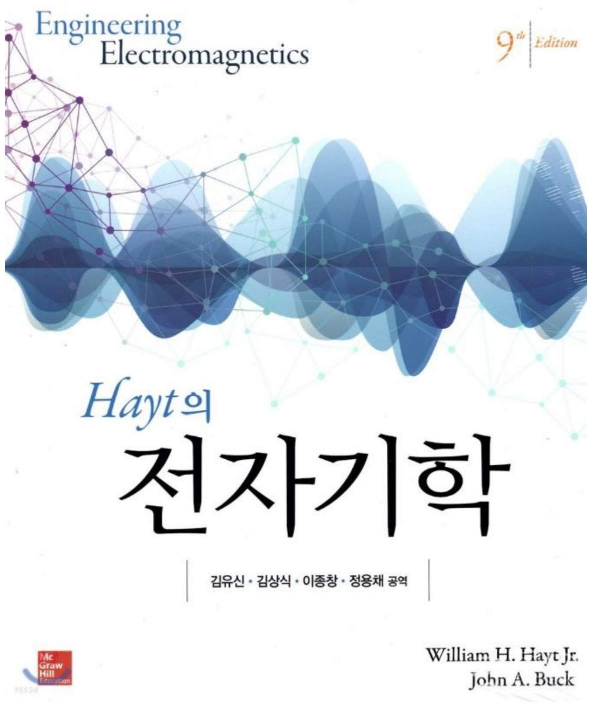
전기자기학 I

(강의자료 #1)



교과목명 : 전기자기학 I

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교재명 : Hayt의 전자기학

수업계획서

교과목명	**전기자기학I**	교과목번호	K3019B, L00009B
담당교수	이 수 형	수강대상	에너지전기공학부 2학년 신재생에너지공학부 3학년
학점 (시수)	3(4)	이수구분	전공
강의시간, 강의실	화3/목8,9 (갈마관 413)	교수연구실	갈마관 405
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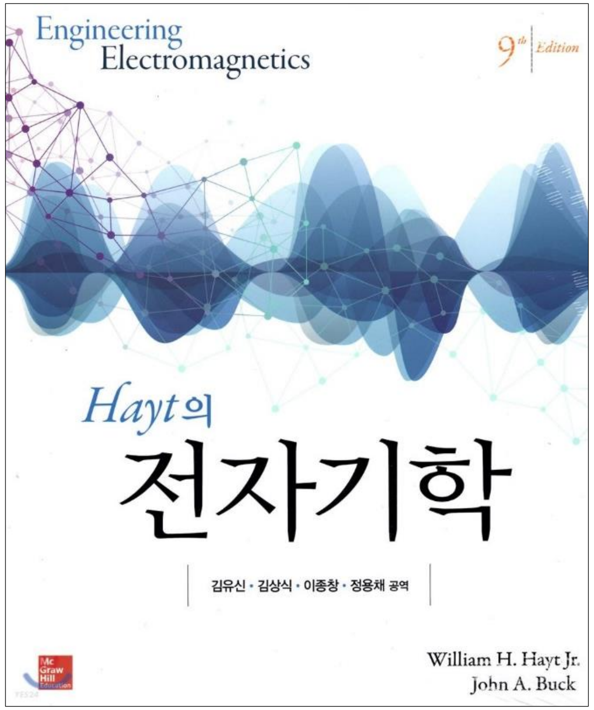
수업 개요

- 전기 및 제어, 원전시스템공학을 전공하는데 중요한 기초과목이므로 전자기 현상에 관한 기초지식과 함께 법칙의 개념파악에 중점을 둔다. 전자장은 다시 전기장과 자기장으로 나누어지는데 전기자기학1에서는 주로 전기장을 다루며 그 내용으로는 벡터해석, 쿨롱의 법칙 및 전계의 세기, 전속밀도와 가우스의 법칙, 전계의 에너지 및 전위 등을 차례로 다룬다.

수업 목표

- 수업 목표
  - 전기자기학에 관련된 현상 및 기본적인 개념을 익힌다.
  - 전기장이 발생하는 원인 및 전하의 분포에 따른 전기장의 해석 방법을 배운다.
  - 전계의 세기 및 전속밀도의 개념을 익히고 계산방법을 배운다.
  - 전기 에너지와 전위의 개념 및 계산방법을 익힌다.

- 교재 및 참고문헌
  - 전자기학 9판, William H. Hayt외 1인 원저, McGraw-Hill Korea



성적 평가 및 참고사항

- 성적 평가 방법 및 기준

평가방법	반영비율	내용
출석 및 수업참여도	20%	- 결석 1회시 1점 감점, 지각 3회시 결석 1회 처리 - 수업시간 중에 참여하지 않을 시에 1점 감점
1차시험	25%	전기자기학의 내용에 대한 이해를 평가하는 서술형 시험
2차시험	25%	전기자기학의 내용에 대한 이해를 평가하는 서술형 시험
기말고사	25%	전기자기학의 내용에 대한 이해를 평가하는 서술형 시험
과제물	5%	수업의 결과를 이해하고 주어진 문제를 해결할 능력이 어느 정도인지 평가

Ch. 1. 벡터해석(Vector Analysis)

CH. 1 : Vector Analysis

- 목적 : 전자장 문제를 간단하게 취급 (전기장 및 자기장이 벡터량)

  1.1 스칼라와 벡터

  1.2 벡터대수

  1.3 직각좌표계

  1.4 벡터 성분 및 단위벡터

  1.5 벡터장

  1.6 내적

  1.7 외적

  1.8 원통좌표계

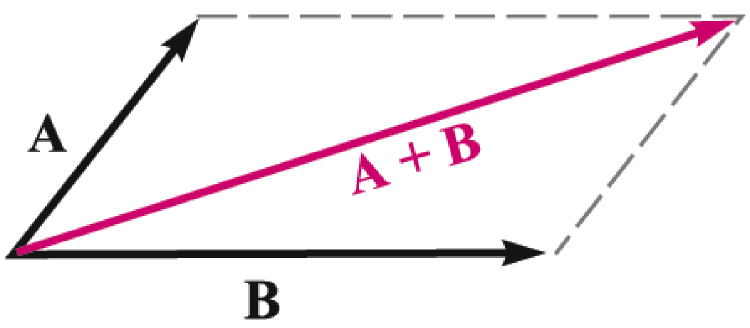
  1.9 구좌표계

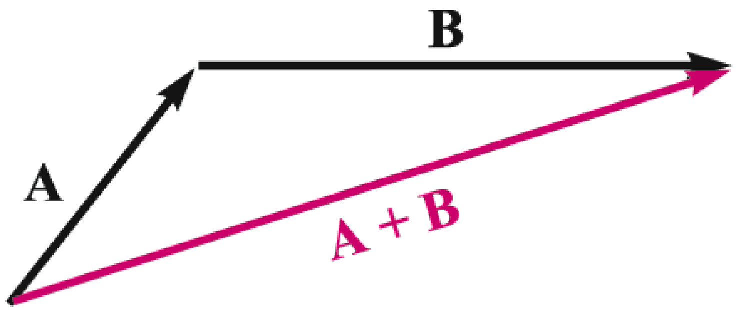
1.1 스칼라와 벡터

- 스칼라(scalar); 하나의 실수값(real value)
  - 무게, 속력, 온도, 부피, 저항, 전압, 전류
  - A, a
- 벡터(vector); 크기(+)와 방향
  - 중력, 힘, 속도, 가속도, 전기장, 자기장
  - $\mathbf{A}, \mathbf{a}$ $\bar{A}, \bar{a}$ $\vec{A}, \vec{a}$

1.2 벡터 대수

- 벡터 대수
  - 계산규칙, 약속
- 덧셈; $\mathbf{A}+\mathbf{B}$
  - (평행사변형법 또는 삼각형법)



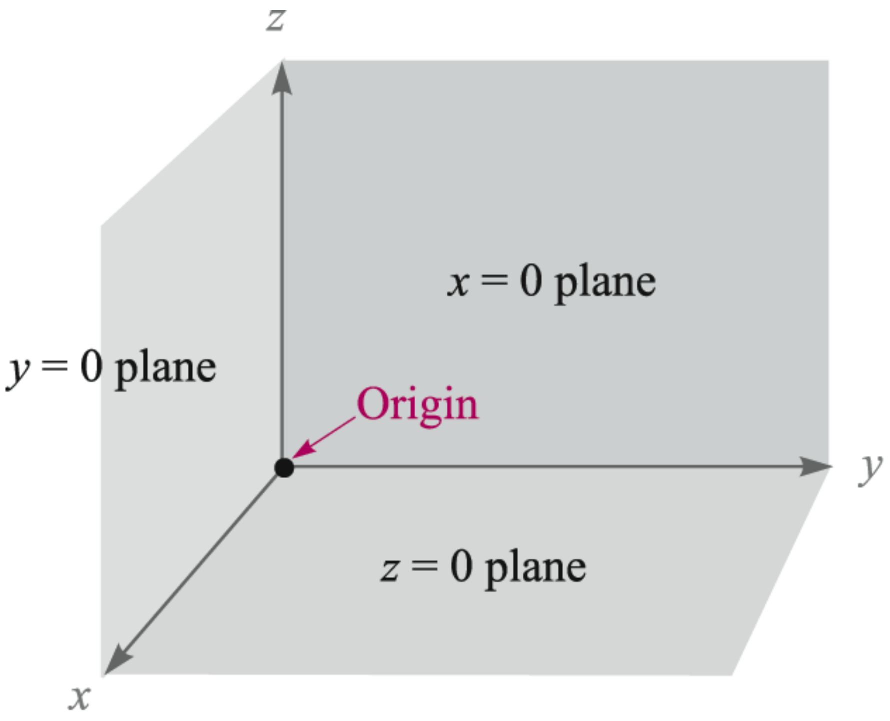


1.2 벡터 대수

- 뺄셈; $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$
- 교환법칙(commutative law); $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$
- 결합법칙(associative law); $\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$
- 배분법칙(distributive law); $r(\mathbf{A} + \mathbf{B}) = r\mathbf{A} + r\mathbf{B}$

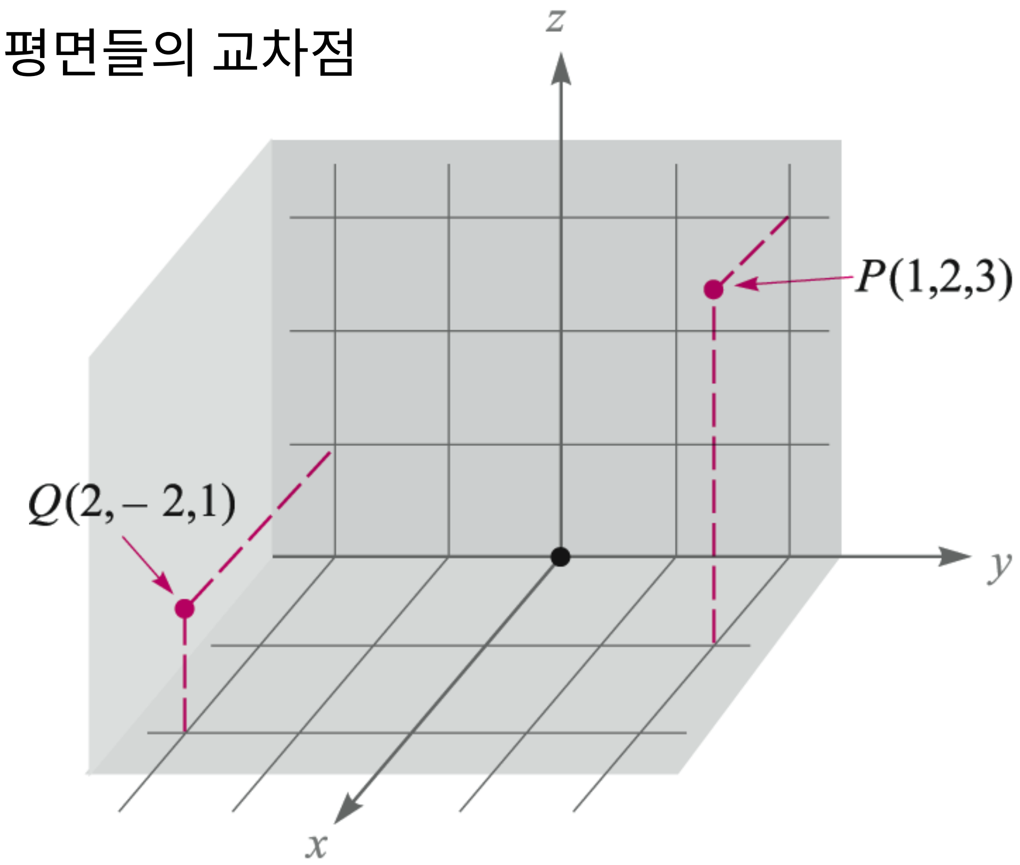
1.3 직각 좌표계

- 직각 좌표계 (cartesian or rectangular coordinate system)
  - 직각 좌표계; x, y, z세 개의 좌표축이 서로 직각인 오른손 좌표계 (right-handed coordinate system)



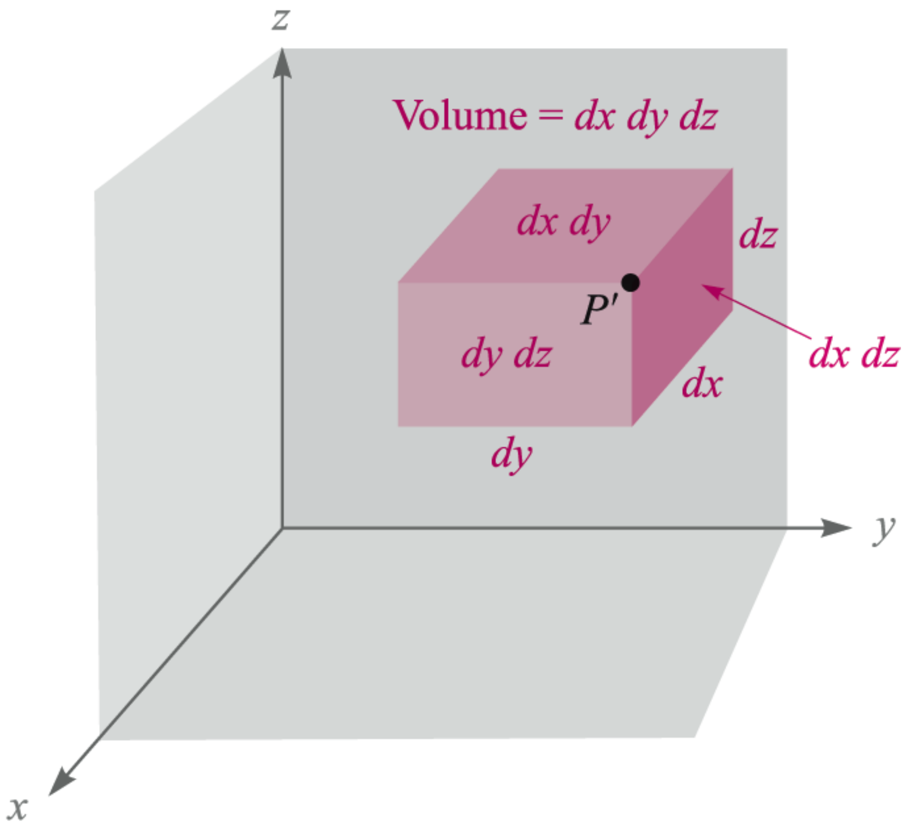
1.3 직각 좌표계

- 점의 표시; 한 점의 위치는 x, y 및 z좌표 값으로 표시
  - $P(1,2,3), Q(2,-2,1)$
  - 점의 의미; $P(1,2,3) \rightarrow x = 1, y = 2, z = 3$인 평면들의 교차점



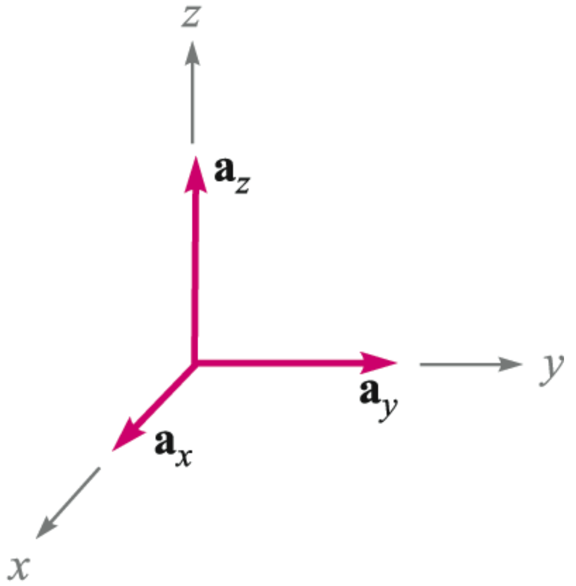
1.3 직각 좌표계

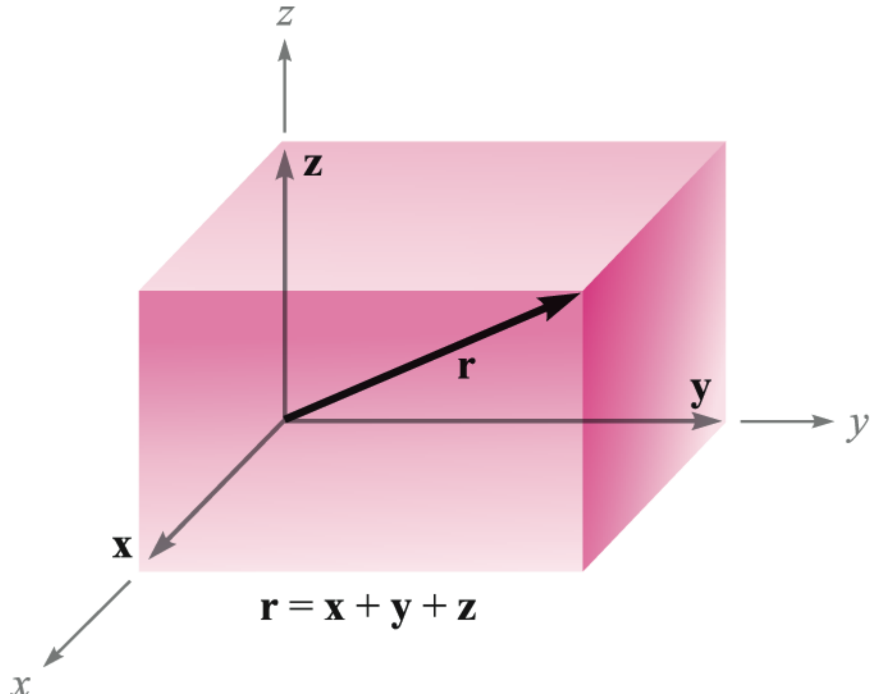
- 미소체적소의 정의;
  - 점 $P(x, y, z)$의 각 좌표값이 각각 미소량만큼 증가한 점이 $P'(x + dx, y + dy, z + dz)$일 때
  - 미소체적소; $dv = dxdydz$
  - 대각선; $\overline{PP'} = dL = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$
  - 면적소(dS); $dxdy, dydz, dzdx$



1.4 벡터성분 및 단위벡터

- 성분벡터(component vector); $\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$
- 단위벡터(unit vector); 크기가 1인 벡터
- 기본 단위벡터; 축방향으로의 단위벡터
  - $\mathbf{a}_x$ $\mathbf{a}_y$ $\mathbf{a}_z$
    $\mathbf{i}$ $\mathbf{j}$ $\mathbf{k}$
    $\mathbf{u}_x$ $\mathbf{u}_y$ $\mathbf{u}_z$





1.4 벡터성분 및 단위벡터

- 벡터의 성분표시법; $\mathbf{F} = F_x\mathbf{a}_x + F_y\mathbf{a}_y + F_z\mathbf{a}_z$
- 벡터의 크기; $|\mathbf{F}| = F = \sqrt{F_x^2 + F_y^2 + F_x^2}$
- $\mathbf{F}$ 방향으로의 단위벡터; $\mathbf{a}_F = \mathbf{F}/|\mathbf{F}|$

- (예제 1.1) 원점 O에서 점 $G(2,-2,-1)$로 향하는 단위벡터 $\mathbf{a}_G$

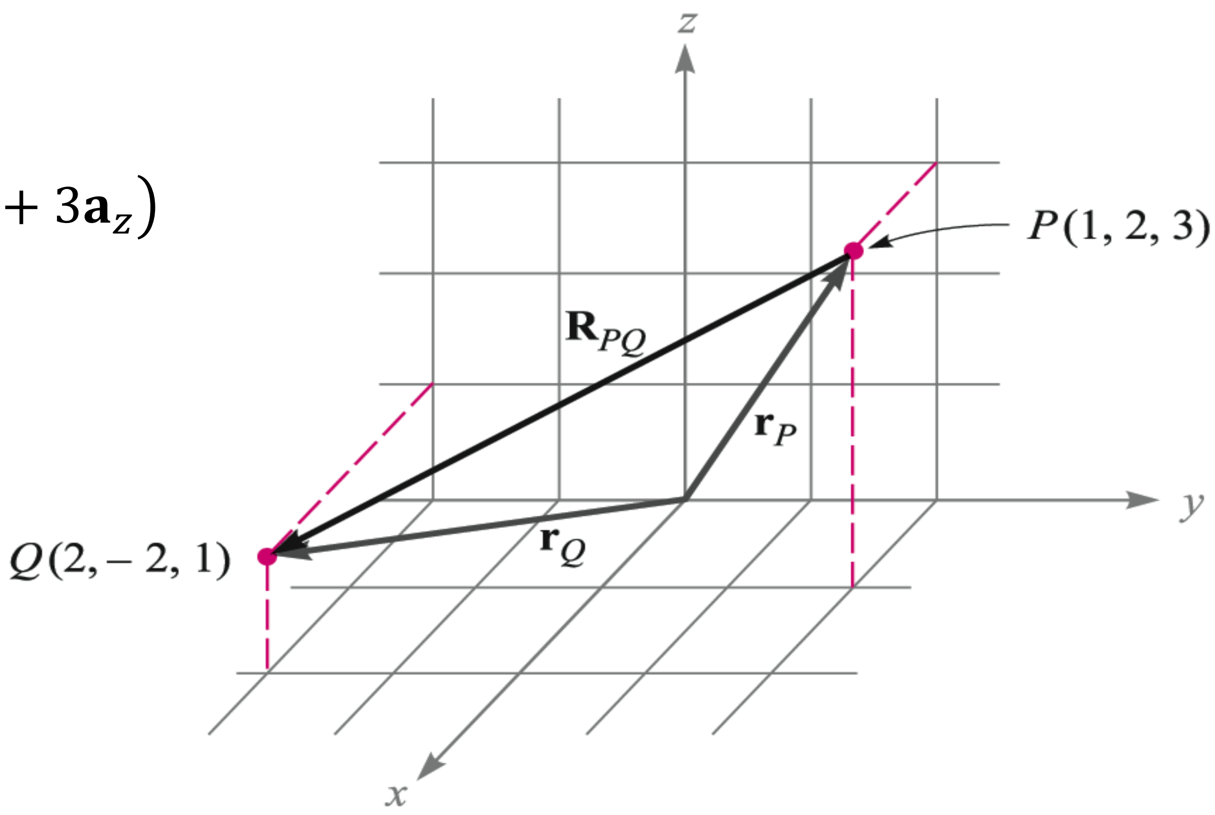
  - $\mathbf{G} = 2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z$
  - $|\mathbf{G}| = \sqrt{G_x^2 + G_y^2 + G_z^2} = \sqrt{4+4+1} = 3$
  - $\therefore \mathbf{a}_G = \frac{\mathbf{G}}{|\mathbf{G}|} = \frac{2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z}{3} = \frac{2}{3}\mathbf{a}_x - \frac{2}{3}\mathbf{a}_y - \frac{1}{3}\mathbf{a}_z$

1.4 벡터성분 및 단위벡터

- (ex 1) 점 $P(1,2,3)$에서 점 $Q(2,-2,1)$까지의 벡터 $\mathbf{R}_{PQ}$
   - △OPQ로부터 $\mathbf{r}_P + \mathbf{R}_{PQ} = \mathbf{r}_Q$
   - $\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P$

$$= (2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z) - (\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)$$

$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$



1.4 벡터성분 및 단위벡터

- (ex 2) $\mathbf{r}_A = -\mathbf{a}_x - 3\mathbf{a}_y - 4\mathbf{a}_z\,,\ \mathbf{r}_B = 2\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z\,,\ \ C(1,3,4)$

(a) $\mathbf{R}_{AB} = \mathbf{r}_B - \mathbf{r}_A =$

(b) $|\mathbf{r}_A| = \sqrt{(-1)^2 + (-3)^2 + (-4)^2} =$

(c) $\mathbf{a}_A = \mathbf{r}_A / |\mathbf{r}_A| =$

1.4 벡터성분 및 단위벡터

(d) $\mathbf{a}_{AB} = \dfrac{\mathbf{R}_{AB}}{|\mathbf{R}_{AB}|} =$

$\mathbf{R}_{AB} = \mathbf{r}_B - \mathbf{r}_A =$

$|\mathbf{R}_{AB}| = \sqrt{\mathbf{R}_x{}^2 + \mathbf{R}_y{}^2 + \mathbf{R}_z{}^2} =$

(e) $\mathbf{a}_{CA} = \dfrac{\mathbf{R}_{CA}}{|\mathbf{R}_{CA}|} =$

$\mathbf{R}_{CA} = \mathbf{r}_A - \mathbf{r}_C =$

$\mathbf{r}_C = \mathbf{a}_x + 3\mathbf{a}_y + 4\mathbf{a}_z$

1.4 벡터성분 및 단위벡터

- (응용예제 1.1) $M(-1,2,1)$, $N(3,-3,0)$, $P(-2,-3,-4)$
  - (a) $\mathbf{R}_{MN}$
  - (b) $\mathbf{R}_{MN} + \mathbf{R}_{MP}$
  - (c) $|\mathbf{r}_M|$
  - (d) $\mathbf{a}_{MP}$
  - (e) $|2\mathbf{r}_P - 3\mathbf{r}_N|$

- (응용예제 1.2)

$$\mathbf{S} = \frac{125}{(x-1)^2+(y-2)^2+(z+1)^2}\left[(x-1)\mathbf{a}_x + (y-2)\mathbf{a}_y + (z+1)\mathbf{a}_z\right]$$

- (a) $\mathbf{S} = ?$ at $P(2,4,3)$ (b) $\mathbf{a}_S = ?$ at $P(2,4,3)$ (c) $|\mathbf{S}| = 1$이 되는 조건

1.6 내적(스칼라곱, scalar product or dot product)

- 정의; $\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$
- 교환법칙; $\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A}$
- 계산방법
  - $\mathbf{A}\cdot\mathbf{B} = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z)$
    $= A_x\mathbf{a}_x\cdot(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z) + A_y\mathbf{a}_y\cdot(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z) + A_z\mathbf{a}_z\cdot(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z)$
    $= A_xB_x + A_yB_y + A_zB_z$
  - $\because\ \mathbf{a}_x\cdot\mathbf{a}_x = |\mathbf{a}_x||\mathbf{a}_x|\cos 0° = 1\times 1\times 1 = 1$
  - $\mathbf{a}_x\cdot\mathbf{a}_y = |\mathbf{a}_x||\mathbf{a}_y|\cos 90° = 1\times 1\times 0 = 0$
- $\mathbf{A}\cdot\mathbf{A} = |\mathbf{A}||\mathbf{A}|\cos 0° = |\mathbf{A}|^2$ or A^2

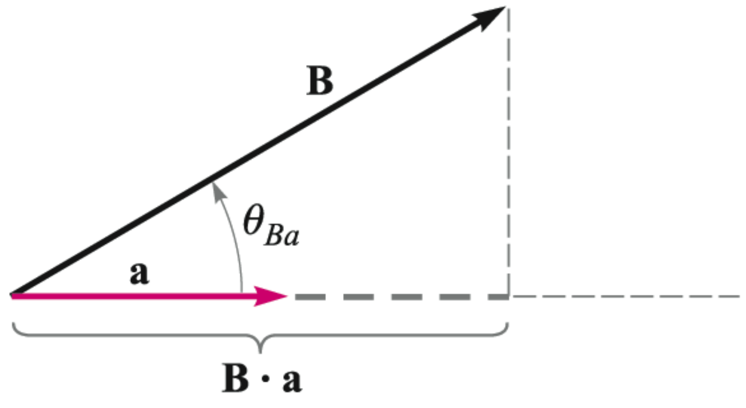
※ 참고 : https://www.youtube.com/watch?v=LyGKycYT2v0

1.6 내적(스칼라곱, scalar product or dot product)

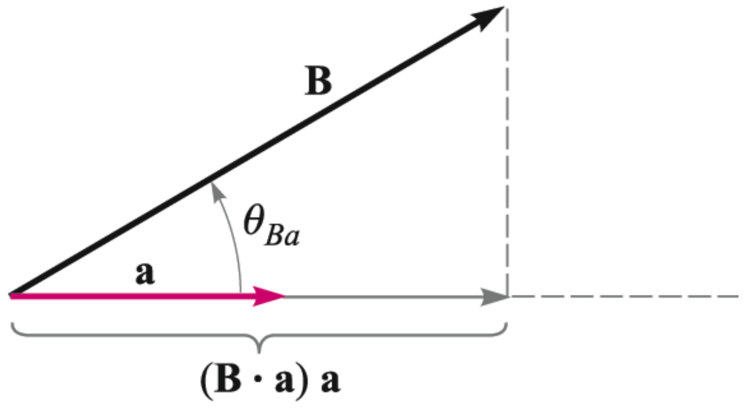
- 투영(projection);

  (a) $\mathbf{B}$의 $\mathbf{a}$방향의 성분; $\mathbf{B} \cdot \mathbf{a} = |\mathbf{B}||\mathbf{a}| \cos\theta_{Ba} = |\mathbf{B}| \cos\theta_{Ba}$

  (b) $\mathbf{B}$의 $\mathbf{a}$방향의 성분벡터; $(\mathbf{B} \cdot \mathbf{a})\mathbf{a}$



B • a gives the component of B in the horizontal direction



(B • a)a gives the *vector* component of B in the horizontal direction

1.6 내적(스칼라곱, scalar product or dot product)

- (예제 1.2) $\mathbf{G} = y\mathbf{a}_x - 2.5x\mathbf{a}_y + 3\mathbf{a}_z$ at $Q(4,5,2)$, $\mathbf{a}_N = \frac{1}{3}(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$

  (a) Q에서 $\mathbf{G}$

  (b) Q에서 $\mathbf{a}_N$ 방향으로 $\mathbf{G}$의 스칼라 성분

  (c) Q에서 $\mathbf{a}_N$ 방향으로 $\mathbf{G}$의 벡터 성분

  (d) Q에서 $\mathbf{G}$와 $\mathbf{a}_N$사이의 각

  - (a)

$$\mathbf{G}_{Q(4,5,2)} = (y\mathbf{a}_x - 2.5x\mathbf{a}_y + 3\mathbf{a}_z)_{Q(4,5,2)} = 5\mathbf{a}_x - 10\mathbf{a}_y + 3\mathbf{a}_z$$

  - (b) Q에서 $\mathbf{G}$의 $\mathbf{a}_N$방향으로의 성분;

$$\mathbf{G} \cdot \mathbf{a}_N = (5\mathbf{a}_x - 10\mathbf{a}_y + 3\mathbf{a}_z) \cdot \frac{2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z}{3} = (10 - 10 - 6)/3 = -2$$

1.6 내적(스칼라곱, scalar product or dot product)

(c) Q에서 $\mathbf{G}$의 $\mathbf{a}_N$방향으로의 성분벡터;

$$(\mathbf{G}\cdot\mathbf{a}_N)\mathbf{a}_N = -2\mathbf{a}_N = -\frac{2}{3}(2\mathbf{a}_x + \mathbf{a}_y - 2\mathbf{a}_z)$$

(d) 정의로부터 $\mathbf{G}\cdot\mathbf{a}_N = |\mathbf{G}||\mathbf{a}_N|\cos\theta_{Ga}$

$$\cos\theta_{Ga} = \frac{\mathbf{G}\cdot\mathbf{a}_N}{|\mathbf{G}||\mathbf{a}_N|} = \frac{-2}{\sqrt{5^2 + (-10)^2 + 3^2}} = \frac{-2}{\sqrt{134}}$$

$$\therefore\ \theta_{Ga} = \cos^{-1}\frac{-2}{\sqrt{134}} = 99.9°$$

1.6 내적(스칼라곱, scalar product or dot product)

- (ex 4) $A(2,5,-1)$, $B(3,-2,4)$, $C(-2,3,1)$

  (a) $\mathbf{R}_{AB} \cdot \mathbf{R}_{AC}$

  - $\mathbf{R}_{AB} \cdot \mathbf{R}_{AC} = (\mathbf{r}_B - \mathbf{r}_A) \cdot (\mathbf{r}_C - \mathbf{r}_A)$

    $= \left(\mathbf{a}_x - 7\mathbf{a}_y + 5\mathbf{a}_z\right) \cdot \left(-4\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z\right)$

    $= -4 + 14 + 10$

    $= 20$

1.6 내적(스칼라곱, scalar product or dot product)

- (ex 4) $A(2,5,-1)$, $B(3,-2,4)$, $C(-2,3,1)$

  (b) $\mathbf{R}_{AB}$와 $\mathbf{R}_{AC}$사이의 각(θ_{BAC})

  ➢ $\mathbf{R}_{AB}\cdot\mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_{BAC}$

$$\cos\theta_{BAC} = \frac{R_{AB}\cdot R_{AC}}{|R_{AB}||R_{AC}|}$$

$$= \frac{20}{\sqrt{1+49+25}\sqrt{16+4+4}}$$

$$= \frac{20}{\sqrt{75}\sqrt{24}} = \frac{2}{\sqrt{18}} = \frac{2}{3\sqrt{2}}$$

  ➢ $\therefore\ \theta_{BAC} = \cos^{-1}\frac{2}{3\sqrt{2}} = 61.9°$

1.6 내적(스칼라곱, scalar product or dot product)

- (ex 4) $A(2,5,-1)$, $B(3,-2,4)$, $C(-2,3,1)$

  (c) $\mathbf{R}_{AB}$의 $\mathbf{R}_{AC}$에 대한 투영의 크기

$$\mathbf{R}_{AB} \cdot \mathbf{a}_{AC} = |\mathbf{R}_{AB}| \cos\theta_{BAC} = 5\sqrt{3}\cos 61.9° = 4.08$$

  (d) $\mathbf{R}_{AB}$의 $\mathbf{R}_{AC}$에 대한 벡터 투영

$$(\mathbf{R}_{AB} \cdot \mathbf{a}_{AC})\mathbf{a}_{AC} = 4.08 \times \frac{-4\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{24}} = -3.33\mathbf{a}_x - 1.667\mathbf{a}_y + 1.667\mathbf{a}_z$$

1.6 내적(스칼라곱, scalar product or dot product)

- (응용예제 1.3) $A(6,-1,2)$, $B(-2,3,-4)$, $C(-3,1,5)$

  (a) $\mathbf{R}_{AB}$

  (b) $\mathbf{R}_{AC}$

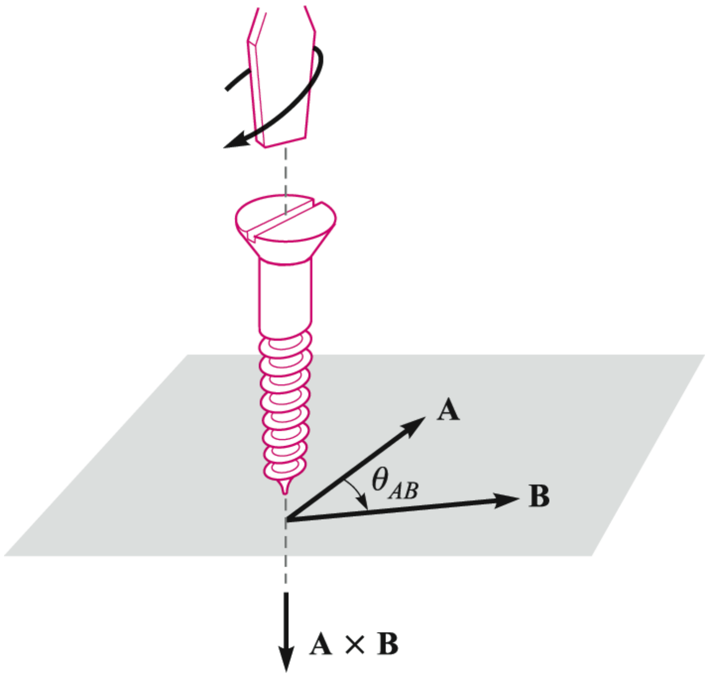
  (c) θ_{BAC}

  (d) $\mathbf{R}_{AB}$를 $\mathbf{R}_{AC}$에 (벡터)투영

1.7 외적(벡터곱, vector product or cross product)

- 정의; $\mathbf{A} \times \mathbf{B} = (|\mathbf{A}||\mathbf{B}| \sin\theta_{AB})\mathbf{a}_N$
- 교환법칙; $\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A}) \neq \mathbf{B} \times \mathbf{A}$

$$\boxed{\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin\theta_{AB}}$$



※ 참고 : https://www.youtube.com/watch?v=eu6i7WJeinw

1.7 외적(벡터곱, vector product or cross product)

- 계산방법;

  - $\mathbf{A} \times \mathbf{B} = \left(A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z\right) \times \left(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z\right)$

$$= A_xB_x(\mathbf{a}_x \times \mathbf{a}_x) + A_xB_y\left(\mathbf{a}_x \times \mathbf{a}_y\right) + A_xB_z(\mathbf{a}_x \times \mathbf{a}_z)$$

$$+A_yB_x\left(\mathbf{a}_y \times \mathbf{a}_x\right) + A_yB_y\left(\mathbf{a}_y \times \mathbf{a}_y\right) + A_yB_z\left(\mathbf{a}_y \times \mathbf{a}_z\right)$$

$$+A_zB_x(\mathbf{a}_z \times \mathbf{a}_x) + A_zB_y\left(\mathbf{a}_z \times \mathbf{a}_y\right) + A_zB_z(\mathbf{a}_z \times \mathbf{a}_z)$$

$$= 0 + A_xB_y\mathbf{a}_z + A_xB_z\left(-\mathbf{a}_y\right) + A_yB_x(-\mathbf{a}_z) + 0 + A_yB_z\mathbf{a}_x + A_zB_x\mathbf{a}_y + A_zB_y(-\mathbf{a}_x) + 0$$

$$= \left(A_yB_z - A_zB_y\right)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + \left(A_xB_y - A_yB_x\right)\mathbf{a}_z$$

$$= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

  - $\therefore\ |\mathbf{a}_x \times \mathbf{a}_x| = |\mathbf{a}_x||\mathbf{a}_x| \sin 0\,° = 1 \times 1 \times 0 = 0$

1.7 외적(벡터곱, vector product or cross product)

- 평행사변형의 면적; $|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin \theta_{AB}$

- (ex 5) $\mathbf{A} = 2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z$, $\mathbf{B} = -4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z$

  - $$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 2 & -3 & 1 \\ -4 & -2 & 5 \end{vmatrix}$$

$$= [(-3)(5) - (-2)(1)]\mathbf{a}_x + [(1)(-4) - (2)(5)]\mathbf{a}_y + [(2)(-2) - (-3)(-4)]\mathbf{a}_z$$

$$= -13\mathbf{a}_x - 14\mathbf{a}_y - 16\mathbf{a}_z$$

1.7 외적(벡터곱, vector product or cross product)

- (ex 6) $A(2,-5,1)$, $B(-3,2,4)$, $C(0,3,1)$

(a) $\mathbf{R}_{BC} \times \mathbf{R}_{BA} = (\mathbf{r}_C - \mathbf{r}_B) \times (\mathbf{r}_A - \mathbf{r}_B)$

$$= (3\mathbf{a}_x + \mathbf{a}_y - 3\mathbf{a}_z) \times (5\mathbf{a}_x - 7\mathbf{a}_y - 3\mathbf{a}_z)$$

$$= \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 3 & 1 & -3 \\ 5 & -7 & -3 \end{vmatrix}$$

$$= -24\mathbf{a}_x - 6\mathbf{a}_y - 26\mathbf{a}_z$$

1.7 외적(벡터곱, vector product or cross product)

(b) $\text{Area}(\Delta ABC)$

$$= \frac{1}{2}|\mathbf{R}_{BC} \times \mathbf{R}_{BA}|$$

$$= \frac{1}{2}\sqrt{(-24)^2 + (-6)^2 + (-26)^2}$$

$$= 17.94$$

(c) ΔABC에 수직인 단위벡터

$$\mathbf{a}_N = \pm \frac{\mathbf{R}_{BC} \times \mathbf{R}_{BA}}{|\mathbf{R}_{BC} \times \mathbf{R}_{BA}|}$$

$$= \pm \frac{-24\mathbf{a}_x - 6\mathbf{a}_y - 26\mathbf{a}_z}{\sqrt{(-24)^2 + (-6)^2 + (-26)^2}}$$

$$= \pm(0.669\mathbf{a}_x + 0.1672\mathbf{a}_y + 0.724\mathbf{a}_z)$$

1.7 외적(벡터곱, vector product or cross product)

- (응용예제 1.4) $A(6,-1,2)$, $B(-2,3,-4)$, $C(-3,1,5)$

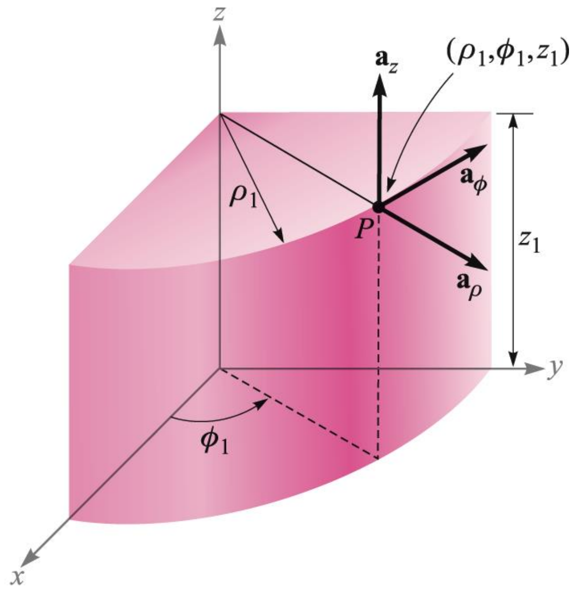
  (a) $\mathbf{R}_{AB} \times \mathbf{R}_{AC}$

  (b) ΔABC의 면적

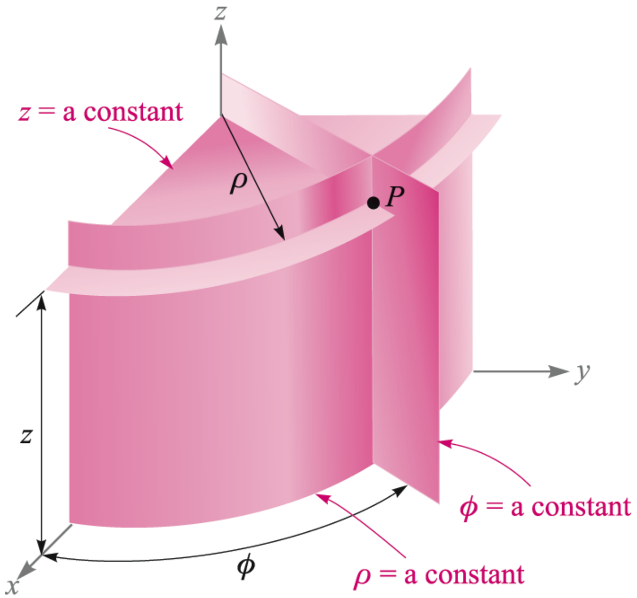
  (c) ΔABC에 수직인 단위벡터

1.8 원통좌표계(cylindrical coordinate system)

- 점의 표시; $P(\rho_1, \phi_1, z_1)$
- $P(\rho_1, \phi_1, z_1)$의 의미; $\rho = \rho_1, \phi = \phi_1, z = z_1$인 평면들의 교차점

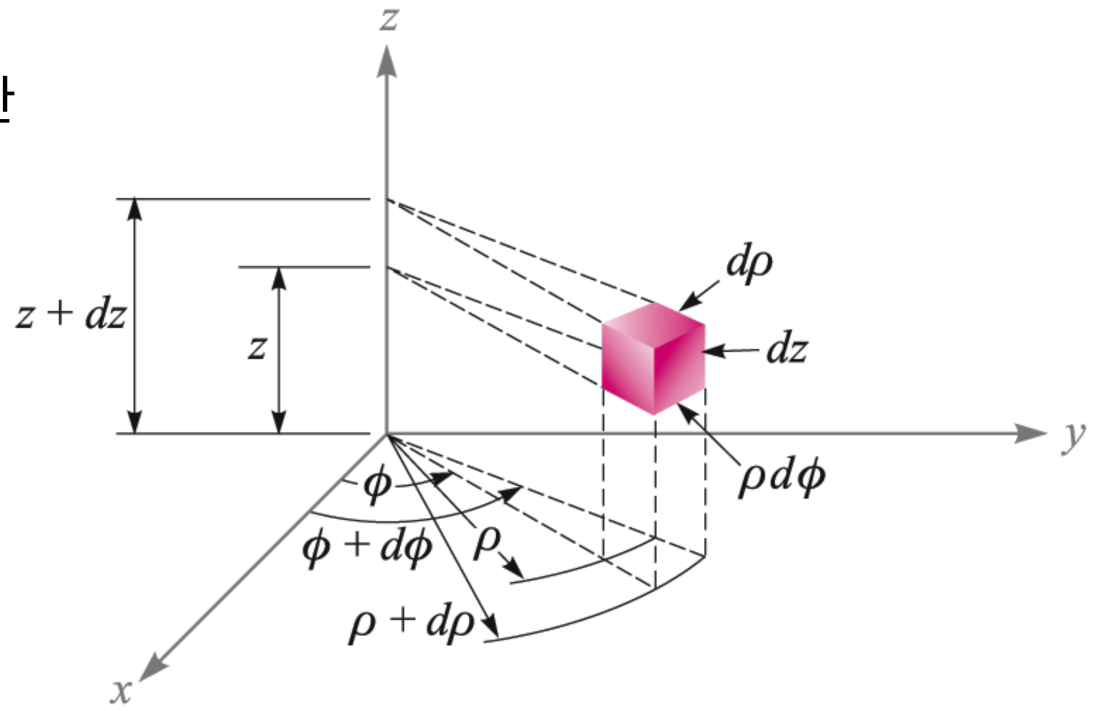


- 기본 단위벡터; $\mathbf{a}_\rho, \mathbf{a}_\phi, \mathbf{a}_z \leftarrow$ 위치에 대한 함수
- z축만 존재하는 오른손 좌표계; $\mathbf{a}_\rho \times \mathbf{a}_\phi = \mathbf{a}_z$



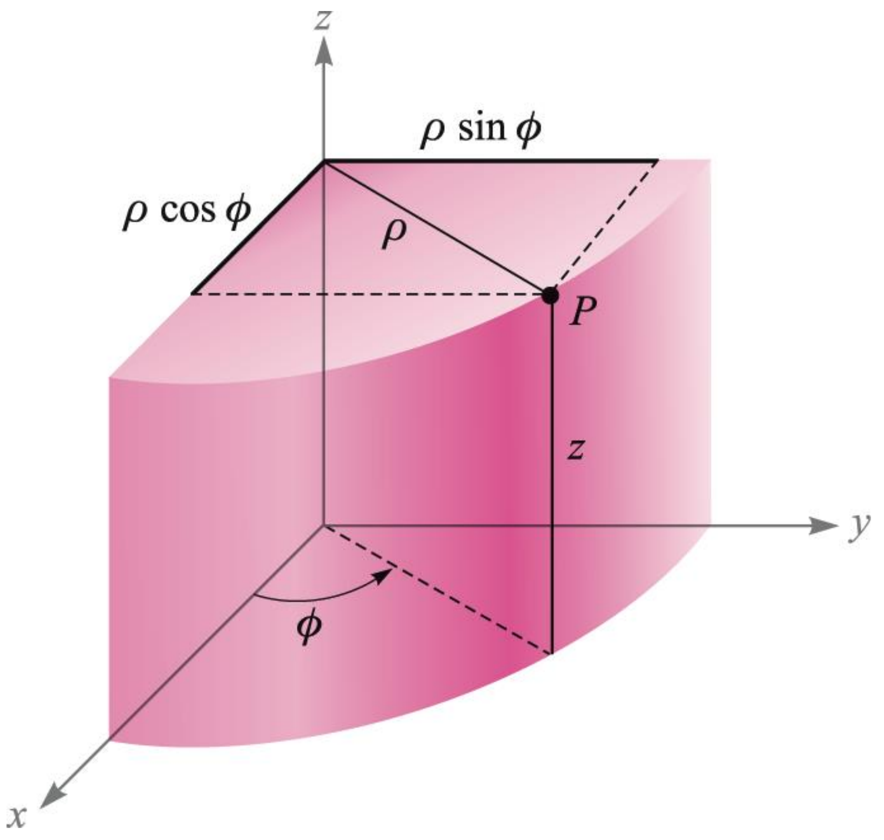
1.8 원통좌표계(cylindrical coordinate system)

- 미소체적소의 정의
  - 점 $P(\rho, \phi, z)$의 각 좌표값이 각각 미소량만큼 증가한 점이 $P'(\rho + d\rho, \phi + d\phi, z + dz)$일 때
  - 미소체적소; $dv = d\rho \cdot \rho d\phi \cdot dz = \rho d\rho d\phi dz$
  - 면적소(dS); $\rho d\rho d\phi$, $\rho d\phi dz$, $dz d\rho$



1.8 원통좌표계(cylindrical coordinate system)

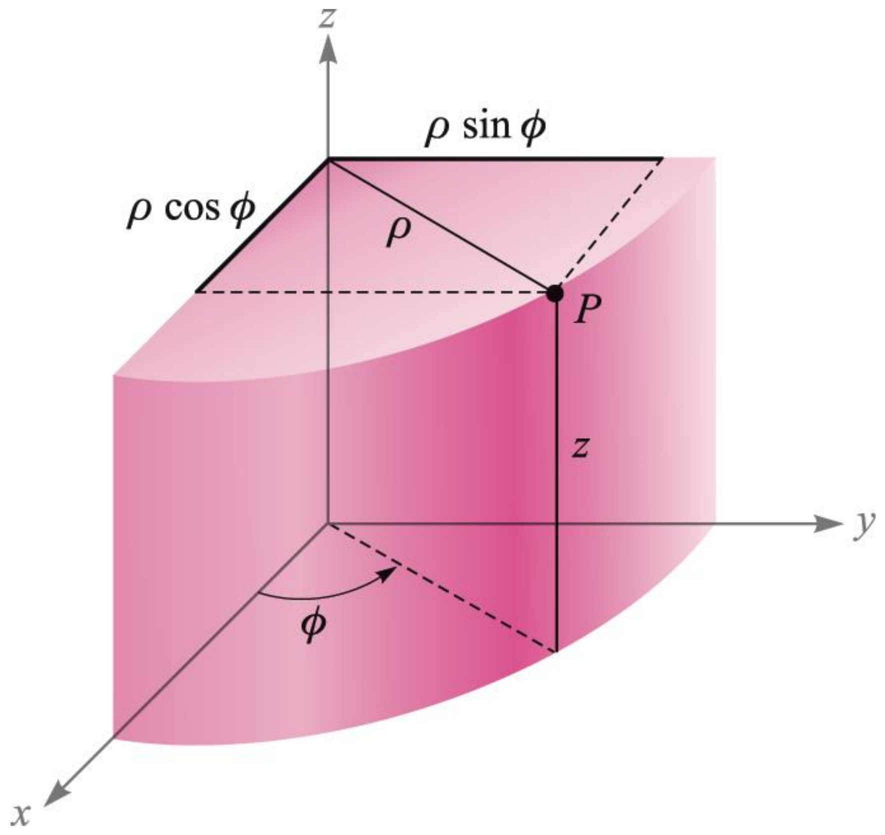
- 원통좌표계 → 직교좌표계의 변환
  - $x = \rho \cos \phi$
  - $y = \rho \sin \phi$
  - $z = z$
- 직교좌표계 → 원통좌표계의 변환
  - $\rho = \sqrt{x^2 + y^2} (\rho \geq 0)$
  - $\phi = \tan^{-1} \frac{y}{x}$
  - $z = z$



1.8 원통좌표계(cylindrical coordinate system)

- 직교, 원통좌표계 기본 단위벡터의 스칼라곱

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1



1.8 원통좌표계(cylindrical coordinate system)

- (ex 7) 벡터 $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$를 원통좌표계로 변환하여라.
  - $\mathbf{A} = A_\rho\mathbf{a}_\rho + A_\phi\mathbf{a}_\phi + A_z\mathbf{a}_z$
  - $A_\rho = \mathbf{A}\cdot\mathbf{a}_\rho = \left(A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z\right)\cdot\mathbf{a}_\rho$
    $= A_x\mathbf{a}_x\cdot\mathbf{a}_\rho + A_y\mathbf{a}_y\cdot\mathbf{a}_\rho + A_z\mathbf{a}_z\cdot\mathbf{a}_\rho = A_x\cos\phi + A_y\sin\phi$
  - $A_\phi = \mathbf{A}\cdot\mathbf{a}_\phi = \left(A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z\right)\cdot\mathbf{a}_\phi$
    $= A_x\mathbf{a}_x\cdot\mathbf{a}_\phi + A_y\mathbf{a}_y\cdot\mathbf{a}_\phi + A_z\mathbf{a}_z\cdot\mathbf{a}_\phi = -A_x\sin\phi + A_y\cos\phi$
  - $A_z = \mathbf{A}\cdot\mathbf{a}_z = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z)\cdot\mathbf{a}_z = A_x\mathbf{a}_x\cdot\mathbf{a}_z + A_y\mathbf{a}_y\cdot\mathbf{a}_z + A_z\mathbf{a}_z\cdot\mathbf{a}_z = A_z$
  - $\therefore\ \mathbf{A} = (A_x\cos\phi + A_y\sin\phi)\mathbf{a}_\rho + (-A_x\sin\phi + A_y\cos\phi)\mathbf{a}_\phi + A_z\mathbf{a}_z$

1.8 원통좌표계(cylindrical coordinate system)

- (ex 8) $A(x = 2, y = 3, z = -1)$, $B(\rho = 4, \phi = -50°, z = 2)$

  (a) 벡터$\mathbf{r}_A$의 크기 (b) 벡터$\mathbf{r}_B$의 크기 (c) 벡터$\mathbf{R}_{AB}$의 크기

  - (a) $|\mathbf{r}_A| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14} = 3.74$
  - (b) $B(x = \rho\cos\phi\,, y = \rho\sin\phi\,, z = z)$

    $= B(4\cos(-50°), 4\sin(-50°), 2)$

    $= B(2.57, -3.064, 2)$

    $\therefore |\mathbf{r}_B| = \sqrt{2.57^2 + (-3.064)^2 + 2^2} = 4.47$
  - (c) $|\mathbf{R}_{AB}| = |\mathbf{r}_B - \mathbf{r}_A| = \left|0.57\mathbf{a}_x - 6.064\mathbf{a}_y + 3\mathbf{a}_z\right|$

    $= \sqrt{0.57^2 + (-6.064)^2 + 3^2} = 6.79$

1.8 원통좌표계(cylindrical coordinate system)

- (ex 9) 직각좌표계 → 원통좌표계

(a) $\mathbf{A} = 5\mathbf{a}_x$ at $P(\rho = 4, \phi = 120°, z = 2)$

➢ $A_\rho = \mathbf{A} \cdot \mathbf{a}_\rho$

$$= 5\mathbf{a}_x \cdot \mathbf{a}_\rho = 5\cos\phi = 5\cos 120°$$

$$= 5\cos(90° + 30°) = 5 \times (-\sin 30°) = -2.5$$

➢ $A_\phi = \mathbf{A} \cdot \mathbf{a}_\phi$

$$= 5\mathbf{a}_x \cdot \mathbf{a}_\phi = -5\sin\phi = -5\sin 120°$$

$$= -5\sin(90° + 30°) = -5\cos 30° = -\frac{5\sqrt{3}}{2} = -4.33$$

➢ $\therefore \mathbf{A} = A_\rho \mathbf{a}_\rho + A_\phi \mathbf{a}_\phi + A_z \mathbf{a}_z = -2.5\mathbf{a}_\rho - 4.33\mathbf{a}_\phi$

(b) $\mathbf{B} = 5\mathbf{a}_x$ at $Q(x = 3, y = 4, z = -1)$

- $Q(x = 3, y = 4, z = -1) = Q\left(\rho = \sqrt{x^2 + y^2}, \phi = \tan^{-1}\frac{y}{x}, z\right)$

  $= Q(\sqrt{3^2 + 4^2}, \tan^{-1}\frac{4}{3}, -1) = Q(5, 53.1°, -1)$

- $B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho$

  $= 5\mathbf{a}_x \cdot \mathbf{a}_\rho = 5\cos\phi = 5\cos 53.1° = 5 \times 0.6 = 3$

- $B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi$

  $= 5\mathbf{a}_x \cdot \mathbf{a}_\phi = -5\sin\phi = -5\sin 53.1° = -5 \times 0.8 = -4$

- $\therefore \mathbf{B} = B_\rho \mathbf{a}_\rho + B_\phi \mathbf{a}_\phi + B_z \mathbf{a}_z$

  $= 3\mathbf{a}_\rho - 4\mathbf{a}_\phi$

1.8 원통좌표계(cylindrical coordinate system)

(c) $\mathbf{C} = 4\mathbf{a}_x - 2\mathbf{a}_y - 4\mathbf{a}_z$ at $A(x = 2, y = 3, z = 5)$

- $A(x = 2, y = 3, z = 5) = A\left(\rho = \sqrt{x^2 + y^2}, \phi = \tan^{-1}\frac{y}{x}, z\right)$

$$= A(\sqrt{2^2 + 3^2}, \tan^{-1}\frac{3}{2}, 5) = A(\sqrt{13}, 56.31°, 5)$$

- $C_\rho = \mathbf{C} \cdot \mathbf{a}_\rho = \left(4\mathbf{a}_x - 2\mathbf{a}_y - 4\mathbf{a}_z\right) \cdot \mathbf{a}_\rho$

$$= 4\mathbf{a}_x \cdot \mathbf{a}_\rho - 2\mathbf{a}_y \cdot \mathbf{a}_\rho - 4\mathbf{a}_z \cdot \mathbf{a}_\rho = 4\cos 56.31° - 2\sin 56.31° = 0.555$$

- $C_\phi = \mathbf{C} \cdot \mathbf{a}_\phi = \left(4\mathbf{a}_x - 2\mathbf{a}_y - 4\mathbf{a}_z\right) \cdot \mathbf{a}_\phi$

$$= 4\mathbf{a}_x \cdot \mathbf{a}_\phi - 2\mathbf{a}_y \cdot \mathbf{a}_\phi - 4\mathbf{a}_z \cdot \mathbf{a}_\phi = -4\sin 56.31° - 2\cos 56.31° = -4.44$$

- $\therefore \mathbf{C} = C_\rho\mathbf{a}_\rho + C_\phi\mathbf{a}_\phi + C_z\mathbf{a}_z = 0.555\mathbf{a}_\rho - 4.44\mathbf{a}_\phi - 4\mathbf{a}_z$

1.8 원통좌표계(cylindrical coordinate system)

- (예제 1.3) $\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$→ 원통좌표계
  - $B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = \left(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z\right) \cdot \mathbf{a}_\rho$

    $= y\mathbf{a}_x \cdot \mathbf{a}_\rho - x\mathbf{a}_y \cdot \mathbf{a}_\rho + z\mathbf{a}_z \cdot \mathbf{a}_\rho$

    $= y\cos\phi - x\sin\phi$

    $= \rho\sin\phi\cos\phi - \rho\cos\phi\sin\phi = 0$
  - $B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = \left(B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z\right) \cdot \mathbf{a}_\phi$

    $= y\mathbf{a}_x \cdot \mathbf{a}_\phi - x\mathbf{a}_y \cdot \mathbf{a}_\phi + z\mathbf{a}_z \cdot \mathbf{a}_\phi$

    $= -y\sin\phi - x\cos\phi$

    $= -\rho\sin^2\phi - \rho\cos^2\phi = -\rho$
  - $\therefore \mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z$

1.8 원통좌표계(cylindrical coordinate system)

- (응용예제 1.5)
  - (a) 점$C(\rho = 4.4, \phi = -115°, z = 2)$→ 직각좌표계
    - $C(x = -1.860, y = -3.99, z = 2)$
  - (b) $D(x = -3.1, y = 2.6, z = -3)$를 원통좌표계로 변환
    - $D(\rho = 4.05, \phi = 140.0°, z = -3)$
  - (c) 점C에서 점D에 이르는 거리
    - 8.36

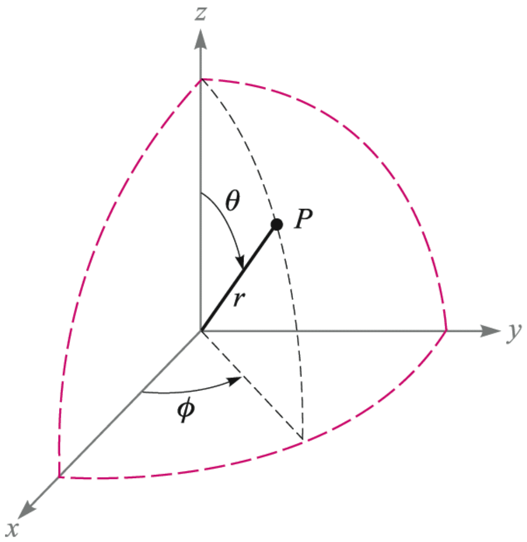
1.8 원통좌표계(cylindrical coordinate system)

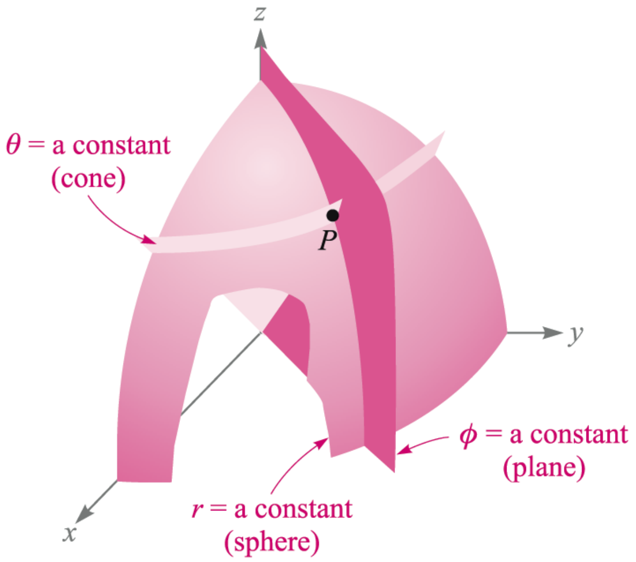
- (응용예제 1.6)
  - (a) 점$P(10,-8,6)$에서 $\mathbf{F} = 10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z$→ 원통좌표계
    - $\mathbf{F} = 12.81\mathbf{a}_\rho + 6\mathbf{a}_z$
  - (b) 점$Q(\rho,\phi,z)$에서 $\mathbf{G} = (2x+y)10\mathbf{a}_x - (y-4x)\mathbf{a}_y$→ 원통좌표계
  - (c) $P(5,2,-1)$에서 $\mathbf{G} = 20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z$→ 직각좌표계

1.9 구좌표계(spherical coordinate system)

- 점의 표시; $P(r_1, \theta_1, \phi_1)$
- $P(r_1, \theta_1, \phi_1)$의 의미;

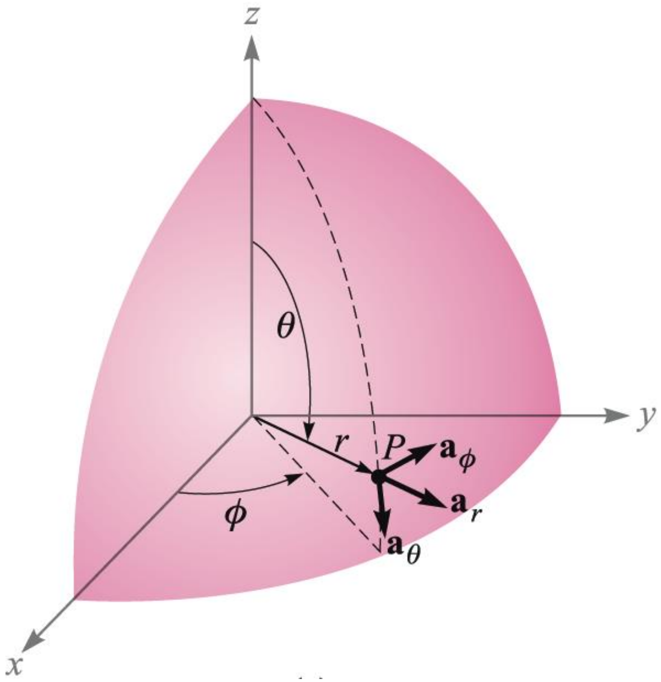
  $r = r_1, \theta = \theta_1, \phi = \phi_1$인 평면들의 교차점





- 기본단위벡터; $\mathbf{a}_r, \mathbf{a}_\theta, \mathbf{a}_\phi$← 위치에 대한 함수

- 오른손 좌표계; $\mathbf{a}_r \times \mathbf{a}_\theta = \mathbf{a}_\phi$

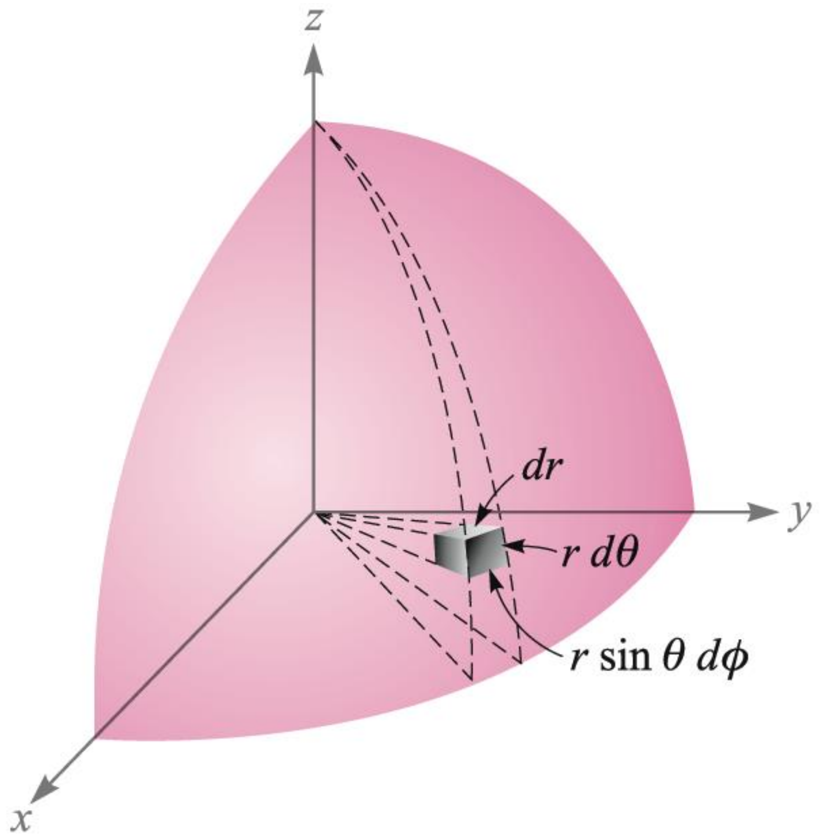


1.9 구좌표계(spherical coordinate system)

- 미소체적소의 정의;
  - 점 $P(r, \theta, \phi)$의 각 좌표값이 각각 미소량만큼 증가한 점이 $P'(r+dr, \theta+d\theta, \phi+d\phi)$일 때
  - 미소체적소;

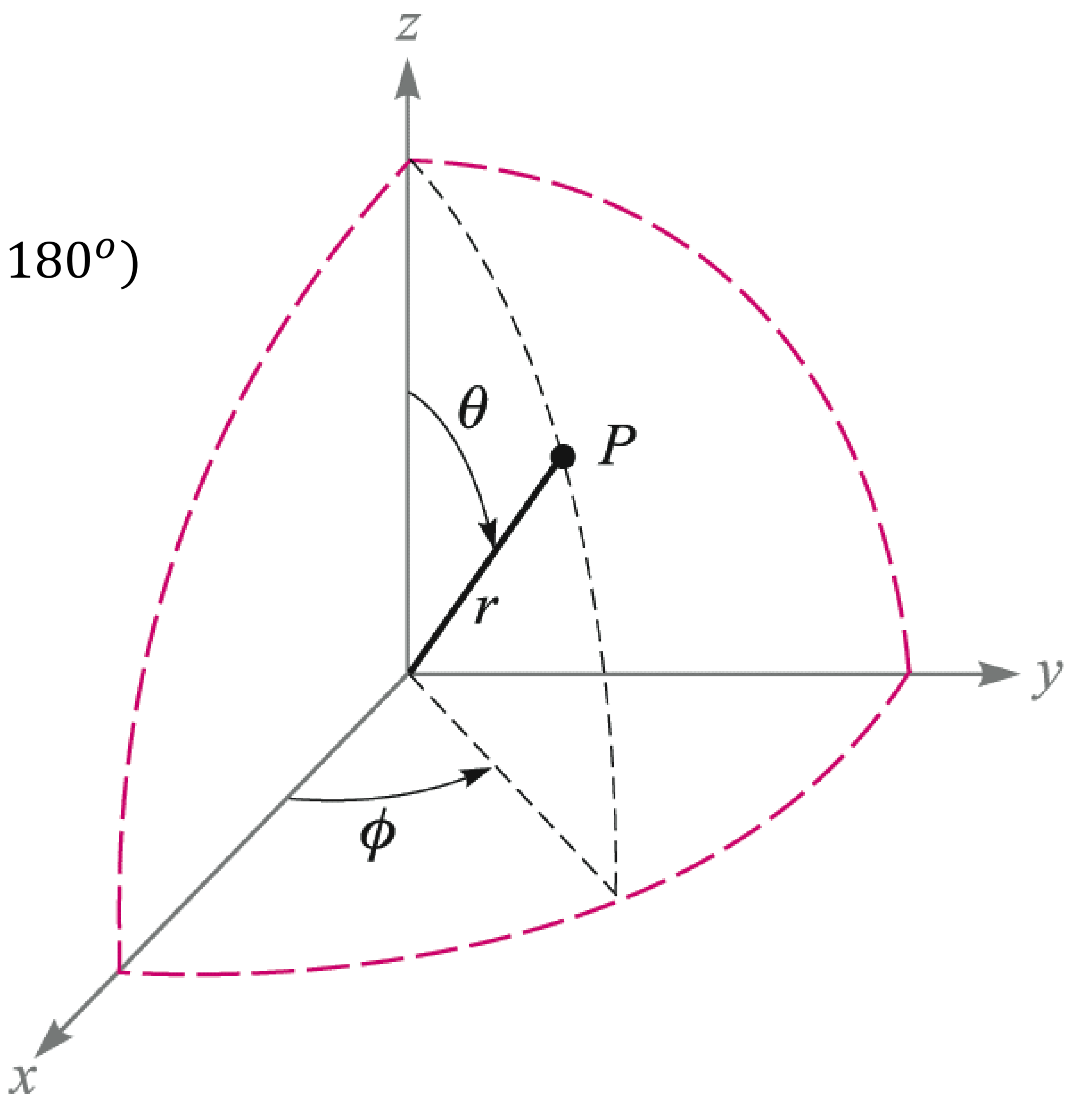
    $dv = dr \cdot r\sin\theta\, d\phi \cdot r d\theta = r^2 \sin\theta\, dr d\theta d\phi$
  - 면적소(dS);

    $r dr d\theta, r^2 \sin\theta\, d\theta d\phi, r\sin\theta\, dr d\phi$



1.9 구좌표계(spherical coordinate system)

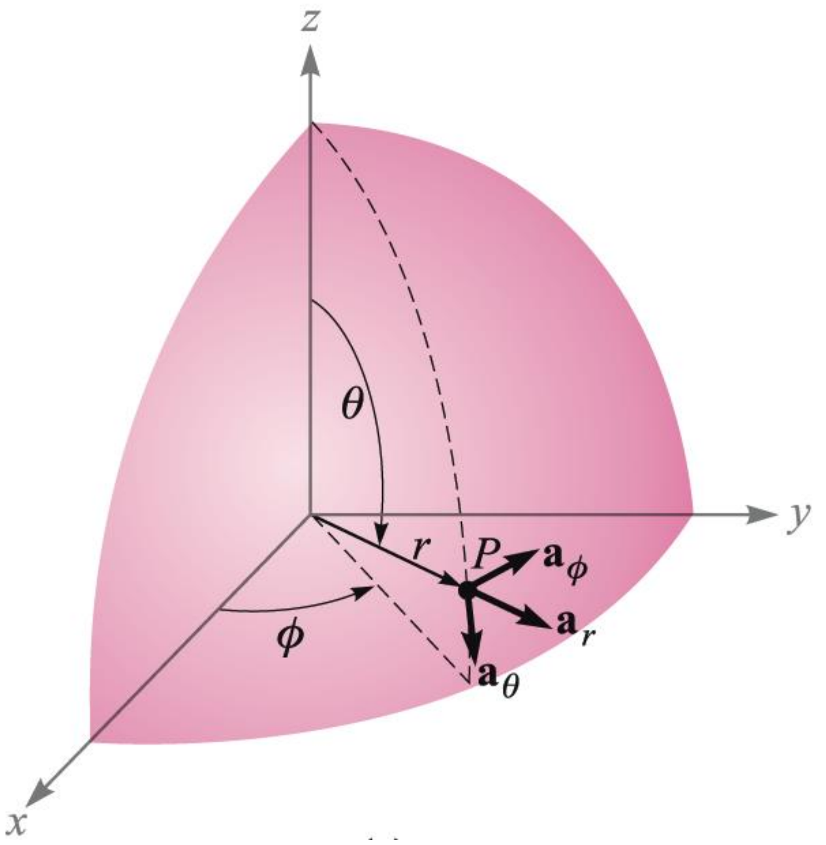
- 직교좌표계 → 구좌표계의 변환
  - $r = \sqrt{x^2 + y^2 + z^2} (r \geq 0)$
  - $\theta = \cos^{-1}\frac{z}{r} = \cos^{-1}\frac{r}{\sqrt{x^2+y^2+z^2}} (0^o \leq \theta \leq 180^o)$
  - $\phi = \tan^{-1}\frac{y}{x}$
- 구좌표계 → 직교좌표계의 변환
  - $x = r\sin\theta\cos\phi$
  - $y = r\sin\theta\sin\phi$
  - $z = r\cos\theta$



1.9 구좌표계(spherical coordinate system)

- 직교, 구좌표계 기본단위벡터의 스칼라곱

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0



1.9 구좌표계(spherical coordinate system)

- (ex 10) $A(x=2, y=3, z=-1)$, $B(r=4, \theta=25°, \phi=120°)$

  (a) 점A→ 구좌표계

  (b) 점B→ 직각좌표계

  (c) 점A와 점B사이의 거리

➢ (a) $r=\sqrt{x^2+y^2+z^2}=\sqrt{2^2+3^2+(-1)^2}=\sqrt{14}=3.74$

$$\theta=\cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}=\cos^{-1}\frac{-1}{\sqrt{14}}=105.5°$$

$$\phi=\tan^{-1}\frac{y}{x}=\tan^{-1}\frac{3}{2} \qquad =56.3°$$

$$\therefore A(r=3.74, \theta=105.5°, \phi=56.3°)$$

1.9 구좌표계(spherical coordinate system)

- (ex 10) $A(x = 2, y = 3, z = -1)$, $B(r = 4, \theta = 25°, \phi = 120°)$
  - (b) $x = r\sin\theta\cos\phi = 4\sin 25\,°\cos 120\,° = -0.845$

    $y = r\sin\theta\sin\phi = 4\sin 25\,°\sin 120\,° = 1.464$

    $z = r\cos\theta = 4\cos 25\,° = 3.63$

    $$\therefore B(x = -0.845, y = 1.464, z = 3.63)$$

  - (c) $|\mathbf{R}_{AB}| = |\mathbf{r}_B - \mathbf{r}_A| = |-2.845\mathbf{a}_x - .536\mathbf{a}_y + 4.63\mathbf{a}_z|$

    $$= \sqrt{(-2.845^2 + (-1.536)^2 + 4.63^2} = 5.64$$

1.9 구좌표계(spherical coordinate system)

- (ex 11) 직각좌표계 → 구좌표계

(a) $\mathbf{A} = 5\mathbf{a}_x$ at $B(r = 4, \theta = 25^o, \phi = 120^o)$

- $A_r = \mathbf{A} \cdot \mathbf{a}_r$

  $= 5\mathbf{a}_x \cdot \mathbf{a}_r = 5 \sin\theta \cos\phi$

  $= 5 \sin 25° \cos 120° = -1.057$

- $A_\theta = \mathbf{A} \cdot \mathbf{a}_\theta$

  $= 5\mathbf{a}_x \cdot \mathbf{a}_\theta = 5 \cos\theta \cos\phi$

  $= 5 \cos 25° \cos 120° = -2.27$

- $A_\phi = \mathbf{A} \cdot \mathbf{a}_\phi$

  $= 5\mathbf{a}_x \cdot \mathbf{a}_\phi = 5(-\sin\phi)$

  $= -5 \sin 120° = -4.33$

- $\therefore \mathbf{A} = A_r\mathbf{a}_r + A_\theta\mathbf{a}_\theta + A_\phi\mathbf{a}_\phi = -1.057\mathbf{a}_r - 2.27\mathbf{a}_\theta - 4.33\mathbf{a}_\phi$

(b) $\mathbf{B} = 5\mathbf{a}_x$ at $A(x = 2, y = 3, z = -1)$

- $A(x = 2, y = 3, z = -1)$

$$\rightarrow A\left(r = \sqrt{x^2 + y^2 + z^2}, \theta = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}, \phi = \tan^{-1}\frac{y}{x}\right)$$

$$\rightarrow A\left(r = \sqrt{2^2 + 3^2 + (-1)^2}, \theta = \cos^{-1} - \frac{1}{\sqrt{2^2+3^2+(-1)^2}}, \phi = \tan^{-1}\frac{3}{2}\right)$$

$$\rightarrow A\left(r = \sqrt{14}, \theta = \cos^{-1}\frac{-1}{\sqrt{14}}, \phi = \tan^{-1}\frac{3}{2}\right)$$

- $B_r = \mathbf{B} \cdot \mathbf{a}_r$

$$= 5\mathbf{a}_x \cdot \mathbf{a}_r = 5\sin\theta\cos\phi$$

$$= 5\sin\left(\cos^{-1}\frac{-1}{\sqrt{14}}\right)\cos\left(\tan^{-1}\frac{3}{2}\right) = 2.67$$

(b) $\mathbf{B} = 5\mathbf{a}_x$ at $A(x = 2, y = 3, z = -1)$

- $A(x = 2, y = 3, z = -1) \rightarrow A\left(r = \sqrt{14}, \theta = \cos^{-1}\frac{-1}{\sqrt{14}}, \phi = \tan^{-1}\frac{3}{2}\right)$

- $B_r = \mathbf{B} \cdot \mathbf{a}_r = 5\mathbf{a}_x \cdot \mathbf{a}_r = 5\sin\theta\cos\phi = 5\sin\left(\cos^{-1}\frac{-1}{\sqrt{14}}\right)\cos\left(\tan^{-1}\frac{3}{2}\right) = 2.67$

- $B_\theta = \mathbf{B} \cdot \mathbf{a}_\theta = 5\mathbf{a}_x \cdot \mathbf{a}_\theta$

  $= 5\cos\theta\cos\phi = 5\cos\left(\cos^{-1}\frac{-1}{\sqrt{14}}\right)\cos\left(\tan^{-1}\frac{3}{2}\right) \quad = -0.741$

- $B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = 5\mathbf{a}_x \cdot \mathbf{a}_\phi$

  $= 5(-\sin\phi) = -5\sin\left(\tan^{-1}\frac{3}{2}\right) = -4.16$

- $\therefore \mathbf{B} = B_r\mathbf{a}_r + B_\theta\mathbf{a}_\theta + B_\phi\mathbf{a}_\phi = 2.67\mathbf{a}_r - 0.741\mathbf{a}_\theta - 4.16\mathbf{a}_\phi$

1.9 구좌표계(spherical coordinate system)

(c) $\mathbf{C} = 4\mathbf{a}_x - 2\mathbf{a}_y - 4\mathbf{a}_z$ at $P(x = -2, y = -3, z = 4)$

- $P(x = -2, y = -3, z = 4) \rightarrow P(r = \quad, \theta = \quad, \phi = \quad)$
- $C_r = \mathbf{C} \cdot \mathbf{a}_r = (4\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z) \cdot \mathbf{a}_r$

$$= 5\mathbf{a}_x \cdot \mathbf{a}_r - 2\mathbf{a}_y \cdot \mathbf{a}_r + 4\mathbf{a}_z \cdot \mathbf{a}_r$$

$$= 5\sin\theta\cos\phi - 2\sin\theta\sin\phi + 4\cos\theta$$

$$= -3.34$$

- $C_\theta = \mathbf{C} \cdot \mathbf{a}_\theta = (4\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z) \cdot \mathbf{a}_\theta =$
- $C_\phi = \mathbf{C} \cdot \mathbf{a}_\phi = (4\mathbf{a}_x - 2\mathbf{a}_y + 4\mathbf{a}_z) \cdot \mathbf{a}_\phi =$
- $\therefore \mathbf{C} = C_r\mathbf{a}_r + C_\theta\mathbf{a}_\theta + C_\phi\mathbf{a}_\phi = -3.34\mathbf{a}_r + 2.27\mathbf{a}_\theta + 4.44\mathbf{a}_\phi$

1.9 구좌표계(spherical coordinate system)

- (예제 1.4) $\mathbf{G} = \frac{xz}{y}\mathbf{a}_x \rightarrow$ 구좌표계
  - $G_r = \mathbf{G} \cdot \mathbf{a}_r$

$$= \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_r = \frac{xz}{y}\sin\theta\cos\phi = \frac{r\sin\theta\cos\phi \cdot r\cos\theta}{r\sin\theta\sin\phi}\sin\theta\cos\phi = r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}$$

  - $G_\theta = \mathbf{G} \cdot \mathbf{a}_\theta$

$$= \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\theta = \frac{xz}{y}\cos\theta\cos\phi = \frac{r\sin\theta\cos\phi \cdot r\cos\theta}{r\sin\theta\sin\phi}\cos\theta\cos\phi = r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}$$

  - $G_\phi = \mathbf{G} \cdot \mathbf{a}_\phi$

$$= \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi) = \frac{r\sin\theta\cos\phi \cdot r\cos\theta}{r\sin\theta\sin\phi}(-\sin\phi) = -r\cos\theta\cos\phi$$

$$\therefore\ \mathbf{G} = r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}\mathbf{a}_r + r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}\mathbf{a}_\theta - r\cos\theta\cos\phi\,\mathbf{a}_\phi$$

$$= r\cos\theta\cos\phi\left(\sin\theta\frac{\cos\phi}{\sin\phi}\mathbf{a}_r + \cos\theta\frac{\cos\phi}{\sin\phi}\mathbf{a}_\theta - \mathbf{a}_\phi\right)$$

$$= r\cos\theta\cos\phi\left(\sin\theta\cot\phi\,\mathbf{a}_r + \cos\theta\cot\phi\,\mathbf{a}_\theta - \mathbf{a}_\phi\right)$$

1.9 구좌표계(spherical coordinate system)

- (응용예제 1.7) $C(-3,2,1)$, $D(r=5, \theta=20°, \phi=-70°)$

  (a) C의 구좌표

  (b) D의 직각좌표

  (c) C와 D사이의 거리

- (응용예제 1.8) 직각좌표계 → 구좌표계

  (a) $10\mathbf{a}_x$ at $P(x=-3, y=2, z=4)$

  (b) $10\mathbf{a}_y$ at $Q(\rho=5, \phi=30°, z=4)$

  (c) $10\mathbf{a}_z$ at $M(r=4, \theta=110°, \phi=120°)$