Ch. 7. 정상자계

7장. 정상자계

- 자계를 정의하고, 전류분포에 따라 형성되는 자계를 다룬다.

1) 비오사바르 법칙

2) 암페어의 주회법칙

3) 벡터회전

4) 스토크스의 정리

5) 자속과 자속밀도

6) 스칼라자위 및 벡터자위

7) 정상자계 법칙의 유도

7.1 Biot-Savart의 법칙

- 전계; 쿨롱의 법칙 → 가우스 법칙
- 자계; 비오사르 법칙 → 암페어의 주회법칙

- 정상자계(직류자계)의 근원; 영구자석

  시간에 대한 변화율이 일정한 전계

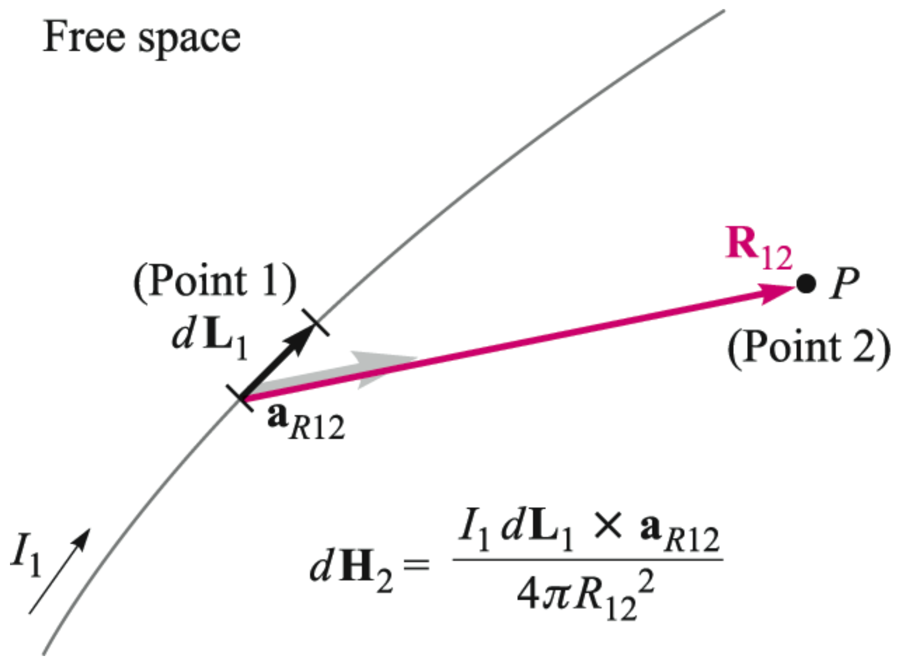
  직류전류

7.1 Biot-Savart의 법칙

- 정상자계(직류자계)의 근원
  - 영구자석
  - 시간에 대한 변화율이 일정한 전계
  - 직류전류

- 직류전류에 의한 자계의 세기
  - Biot-Savart의 실험법칙

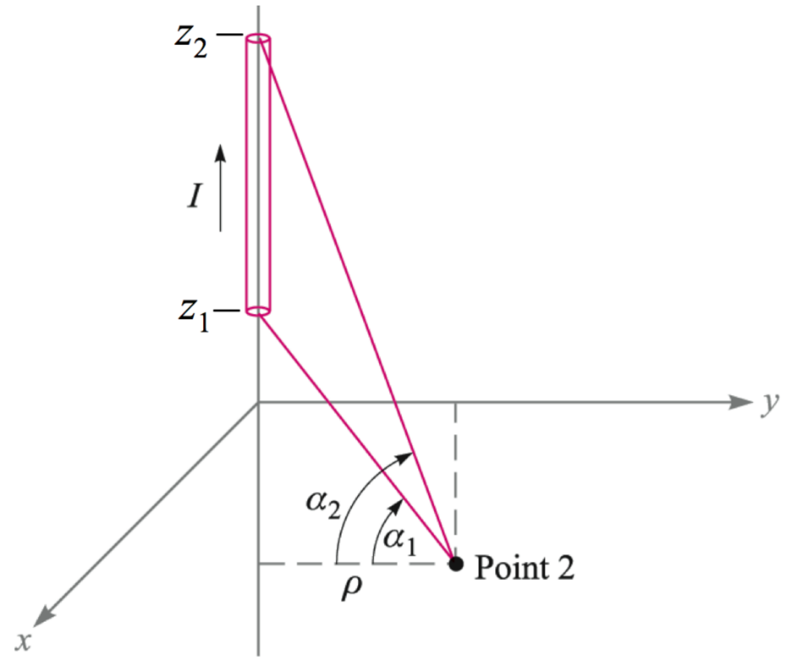
$$\mathbf{H}_2 = \oint \frac{\mathbf{I}_1 d\mathbf{L}_1 \times \mathbf{a}_\mathbf{R}}{4\pi R_{12}^2} = \oint \frac{\mathbf{I}_1 d\mathbf{L}_1 \times \mathbf{R}_{12}}{4\pi R_{12}^3} \text{ [A/m]}$$



7.1 Biot-Savart의 법칙

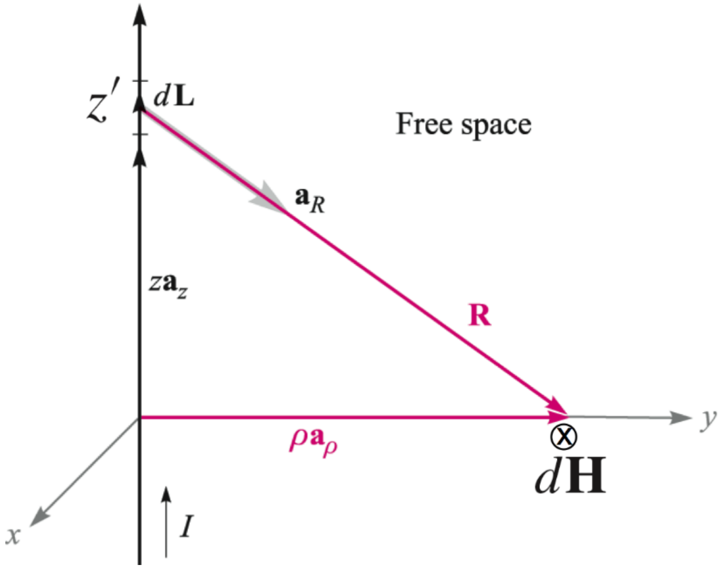
- 유한길이의 선전류에 의한 자계의 세기

$$\mathbf{H} = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\phi [\mathrm{A/m}]$$



- 무한길이의 선전류에 의한 자계의 세기(그림 7.3)

$$\mathbf{H} = \frac{I}{4\pi\rho}[\sin 90^o - \sin(-90^o)]\mathbf{a}_\phi = \frac{I}{2\pi\rho}\mathbf{a}_\phi$$



- (예제 7.1) 두 개의 반무한 선전류에 의한 자계의 세기

  - $I = 8A, P_2(0.4,0.3,0) \rightarrow \mathbf{H}_2 = ?$

$$\mathbf{H}_2 = \mathbf{H}_{2(x)} + \mathbf{H}_{2(y)}$$

$$\alpha_{1x} = -90^o, \qquad \alpha_{2x} = \tan^{-1}(0.4/0.3) = 53.1^o, \qquad \rho_x = 0.3$$

$$\mathbf{H}_{2(x)} = \frac{I_x}{4\pi\rho_x}(\sin\alpha_{2x} - \sin\alpha_{1x})\mathbf{a}_\phi$$

$$= \frac{8}{4\pi \times 0.3}(\sin 53.1^0 - \sin(-90^0))\mathbf{a}_\phi$$

$$= \frac{2}{0.3\pi}(1.8)\mathbf{a}_\phi = \frac{12}{\pi}\mathbf{a}_\phi$$

$$= -\frac{12}{\pi}\mathbf{a}_z[\mathrm{A/m}]$$

7.1 Biot-Savart의 법칙

$$\alpha_{1y} = -\tan^{-1}(0.3/0.4) = -36.9^o, \qquad \alpha_{2y} = 90^o, \qquad \rho_y = 0.4$$

$$\mathbf{H}_{2(y)} = \frac{I_y}{4\pi\rho_y}(\sin\alpha_{2y} - \sin\alpha_{1y})\mathbf{a}_\phi$$

$$= \frac{8}{4\pi \times 0.4}(\sin 90^o - \sin(-36.9^0))\mathbf{a}_\phi$$

$$= \frac{2}{0.4\pi}(1.6)\mathbf{a}_\phi = \frac{8}{\pi}\mathbf{a}_\phi$$

$$= -\frac{8}{\pi}\mathbf{a}_z \ \ [\mathrm{A/m}]$$

$$\therefore \mathbf{H}_2 = \left(\frac{-12}{\pi}\right)\mathbf{a}_z + \left(\frac{-8}{\pi}\right)\mathbf{a}_z$$

$$= -\frac{20}{\pi}\mathbf{a}_z = -6.37\mathbf{a}_z \ \ [\mathrm{A/m}]$$

7.1 Biot-Savart의 법칙

- (ex 1) $I = 24A, P_2(1.5,2,3) \rightarrow \mathbf{H}_2 = ?$

$$\rho = \sqrt{1.5^2 + 2^2} = 2.5$$

$$\mathbf{H}_2 = \frac{I}{4\pi\rho}(\sin\alpha_2 - \sin\alpha_1)\mathbf{a}_\phi$$

  - (a) from $z = 0$ to $z = 6$

$$\alpha_1 = -\tan^{-1}(3/\rho) = -50.2°, \alpha_2 = 50.2°$$

$$\mathbf{H}_2 = \frac{24}{4\pi \times 2.5}(\sin 50.2° + \sin 50.2°)\mathbf{a}_\phi$$

$$= \frac{12}{2.5\pi}\sin 50.2°\mathbf{a}_\phi$$

$$= 1.174\mathbf{a}_\phi$$

$$= H_x\mathbf{a}_x + H_y\mathbf{a}_y$$

7.1 Biot-Savart의 법칙

$$H_x = \mathbf{H}_2 \cdot \mathbf{a}_x$$
$$= 1.174\mathbf{a}_\phi \cdot \mathbf{a}_x$$
$$= 1.174(-\sin\phi)$$
$$= 1.174 \times (-2/2.5) = -0.9392$$

$$H_y = \mathbf{H}_2 \cdot \mathbf{a}_y$$
$$= 1.174\mathbf{a}_\phi \cdot \mathbf{a}_y$$
$$= 1.174\cos\phi$$
$$= 1.174 \times (1.5/2.5) = 0.7044$$

$$\therefore \mathbf{H}_2 = H_x\mathbf{a}_x + H_y\mathbf{a}_y = = -0.9392\mathbf{a}_x + 0.7044\mathbf{a}_y \ \ [\mathrm{A/m}]$$

- (b) from $z = 6$ to $z = \infty$, (c) from $z = -\infty$ to $z = \infty$

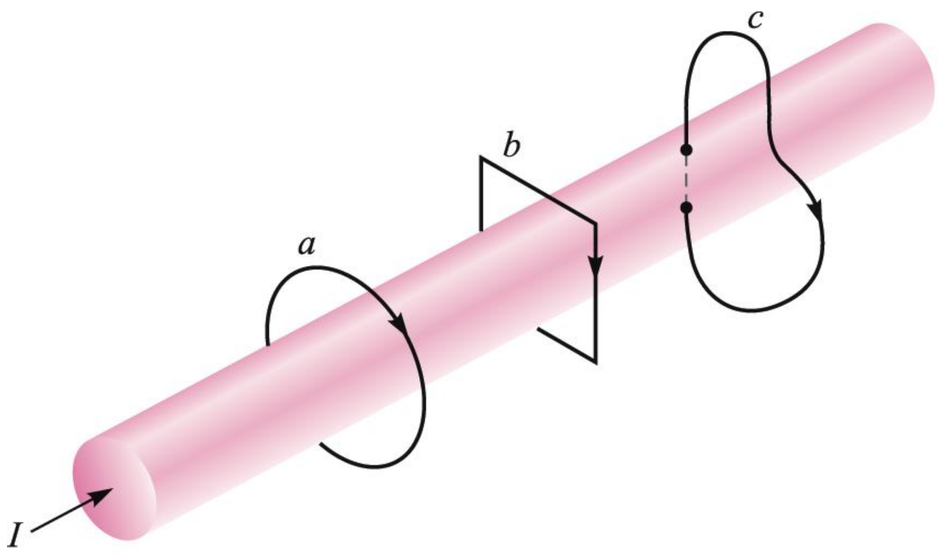
- 응용예제 7.1 & 7.2

7.2 Ampere의 주회법칙(Circuital Law)

- Ampere의 주회법칙

$$\oint \mathbf{H} \cdot d\mathbf{L} = I$$

➢ 폐곡로를 따라 **H**를 선적분한 결과는
폐곡로로 둘러싸인 직류전류와 같다.



- 무한길이의 선전류에 의한 자계의 세기

$$\oint \mathbf{H} \cdot d\mathbf{L} = \int_0^{2\pi} H_\phi \, \rho d\phi = H_\phi \rho \int_0^{2\pi} d\,\phi = H_\phi 2\pi\rho = I$$

$$\boxed{\therefore H_\phi = \frac{I}{2\pi\rho} \quad [\mathrm{A/m}]}$$

7.2 Ampere의 주회법칙(Circuital Law)

- 동축케이블에서의 자계의 세기

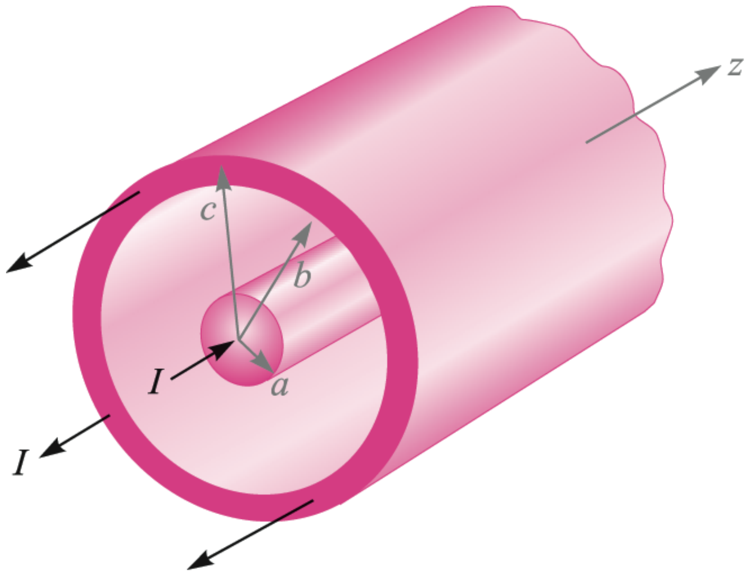
i) $\rho < a$

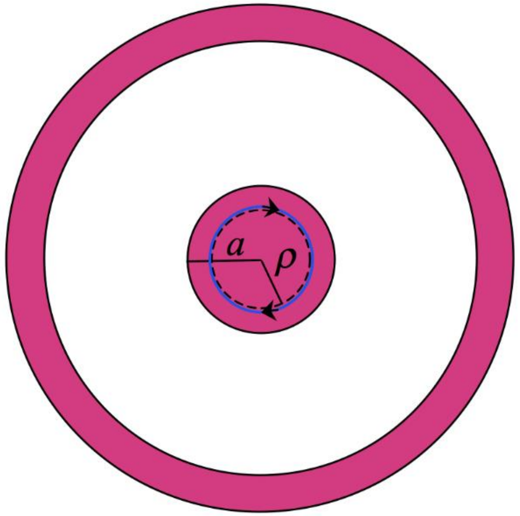
$$H_\phi = \frac{I'}{2\pi\rho} \quad \leftarrow \quad \frac{I'}{I} = \frac{\pi\rho^2}{\pi a^2}$$

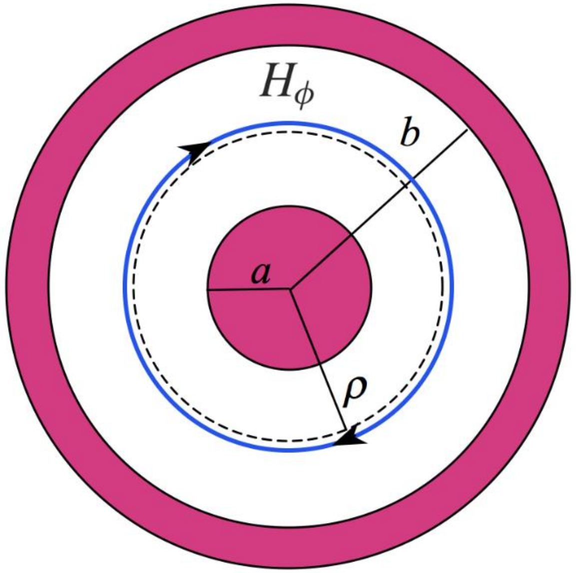
$$= \frac{I}{2\pi\rho}\frac{\rho^2}{a^2} = \frac{I}{2\pi a^2}\rho \ \ [\mathrm{A/m}]$$

ii) $a \le \rho < b$

$$H_\phi = \frac{I}{2\pi\rho} = \frac{I}{2\pi}\frac{1}{\rho} \ \ [\mathrm{A/m}]$$







7.2 Ampere의 주회법칙(Circuital Law)

- 동축케이블에서의 자계의 세기

iii) $b \le \rho < c$

$$H_\phi = \frac{I - I''}{2\pi\rho} \quad \leftarrow \quad \frac{I''}{I} = \frac{\pi\rho^2 - \pi b^2}{\pi c^2 - \pi b^2}$$

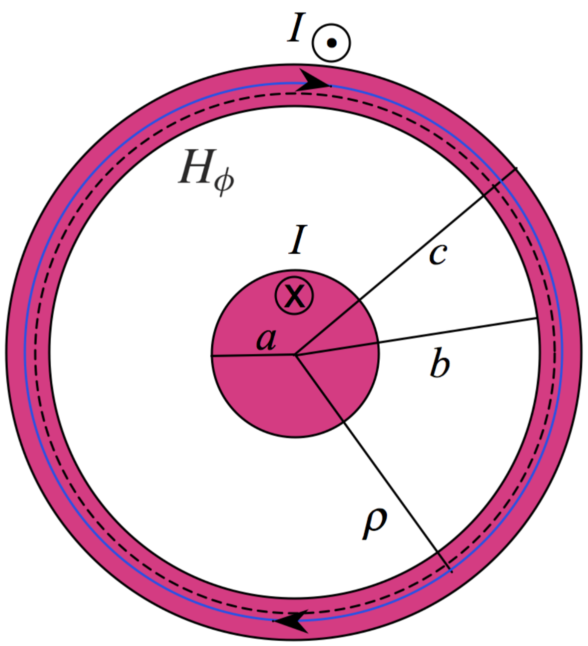
$$= \frac{I}{2\pi\rho}\left(1 - \frac{\rho^2 - b^2}{c^2 - b^2}\right)$$

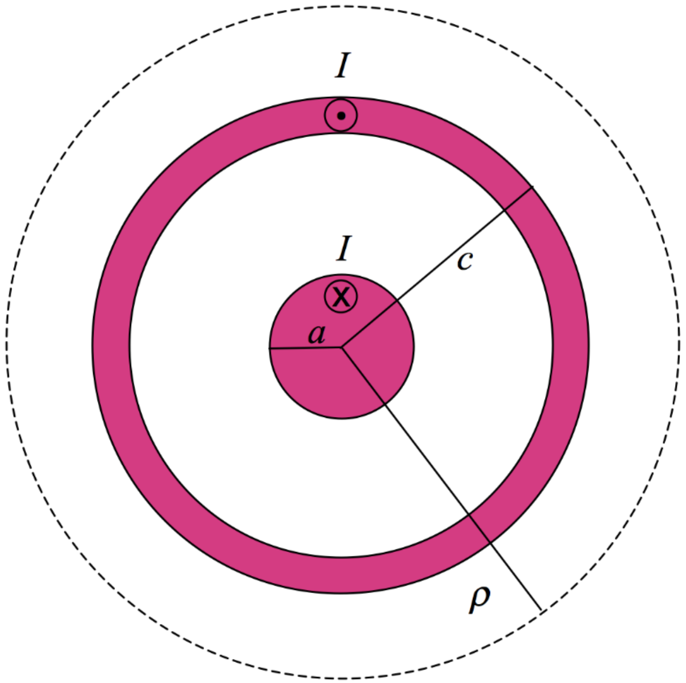
$$= \frac{I}{2\pi\rho}\left(\frac{c^2 - b^2 - \rho^2 + b^2}{c^2 - b^2}\right)$$

$$= \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right) \ [\mathrm{A/m}]$$

iv) $\rho \ge d$

$$H_\phi = \frac{0}{2\pi\rho} = 0$$





- 동축케이블에서의 자계의 세기

i) $\rho < a$

$$H_\phi = \frac{I}{2\pi a^2}\rho \ \ [\mathrm{A/m}]$$

ii) $a \le \rho < b$

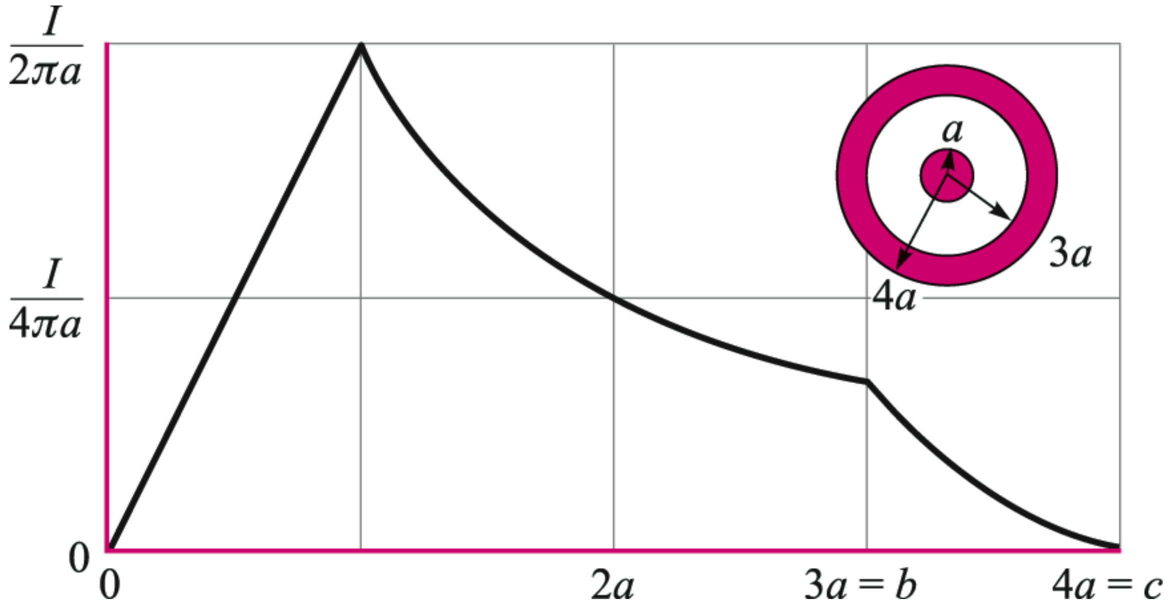
$$H_\phi = \frac{I}{2\pi\rho} = \frac{I}{2\pi}\frac{1}{\rho} \ \ [\mathrm{A/m}]$$

iii) $b \le \rho < c$

$$H_\phi = \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right) \ \ [\mathrm{A/m}]$$

iv) $\rho \ge d$

$$H_\phi = \frac{0}{2\pi\rho} = 0$$



7.2 Ampere의 주회법칙(Circuital Law)

- 솔레노이드(solenoid)에서의 자계의 세기

$$\oint \mathbf{H} \cdot d\mathbf{L} = NI$$

$$\mathbf{H}_1 \cdot d\mathbf{L}_1 + \mathbf{H}_2 \cdot d\mathbf{L}_2 + \mathbf{H}_3 \cdot d\mathbf{L}_3 + \mathbf{H}_4 \cdot d\mathbf{L}_4 = NI$$

$$\mathbf{H}_2 \perp d\mathbf{L}_2, \mathbf{H}_4 \perp d\mathbf{L}_4$$

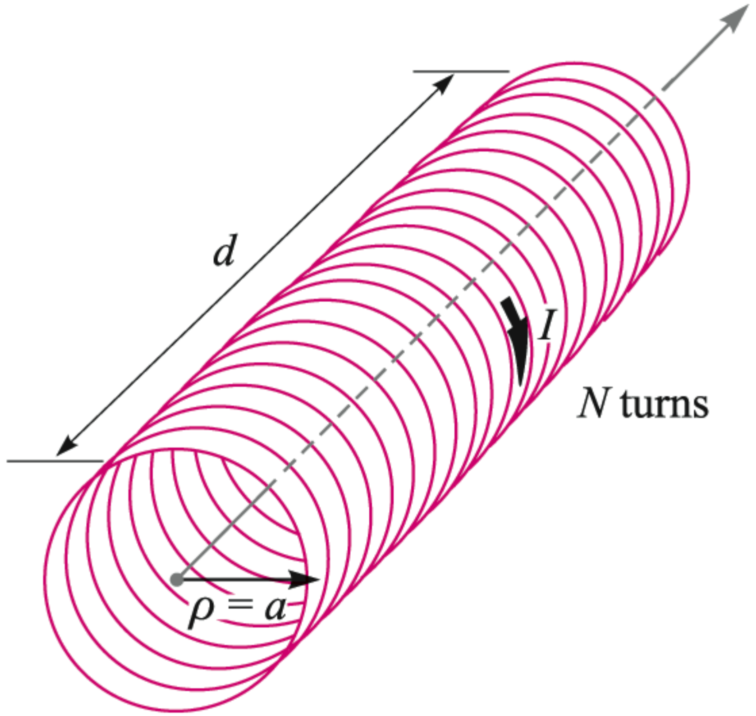
$$\mathbf{H}_1 \cdot d\mathbf{L}_1 + \mathbf{H}_3 \cdot d\mathbf{L}_3 = NI \leftarrow \mathbf{H}_3 = 0$$

$$\mathbf{H}_1 \cdot d\mathbf{L}_1 = NI \leftarrow \mathbf{H}_1 \; parallel \; d\mathbf{L}_1$$

$$H_1 dL_1 = NI$$

$$H_1 = \frac{NI}{dL_1} \leftarrow H_1 = H_z, dL_1 = d$$

$$\therefore H_z = \frac{NI}{d} = \frac{nIA}{m} \leftarrow \frac{N}{d} = n(\text{권선밀도})$$



$\mathbf{H} = \frac{NI}{d}\mathbf{a}_z$

(well inside coil)

- 토로이드(toroid)에서의 자계의 세기

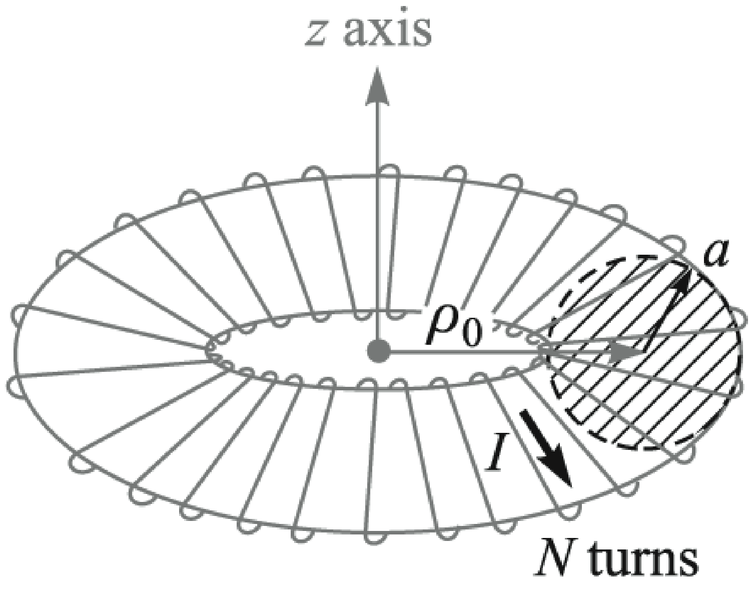
  i) 토로이드 외부 : $\oint \mathbf{H} \cdot d\mathbf{L} = 0$

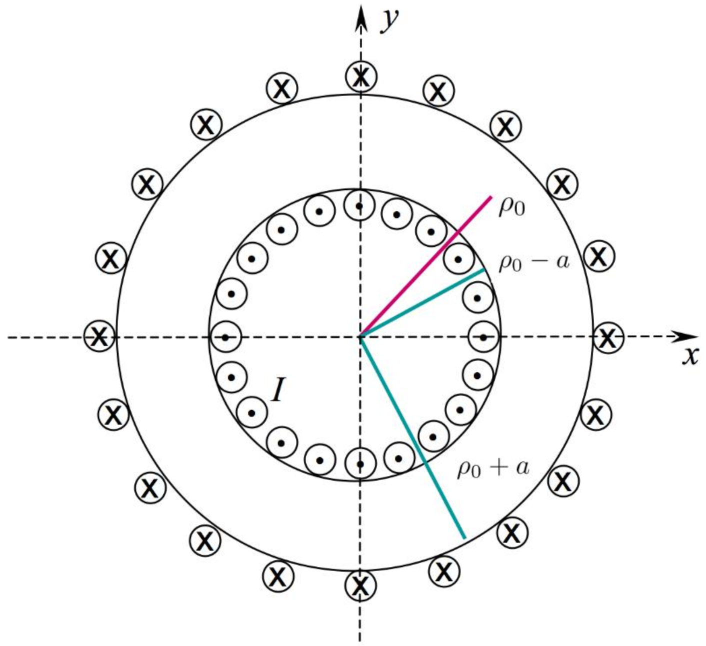
  ii) 토로이드 내부 : $\oint \mathbf{H} \cdot d\mathbf{L} = NI$

$$\int_0^{2\pi} H_\phi\, \rho d\phi = NI$$

$$H_\phi 2\pi\rho = NI$$

$$\therefore H_\phi = \frac{NI}{2\pi\rho} \quad [\mathrm{A/m}]$$





- (ex 2) $\mathbf{H}$ =? at $P(0.01,0,0)$

(a) $0.08A$ at z축 & $-0.08A$ at $x = 0.015$, $y = 0$

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2$$

$$= \frac{0.08}{2\pi \times 0.01}\mathbf{a}_y + \frac{0.08}{2\pi \times 0.005}\mathbf{a}_y$$

$$= \frac{0.08}{2\pi}\left(\frac{1}{0.01} + \frac{1}{0.005}\right)\mathbf{a}_y = 3.82\mathbf{a}_y \quad [\mathrm{A/m}]$$

(b) $a = 6$㎜, $b = 9$㎜, $c = 12$㎜인 동축케이블의 내부도체에 $I = 0.8\mathbf{a}_z A$

$b < \rho = 0.01 < c$이므로

$$\mathbf{H} = \frac{I}{2\pi\rho}\left(\frac{c^2 - \rho^2}{c^2 - b^2}\right)\mathbf{a}_\phi$$

$$= \frac{0.8}{2\pi \times 0.01}\frac{0.012^2 - 0.01^2}{0.012^2 - 0.009^2}\mathbf{a}_y \ = 8.8925\mathbf{a}_y \quad [\mathrm{A/m}]$$

7.2 Ampere의 주회법칙(Circuital Law)

- (ex 2) $\mathbf{H} =$? at $P(0.01,0,0)$

(d) 원점을 중심으로 하고, 중심선이 y축인 토로이드($\rho_0 = 12$㎜, $a = 3$㎜, $N = 250$회)에서, 밖에서 $I = 2\mathbf{a}_y$ [mA]

$\rho_0 - a < \rho = 0.01 < \rho_0 + a$ (토로이드 내부)이므로

$$\mathbf{H} = \frac{NI}{2\pi\rho}\mathbf{a}_\phi$$

$$= \frac{250 \times 0.002}{2\pi \times 0.01}\mathbf{a}_z$$

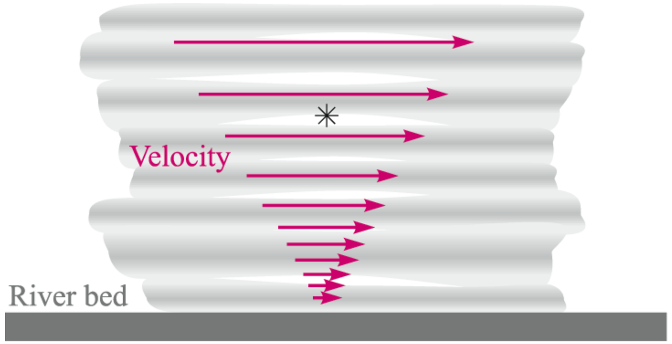
$$= 7.958\mathbf{a}_z \ \text{[A/m]}$$

7.3 벡터의 회전(Curl)

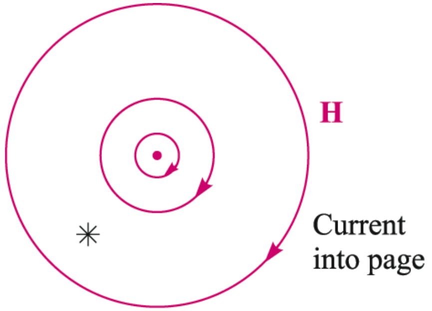
- 벡터의 회전(curl)
  - Gauss의 법칙 → 벡터의 발산(divergence)
  - Ampere의 주회법칙 → 벡터의 회전(curl)

$$curl\ \mathbf{H} = \nabla \times \mathbf{H} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = \left(\frac{\partial H}{\partial y} - \frac{\partial H}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial H}{\partial z} - \frac{\partial H_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\mathbf{a}_z = \mathbf{J}$$

- Ampere의 주회법칙 미분형
  - $\oint \mathbf{H} \cdot d\mathbf{L} = I$ (적분형) $\leftrightarrow \nabla \times \mathbf{H} = \mathbf{J}$(미분형)



(a)



(b)

7.3 벡터의 회전(Curl)

- 응용예제 7.5) $\nabla \times \mathbf{G} = ?$
  - a) $\mathbf{G} = xyz(\mathbf{a}_x + \mathbf{a}_y)$at P(3, 2, 1)

$$\nabla \times \mathbf{G} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ G_x & G_y & G_z \end{vmatrix} = \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}\right)\mathbf{a}_x + \left(\frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}\right)\mathbf{a}_y + \left(\frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)\mathbf{a}_z$$

$$= (0 - xy)\mathbf{a}_x + (xy - 0)\mathbf{a}_y + (yz - xz)\mathbf{a}_z = -6\mathbf{a}_x + 6\mathbf{a}_y - \mathbf{a}_z$$

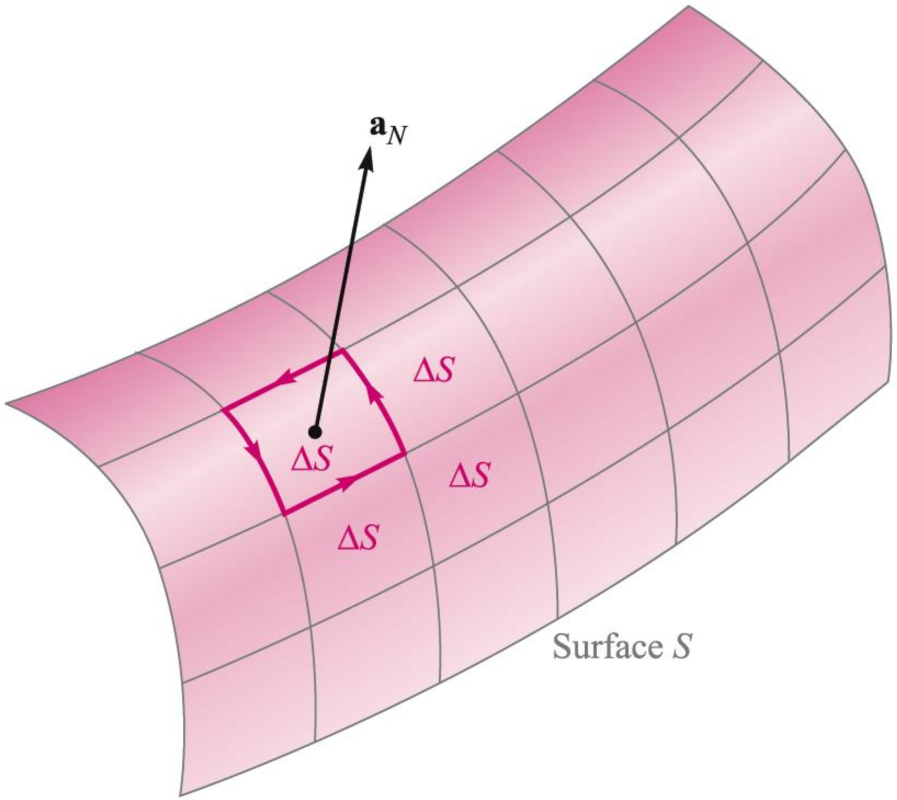
7.4 Stokes의 정리

$$\oint \mathbf{H} \cdot d\mathbf{L} \equiv \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$

$$\because \oint \mathbf{H} \cdot d\mathbf{L} = I$$

$$= \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$= \int_S (\nabla \times \mathbf{H}) \cdot d\mathbf{S}$$



- 전류의 연속성; 임의의 폐곡면을 통하여 나가는 전류의 합은 항상 0

$$\nabla \cdot \mathbf{J} = \nabla \cdot (\nabla \times \mathbf{H}) \equiv 0$$

7.5 자속과 자속밀도

- 자속밀도(magnetic flux density)의 정의
  - $\mathbf{B} = \mu_0 \mathbf{H}$ $[\mathrm{Wb/m^2} = \mathrm{T} = 10000\mathrm{G}]$(자유공간에서)

    where, $\mu_0 = 4\pi \times 10^{-7}$ $[\mathrm{H/m}]$
- 자속
  - 전계의 세기 $\mathbf{E}$ → 전속밀도 $\mathbf{D} = \varepsilon \mathbf{E}$ → 전속 $\Psi = \oint_S \mathbf{D} \cdot d\mathbf{S} = Q$
  - 자계의 세기 $\mathbf{H}$ → 자속밀도 $\mathbf{B} = \mu \mathbf{H}$ → 자속 $\Phi = \oint_S \mathbf{B} \cdot d\mathbf{S} = 0$