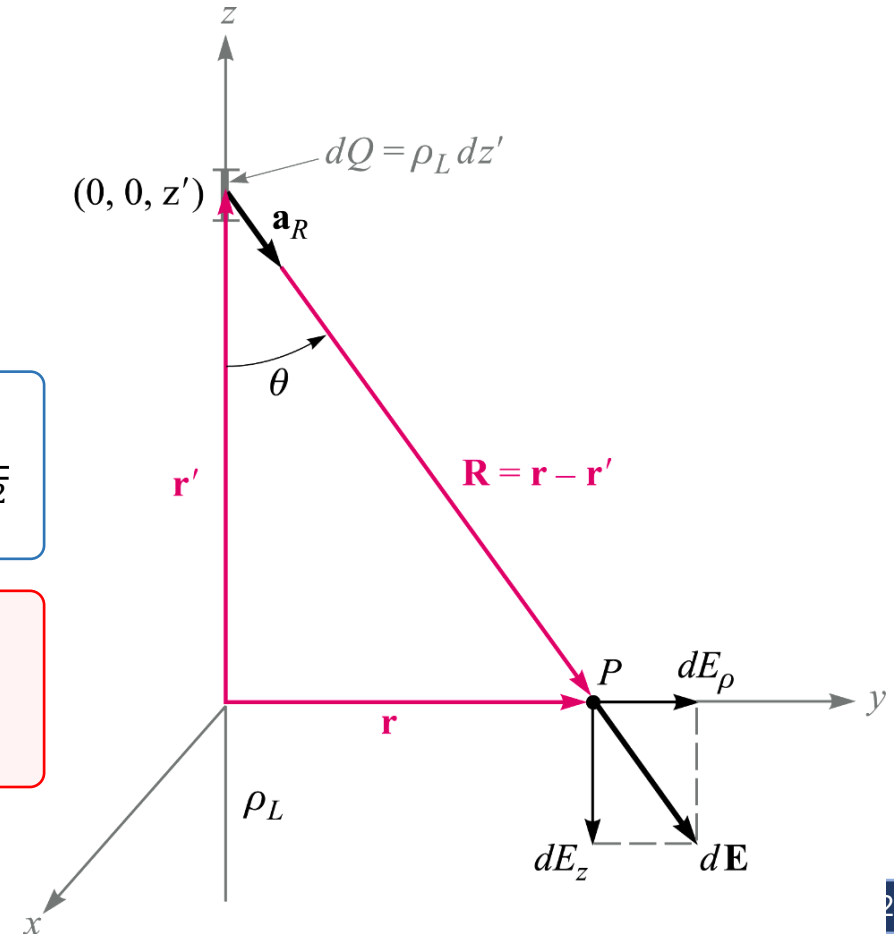
$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a\sqrt{a^2 + x^2}}$$

$$E_\rho = \frac{\rho_L}{2\pi\epsilon_0 \rho} \text{ [V/m]}$$

$$dQ = \rho_L dz'$$

$$\mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho, \quad \mathbf{r}' = z'\mathbf{a}_z$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z$$



2.4 선전하에 의한 전계

$$d\mathbf{E} = \frac{dQ}{4\pi\epsilon_0 R^2} \mathbf{a}_R = \frac{dQ(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3} = \frac{\rho_L dz'(\mathbf{r} - \mathbf{r}')}{4\pi\epsilon_0 |\mathbf{r} - \mathbf{r}'|^3}$$

$$dQ = \rho_L dz'$$

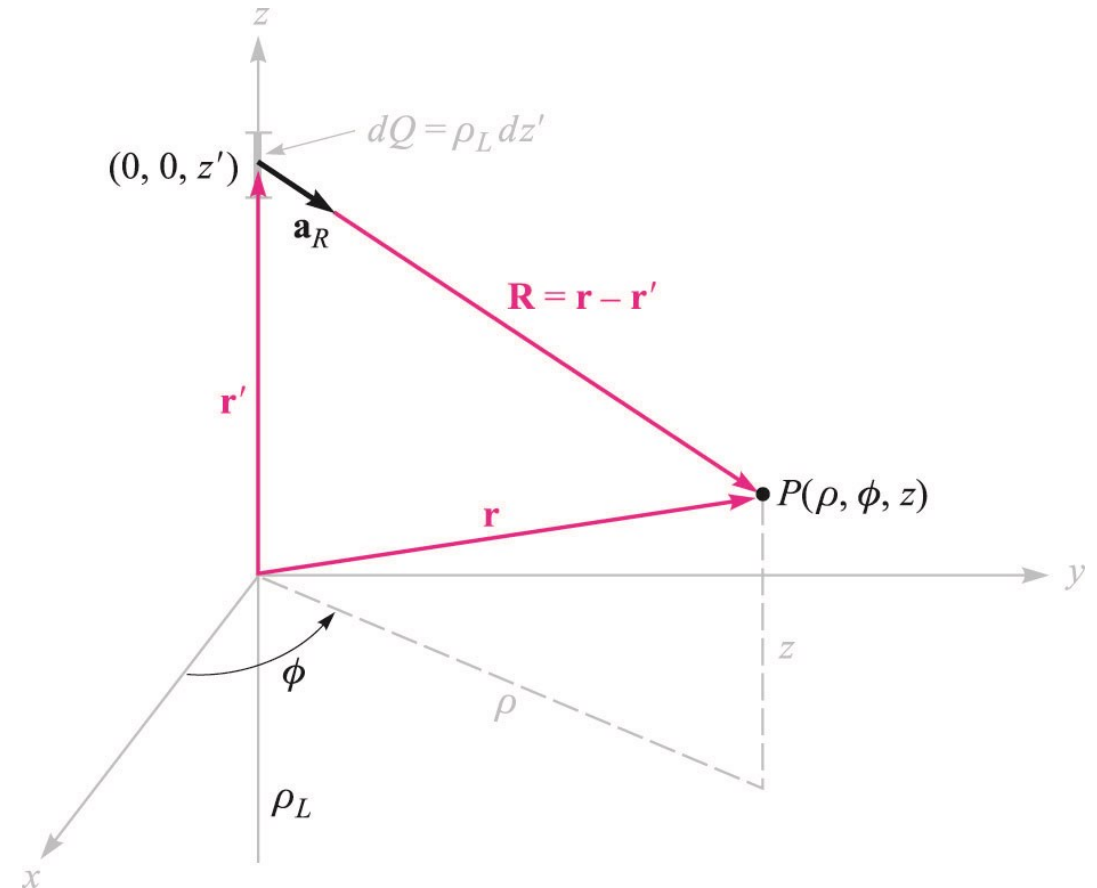
$$\mathbf{r} = \rho \mathbf{a}_\rho + z \mathbf{a}_z$$

$$\mathbf{r}' = z' \mathbf{a}_z$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \rho \mathbf{a}_\rho + (z - z') \mathbf{a}_z$$

$$R = \sqrt{\rho^2 + (z - z')^2}$$

$$\mathbf{a}_R = \frac{\rho \mathbf{a}_\rho + (z - z') \mathbf{a}_z}{\sqrt{\rho^2 + (z - z')^2}}$$



2.4 선전하에 의한 전기

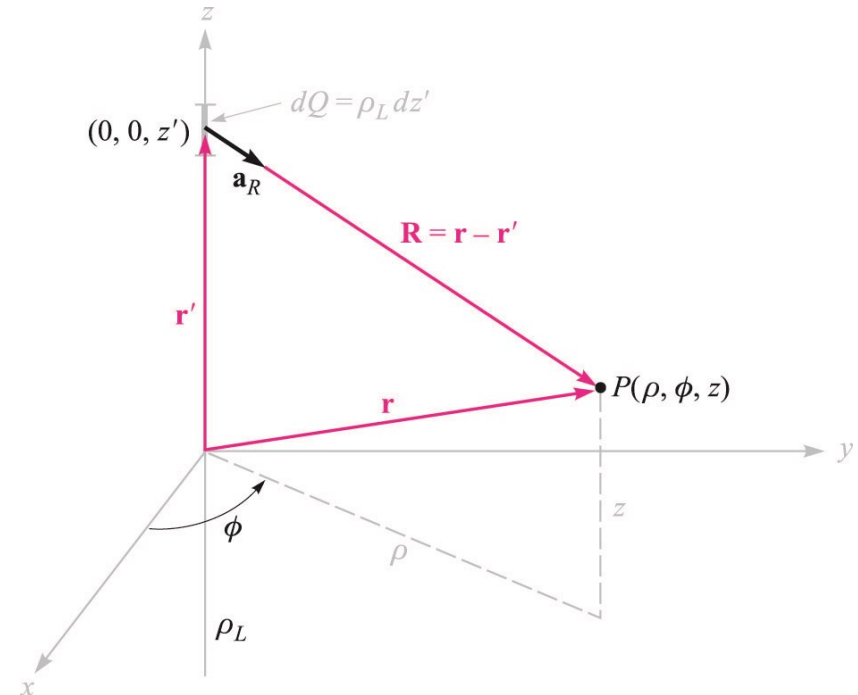
$$\mathbf{E} = \int_{-\infty}^{\infty} \frac{\rho_L dz' [\rho \mathbf{a}_\rho + (z - z') \mathbf{a}_z]}{4\pi\epsilon_0 [\rho^2 + (z - z')^2]^{3/2}} = \frac{\rho_L}{4\pi\epsilon_0} \left\{ \int_{-\infty}^{\infty} \frac{\rho dz' \mathbf{a}_\rho}{[\rho^2 + (z - z')^2]^{3/2}} + \int_{-\infty}^{\infty} \frac{(z - z') dz' \mathbf{a}_z}{[\rho^2 + (z - z')^2]^{3/2}} \right\}$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left\{ \left[\rho \frac{1}{\rho^2} \frac{-(z - z')}{\sqrt{\rho^2 + (z - z')^2}} \mathbf{a}_\rho \right]_{-\infty}^{\infty} + \left[\frac{\mathbf{a}_z}{\sqrt{\rho^2 + (z - z')^2}} \right]_{-\infty}^{\infty} \right\}$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \left[\frac{2}{\rho} \mathbf{a}_\rho \right] = \frac{\rho_L}{2\pi\epsilon_0 \rho} \mathbf{a}_\rho$$

$$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a\sqrt{a^2 + x^2}}$$

$$\mathbf{E} = \frac{\rho_L}{2\pi\epsilon_0 \rho} \mathbf{a}_\rho \quad [\text{V/m}]$$



2.4 선전하에 의한 전기

- (응용예제 2.5) $\rho_L = 5 \text{ [nC/m]}$ at x 축 & y 축
 - $\mathbf{E} = ?$ at (a) $P_A(0,0,4)$ (b) $P_B(0,3,4)$

2.5 면전하(판전하)에 의한 전기

- 전하가 점이 아닌 판에 분포 $\rightarrow$ 면적전하밀도(ρ_s [C/m²])로 표시

$$\rho_s = \frac{Q}{S} \rightarrow \rho_s = \frac{\Delta Q}{\Delta} S \rightarrow \rho_s = \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

$$\therefore Q = \int_S dQ = \int_S \rho_s dS$$

2.5 면전하(판전하)에 의한 전기

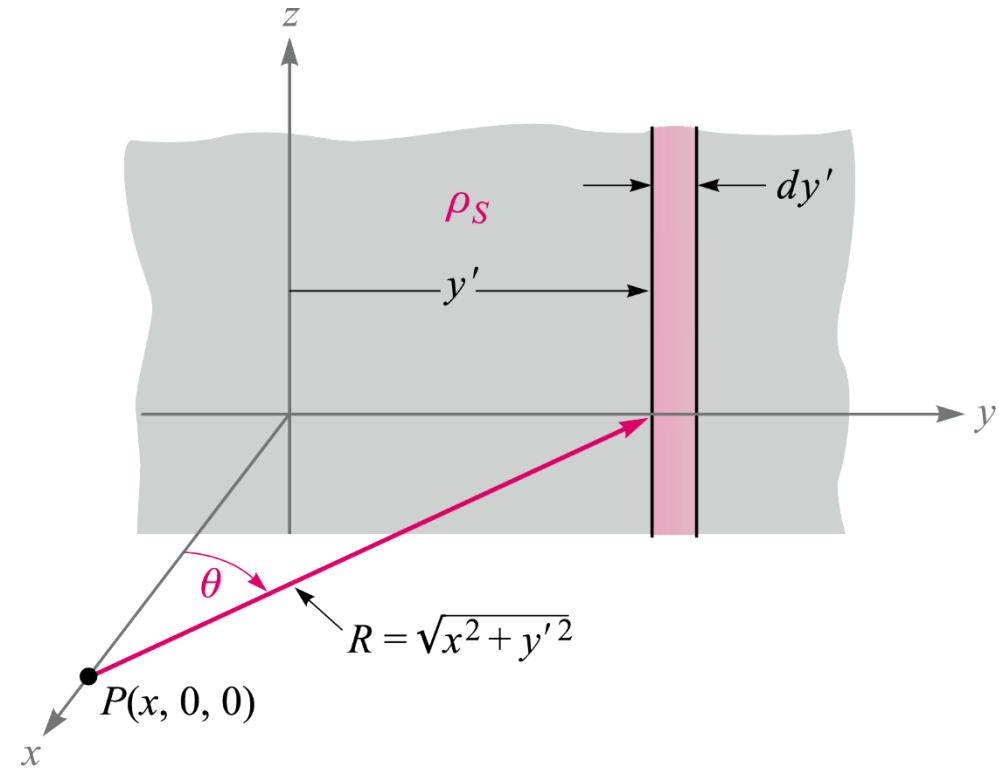
$$d\mathbf{E} = \frac{\rho_S dy'}{2\pi\epsilon_0 R} \mathbf{a}_r \quad \left(R = \sqrt{x^2 + y'^2}\right)$$

$$dE = d\mathbf{E} \cdot \mathbf{a}_x = \frac{\rho_S dy'}{2\pi\epsilon_0 \sqrt{x^2 + y'^2}} \cos \theta = \frac{\rho_S}{2\pi\epsilon_0} \frac{xdy'}{x^2 + y'^2}$$

$$E_x = \frac{\rho_S}{2\pi\epsilon_0} \int_{-\infty}^{\infty} \frac{xdy'}{x^2 + y'^2} = \frac{\rho_S}{2\pi\epsilon_0} \left[\tan^{-1} \frac{y'}{x} \right]_{-\infty}^{\infty} = \frac{\rho_S}{2\pi\epsilon_0}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\mathbf{E} = \frac{\rho_S}{2\pi\epsilon_0} \mathbf{a}_N \quad [\text{V/m}]$$



2.5 면전하(판전하)에 의한 전기

- 음의 전하밀도 $-\rho_S$ 를 갖는 또 다른 두번째 무한 넓이의 대전판이 $x = a$ 인 평면상에 위치한다면

$$\text{i) } x > a \quad \mathbf{E}_+ = \frac{\rho_S}{2\epsilon_0} \mathbf{a}_x \quad \mathbf{E}_- = -\frac{\rho_S}{2\epsilon_0} \mathbf{a}_x \quad \therefore \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 0$$

$$\text{ii) } x < 0 \quad \mathbf{E}_+ = -\frac{\rho_S}{2\epsilon_0} \mathbf{a}_x \quad \mathbf{E}_- = \frac{\rho_S}{2\epsilon_0} \mathbf{a}_x \quad \therefore \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 0$$

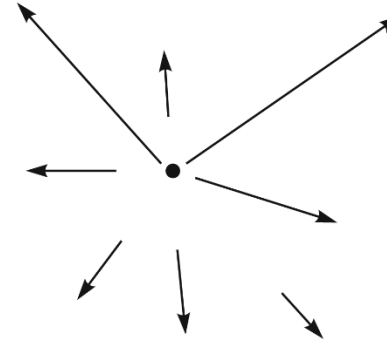
$$\text{iii) } 0 < x < a \quad \mathbf{E}_+ = \frac{\rho_S}{2\epsilon_0} \mathbf{a}_x \quad \mathbf{E}_- = \frac{\rho_S}{2\epsilon_0} \mathbf{a}_x \quad \therefore \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho_S}{\epsilon_0} \mathbf{a}_x$$

- (응용예제 2.6) $\rho_S = 3 \text{ nC/m}^2$ at $z = 1$ & $\rho_S = -8 \text{ nC/m}^2$ at $z = 4$

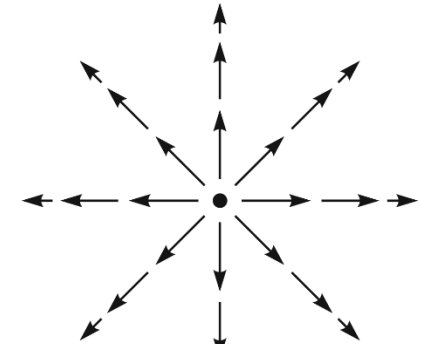
$$\text{➤ } \mathbf{E} = ? \text{ at (a) } P_A(2, 5, -5) \quad \text{(b) } P_B(4, 2, -3) \quad \text{(c) } P_C(-1, -5, 2) \quad \text{(d) } P_D(-2, 4, 5)$$

2.6 전계의 유선(streamline)과 스케치

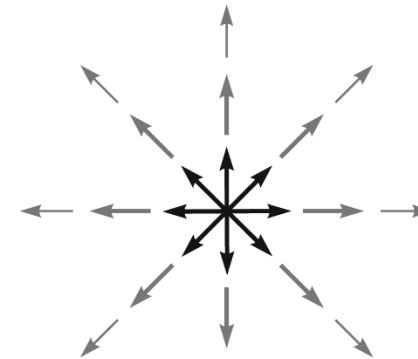
- 전기장의 선; 전기력선 또는 전력선(lines of electric force)
 - 선속 또는 속선(flux line)
 - 방향선(directional line)
 - 유선(streamline)
- 전기력선의 표시방법
 - + 전하가 있는 곳으로부터 먼 곳으로 화살표
 - 먼 곳에서 - 전하가 있는 곳으로 화살표
 - → 전기력선 사이의 간격과 전기장의 세기는 반비례



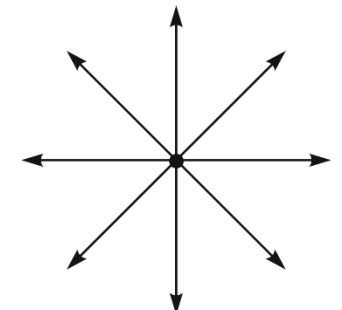
(a)



(b)



(c)



(d)