1. Define percent error as $100\left[e^{x}-(1+x)\right] / e^{x}$

| $x$ | $1+x$ | $e^{x}$ | \% error |
| :--- | :--- | :--- | :--- |
| 0.001 | 1.001 | 1.001 | $5 \times 10^{-5}$ |
| 0.005 | 1.005 | 1.005 | $1 \times 10^{-3}$ |
| 0.01 | 1.01 | 1.010 | $5 \times 10^{-3}$ |
| 0.05 | 1.05 | 1.051 | 0.1 |
| 0.10 | 1.10 | 1.105 | 0.5 |
| 0.50 | 1.50 | 1.649 | 9 |
| 1.00 | 2.00 | 2.718 | 26 |
| 5.00 | 6.00 | 148.4 | 96 |

Of course, "reasonable" is a very subjective term. However, if we choose $x<0.1$, we ensure that the error is less than $1 \%$.
2. (a) Short-circuit the 10 V source.

Note that $6 \| 4=2.4 \Omega$. By voltage division, the voltage across the $6 \Omega$ resistor is then

$$
4 \frac{2.4}{3+2.4}=1.778 \mathrm{~V}
$$

So that $i_{1}^{\prime}=\frac{1.778}{6}=0.2963 \mathrm{~A}$.
(b) Short-circuit the 4 V source.

Note that $3 \| 6=2 \Omega$. By voltage division, the voltage across the $6 \Omega$ resistor is then

$$
-10 \frac{2}{6}=-3.333 \mathrm{~V}
$$

So that $i_{1}^{\prime \prime}=\frac{-3.333}{6}=-0.5556 \mathrm{~A}$.
(c) $i_{1}=i_{1}^{\prime}+i_{2}^{\prime \prime}=-259.3 \mathrm{~mA}$
3. Open circuit the 4 A source. Then, since
$(7+2) \|(5+5)=4.737 \Omega$, we can calculate $v_{1}^{\prime}=(1)(4.737)=4.737 \mathrm{~V}$.
To find the total current flowing through the $7 \Omega$ resistor, we first determine the total voltage $v_{1}$ by continuing our superposition procedure. The contribution to $v_{1}$ from the 4 A source is found by first open-circuiting the 1 A source, then noting that current division yields:

$$
4 \frac{5}{5+(5+7+2)}=\frac{20}{19}=1.053 \mathrm{~A}
$$

Then, $v_{1}^{\prime \prime}=(1.053)(9)=9.477 \mathrm{~V}$. Hence, $v_{1}=v_{1}^{\prime}+v_{1}^{\prime \prime}=14.21 \mathrm{~V}$.

We may now find the total current flowing downward through the $7 \Omega$ resistor as
$14.21 / 7=2.03 \mathrm{~A}$.
4. One approach to this problem is to write a set of mesh equations, leaving the voltage source and current source as variables which can be set to zero.

We first rename the voltage source as $V_{\mathrm{x}}$. We next define three clockwise mesh currents in the bottom three meshes: $i_{1}, i_{y}$ and $i_{4}$. Finally, we define a clockwise mesh current $i_{3}$ in the top mesh, noting that it is equal to -4 A .

Our general mesh equations are then:

$$
\begin{aligned}
&-\mathrm{V}_{\mathrm{x}}+ 18 \mathrm{i}_{1}-10 \mathrm{i}_{\mathrm{y}}=0 \\
&-10 \mathrm{i}_{1}+ 15 \mathrm{i}_{\mathrm{y}}-3 \mathrm{i}_{4}=0 \\
&-3 \mathrm{i}_{\mathrm{y}}+16 \mathrm{i}_{4}-5 \mathrm{i}_{3}=0
\end{aligned}
$$

** Set $V_{x}=10 V, i_{3}=0$. Our mesh equations then become:

$$
18 i_{1}-10 i_{y}^{\prime}=10
$$

$$
-10 i_{1}+15 i_{y}^{\prime}-3 i_{4}=0
$$

$$
-3 i_{y}^{\prime}+16 i_{4}=0
$$

Solving, $\underline{i}_{y}^{\prime}=0.6255 \mathrm{~A}$.
** Set $\mathrm{V}_{\mathrm{x}}=0 \mathrm{~V}, \mathrm{i}_{3}=-4 \mathrm{~A}$. Our mesh equations then become:

$$
\begin{aligned}
& 18 i_{1}-10 i_{y}^{\prime \prime} \quad=0 \\
&-10 i_{1}+15 i_{y}^{\prime \prime}-3 i_{4}=0 \\
&-3 i_{y}^{\prime \prime}+16 i_{4}=-20
\end{aligned}
$$

Solving, $i_{y}^{\prime \prime}=-0.4222 \mathrm{~A}$.

Thus, $\mathrm{i}_{\mathrm{y}}=i_{y}^{\prime}+i_{y}^{\prime \prime}=203.3 \mathrm{~mA}$.
5. We may solve this problem without writing circuit equations if we first realise that the current $i_{1}$ is composed of two terms: one that depends solely on the 4 V source, and one that depends solely on the 10 V source.

This may be written as $i_{1}=4 \mathrm{~K}_{1}+10 \mathrm{~K}_{2}$, where $\mathrm{K}_{1}$ and $\mathrm{K}_{2}$ are constants that depend on the circuit topology and resistor values.

We may not change $\mathrm{K}_{1}$ or $\mathrm{K}_{2}$, as only the source voltages may be changed. If we increase both sources by a factor of 10 , then $i_{1}$ increases by the same amount.
Thus, $4 \mathrm{~V} \rightarrow 40 \mathrm{~V}$ and $10 \mathrm{~V} \rightarrow 100 \mathrm{~V}$.
6. $\quad i_{\mathrm{A}}, v_{\mathrm{B}}$ "on", $v_{\mathrm{C}}=0: \quad i_{\mathrm{x}}=20 \mathrm{~A}$
$i_{\mathrm{A}}, v_{\mathrm{C}}$ "on", $v_{\mathrm{B}}=0: \quad i_{\mathrm{x}}=-5 \mathrm{~A}$
$i_{\mathrm{A}}, v_{\mathrm{B}}, v_{\mathrm{C}} " \mathrm{on} ": \quad i_{\mathrm{x}}=12 \mathrm{~A}$
so, we can write $\quad i_{\mathrm{x}}{ }^{\prime}+i_{\mathrm{x}}{ }^{\prime \prime}+i_{\mathrm{x}}{ }^{\prime \prime}=12$
$i_{\mathrm{x}}{ }^{\prime}+i_{\mathrm{x}}{ }^{\prime \prime}=20$
$i_{x}{ }^{\prime}+i_{x}{ }^{\prime \prime}=-5$

In matrix form,

$$
\left[\begin{array}{lll}
1 & 1 & 1 \\
1 & 1 & 0 \\
1 & 0 & 1
\end{array}\right]\left[\begin{array}{l}
i_{x}^{\prime} \\
i_{x}^{\prime \prime} \\
i_{x}^{\prime \prime \prime}
\end{array}\right]=\left[\begin{array}{c}
12 \\
20 \\
-5
\end{array}\right]
$$

(a) with $i_{\mathrm{A}}$ on only, the response $i_{\mathrm{x}}=i_{\mathrm{x}}{ }^{\prime}=3 \mathrm{~A}$.
(b) with $\nu_{\mathrm{B}}$ on only, the response $i_{\mathrm{x}}=i_{\mathrm{x}}{ }^{\prime \prime}=17 \mathrm{~A}$
(c) with $v_{\mathrm{C}}$ on only, the response $i_{\mathrm{x}}=i_{\mathrm{x}}$ " $=-8 \mathrm{~A}$.
(d) $i_{\mathrm{A}}$ and $v_{\mathrm{C}}$ doubled, $v_{\mathrm{B}}$ reversed: $2(3)+2(-8)+(-1)(17)=-27 \mathrm{~A}$.
7. One source at a time:

The contribution from the $24-\mathrm{V}$ source may be found by shorting the $45-\mathrm{V}$ source and open-circuiting the $2-\mathrm{A}$ source. Applying voltage division,

$$
v_{\mathrm{x}}^{\prime}=24 \frac{20}{10+20+45 \| 30}=24 \frac{20}{10+20+18}=10 \mathrm{~V}
$$

We find the contribution of the 2-A source by shorting both voltage sources and applying current division:

$$
v_{\mathrm{x}}^{\prime \prime}=20\left[2 \frac{10}{10+20+18}\right]=8.333 \mathrm{~V}
$$

Finally, the contribution from the $45-\mathrm{V}$ source is found by open-circuiting the 2-A source and shorting the $24-\mathrm{V}$ source. Defining $v_{30}$ across the $30-\Omega$ resistor with the " + " reference on top:

$$
0=v_{30} / 20+v_{30} /(10+20)+\left(v_{30}-45\right) / 45
$$

solving, $v_{30}=11.25 \mathrm{~V}$ and hence $v_{\mathrm{x}} ">=-11.25(20) /(10+20)=-7.5 \mathrm{~V}$
Adding the individual contributions, we find that $v_{\mathrm{x}}=v_{\mathrm{x}}{ }^{\prime}+v_{\mathrm{x}}{ }^{\prime \prime}+v_{\mathrm{x}}{ }^{\prime \prime}=10.83 \mathrm{~V}$.
8. The contribution of the 8 -A source is found by shorting out the two voltage sources and employing simple current division:

$$
i_{3}{ }^{\prime}=-8 \frac{50}{50+30}=-5 \mathrm{~A}
$$

The contribution of the voltage sources may be found collectively or individually. The contribution of the $100-\mathrm{V}$ source is found by open-circuiting the $8-\mathrm{A}$ source and shorting the $60-\mathrm{V}$ source. Then,

$$
i_{3}{ }^{\prime \prime}=\frac{100}{(50+30)\|60\| 30}=6.25 \mathrm{~A}
$$

The contribution of the $60-\mathrm{V}$ source is found in a similar way as $i_{3}{ }^{\prime \prime}=-60 / 30=-2 \mathrm{~A}$.
The total response is $i_{3}=i_{3}{ }^{\prime}+i_{3}{ }^{\prime \prime}+i_{3}{ }^{\prime \prime \prime}=-750 \mathrm{~mA}$.
9. (a) By current division, the contribution of the 1-A source $i_{2}$ ' is $i_{2}{ }^{\prime}=1(200) / 250=800 \mathrm{~mA}$.

The contribution of the $100-\mathrm{V}$ source is $i_{2} "=100 / 250=400 \mathrm{~mA}$.
The contribution of the $0.5-\mathrm{A}$ source is found by current division once the $1-\mathrm{A}$ source is open-circuited and the voltage source is shorted. Thus,

$$
i_{2}{ }^{\prime \prime \prime}=0.5(50) / 250=100 \mathrm{~mA}
$$

Thus, $i_{2}=i_{2}{ }^{\prime}+i_{2} "+i_{2} ",=1.3 \mathrm{~A}$
(b) $\mathrm{P}_{1 \mathrm{~A}}=(1)[(200)(1-1.3)]=60 \mathrm{~W}$
$\mathrm{P}_{200}=(1-1.3)^{2}(200)=18 \mathrm{~W}$
$\mathrm{P}_{100 \mathrm{~V}}=-(1.3)(100)=-130 \mathrm{~W}$
$\mathrm{P}_{50}=(1.3-0.5)^{2}(50)=32 \mathrm{~W}$
$\mathrm{P}_{0.5 \mathrm{~A}}=(0.5)[(50)(1.3-0.5)]=20 \mathrm{~W}$
Check: $60+18+32+20=+130$.
10. We find the contribution of the 4-A source by shorting out the $100-\mathrm{V}$ source and analysing the resulting circuit:


$$
\begin{align*}
& 4=\mathrm{V}_{1}{ }^{\prime} / 20+\left(\mathrm{V}_{1}{ }^{\prime}-\mathrm{V}^{\prime}\right) / 10  \tag{1}\\
& 0.4 i_{1}{ }^{\prime}=\mathrm{V}_{1}{ }^{\prime} / 30+\left(\mathrm{V}^{\prime}-\mathrm{V}^{\prime}{ }^{\prime}\right) / 10 \tag{2}
\end{align*}
$$

where $i_{1}{ }^{\prime}=V_{1}{ }^{\prime} / 20$
Simplifying \& collecting terms, we obtain $\quad 30 \mathrm{~V}_{1}{ }^{\prime}-20 \mathrm{~V}^{\prime}=800$

$$
\begin{equation*}
-7.2 \mathrm{~V}_{1}^{\prime}+8 \mathrm{~V}^{\prime}=0 \tag{1}
\end{equation*}
$$

Solving, we find that $\mathrm{V}^{\prime}=60 \mathrm{~V}$. Proceeding to the contribution of the $60-\mathrm{V}$ source, we analyse the following circuit after defining a clockwise mesh current $i_{a}$ flowing in the left mesh and a clockwise mesh current $i_{\mathrm{b}}$ flowing in the right mesh.


Solving, we find that $i_{\mathrm{a}}=1.25 \mathrm{~A}$ and so $\mathrm{V}^{\prime \prime}=30\left(i_{\mathrm{a}}-i_{\mathrm{b}}\right)=22.5 \mathrm{~V}$.
Thus, $\mathrm{V}=\mathrm{V}^{\prime}+\mathrm{V}^{\prime \prime}=82.5 \mathrm{~V}$.
11. (a) Linearity allows us to consider this by viewing each source as being scaled by $25 / 10$. This means that the response $\left(v_{3}\right)$ will be scaled by the same factor:

$$
\begin{array}{r}
25 i_{\mathrm{A}}{ }^{\prime} / 10+25 i_{\mathrm{B}}^{\prime} / 10=25 v_{3}^{\prime} / 10 \\
\therefore v_{3}=25 v_{3}^{\prime} / 10=25(80) / 10=200 \mathrm{~V}
\end{array}
$$

(b) $\quad i_{\mathrm{A}}{ }^{\prime}=10 \mathrm{~A}, i_{\mathrm{B}}{ }^{\prime}=25 \mathrm{~A} \quad \rightarrow v_{4}{ }^{\prime}=100 \mathrm{~V}$
$i_{\mathrm{A}}{ }^{\prime \prime}=10 \mathrm{~A}, i_{\mathrm{B}}{ }^{\prime \prime}=25 \mathrm{~A} \quad \rightarrow v_{4} "=-50 \mathrm{~V}$
$i_{\mathrm{A}}=20 \mathrm{~A}, i_{\mathrm{B}}=-10 \mathrm{~A} \quad \rightarrow v_{4}=$ ?
We can view this in a somewhat abstract form: the currents $i_{\mathrm{A}}$ and $i_{\mathrm{B}}$ multiply the same circuit parameters regardless of their value; the result is $v_{4}$.

Writing in matrix form, $\left[\begin{array}{ll}10 & 25 \\ 25 & 10\end{array}\right]\left[\begin{array}{l}\mathrm{a} \\ \mathrm{b}\end{array}\right]=\left[\begin{array}{c}100 \\ -50\end{array}\right]$, we can solve to find
$a=-4.286$ and $b=5.714$, so that $20 a-10 b$ leads to $v_{4}=-142.9 \mathrm{~V}$
12. With the current source open-circuited and the 7-V source shorted, we are left with $100 \mathrm{k} \|(22 \mathrm{k}+4.7 \mathrm{k})=21.07 \mathrm{k} \Omega$.

Thus, $\mathrm{V}_{3 \mathrm{~V}}=3(21.07) /(21.07+47)=0.9286 \mathrm{~V}$.
In a similar fashion, we find that the contribution of the 7-V source is:
$\mathrm{V}_{7 \mathrm{~V}}=7(31.97) /(31.97+26.7)=3.814 \mathrm{~V}$
Finally, the contribution of the current source to the voltage V across it is:
$\mathrm{V}_{5 \mathrm{~mA}}=\left(5 \times 10^{-3}\right)(47 \mathrm{k}\|100 \mathrm{k}\| 26.7 \mathrm{k})=72.75 \mathrm{~V}$.
Adding, we find that $\mathrm{V}=0.9286+3.814+72.75=77.49 \mathrm{~V}$.
13. We must find the current through the $500-\mathrm{k} \Omega$ resistor using superposition, and then calculate the dissipated power.

The contribution from the current source may be calculated by first noting that $1 \mathrm{M}\|2.7 \mathrm{M}\| 5 \mathrm{M}=636.8 \mathrm{k} \Omega$. Then,

$$
i_{60 \mu \mathrm{~A}}=60 \times 10^{-6}\left(\frac{3}{0.5+3+0.6368}\right)=43.51 \mu \mathrm{~A}
$$

The contribution from the voltage source is found by first noting that $2.7 \mathrm{M} \| 5 \mathrm{M}=$ $1.753 \mathrm{M} \Omega$. The total current flowing from the voltage source (with the current source open-circuited) is $-1.5 /(3.5 \| 1.753+1) \mu \mathrm{A}=-0.6919 \mu \mathrm{~A}$. The current flowing through the $500-\mathrm{k} \Omega$ resistor due to the voltage source acting alone is then

$$
i_{1.5 \mathrm{~V}}=0.6919(1.753) /(1.753+3.5) \mathrm{mA}=230.9 \mathrm{nA} .
$$

The total current through the $500-\mathrm{k} \Omega$ resistor is then $i_{60 \mu \mathrm{~A}}+i_{1.5 \mathrm{~V}}=43.74 \mu \mathrm{~A}$ and the dissipated power is $\left(43.74 \times 10^{-9}\right)^{2}\left(500 \times 10^{3}\right)=956.6 \mu \mathrm{~W}$.
14. We first determine the contribution of the voltage source:


Via mesh analysis, we write: $5=18000 \mathrm{I}_{1}{ }^{\prime}-17000 \mathrm{I}_{\mathrm{x}}{ }^{\prime}$

$$
-6 \mathrm{I}_{\mathrm{x}}{ }^{\prime}=-17000 \mathrm{I}_{\mathrm{x}}{ }^{\prime}+39000 \mathrm{I}_{\mathrm{x}}{ }^{\prime}
$$

Solving, we find $\mathrm{I}_{1}{ }^{\prime}=472.1 \mathrm{~mA}$ and $\mathrm{I}_{\mathrm{x}}{ }^{\prime}=205.8 \mathrm{~mA}$, so $\mathrm{V}^{\prime}=17 \times 10^{3}\left(\mathrm{I}_{1}{ }^{\prime}-\mathrm{I}_{\mathrm{x}}{ }^{\prime}\right)$ $=4.527 \mathrm{~V}$. We proceed to find the contribution of the current source:


Via supernode: $-20 \times 10^{-3}=\mathrm{V}_{\mathrm{x}}$ "/ $22 \times 10^{3}+\mathrm{V}^{\prime \prime} / 0.9444 \times 10^{3}$
and $\quad \mathrm{V}^{\prime \prime}-\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=6 \mathrm{I}_{\mathrm{x}}{ }^{\prime \prime}$ or $\mathrm{V}^{\prime \prime}-\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=6 \mathrm{~V}_{\mathrm{x}} " / 22 \times 10^{3}$
Solving, we find that $\mathrm{V}^{\prime \prime}=-18.11 \mathrm{~V}$. Thus, $\mathrm{V}=\mathrm{V}^{\prime}+\mathrm{V}^{\prime \prime}=-13.58 \mathrm{~V}$.
The maximum power is $\mathrm{V}^{2} / 17 \times 10^{3}=\mathrm{V}^{2} / 17 \mathrm{~mW}=250 \mathrm{~mW}$, so $\mathrm{V}=\sqrt{(17)(250)}=65.19=\mathrm{V}^{\prime}-(-18.11)$. Solving, we find $\mathrm{V}_{\text {max }}^{\prime}=83.3 \mathrm{~V}$. The 5-V source may then be increased by a factor of $83.3 / 4.527$, so that its maximum positive value is 92 V ; past this value, and the resistor will overheat.
15. It is impossible to identify the individual contribution of each source to the power dissipated in the resistor; superposition cannot be used for such a purpose.

Simplifying the circuit, we may at least determine the total power dissipated in the resistor:


Via superposition in one step, we may write

$$
i=\frac{5}{2+2.1}-2 \frac{2.1}{2+2.1}=195.1 \mathrm{~mA}
$$

Thus,

$$
\mathrm{P}_{2 \Omega}=i^{2} \cdot 2=76.15 \mathrm{~mW}
$$

16. We will analyse this circuit by first considering the combined effect of both dc sources (left), and then finding the effect of the single ac source acting alone (right).


1, 3 supernode: $\quad \mathrm{V}_{1} / 100+\mathrm{V}_{1} / 17 \times 10^{3}+\left(\mathrm{V}_{1}-15\right) / 33 \times 10^{3}+\mathrm{V}_{3} / 10^{3}=20 \mathrm{I}_{\mathrm{B}}[1]$ and:

$$
\begin{equation*}
\mathrm{V}_{1}-\mathrm{V}_{3}=0.7 \tag{2}
\end{equation*}
$$

Node 2: $\quad-20 \mathrm{I}_{\mathrm{B}}=\left(\mathrm{V}_{2}-15\right) / 1000$
We require one additional equation if we wish to have $\mathrm{I}_{\mathrm{B}}$ as an unknown:

$$
\begin{equation*}
20 \mathrm{I}_{\mathrm{B}}+\mathrm{I}_{\mathrm{B}}=\mathrm{V}_{3} / 1000 \tag{4}
\end{equation*}
$$

Simplifying and collecting terms,

$$
\begin{align*}
10.08912 \mathrm{~V}_{1}+\mathrm{V}_{3}-20 \times 10^{3} \mathrm{I}_{\mathrm{B}} & =0.4545  \tag{1}\\
\mathrm{~V}_{1}-\mathrm{V}_{3} & =0.7  \tag{2}\\
\mathrm{~V}_{2}+20 \times 10^{3} \mathrm{I}_{\mathrm{B}} & =15  \tag{3}\\
-\mathrm{V}_{3}+21 \times 103 \mathrm{I}_{\mathrm{B}} & =0 \tag{4}
\end{align*}
$$

Solving, we find that $\mathrm{I}_{\mathrm{B}}=-31.04 \mu \mathrm{~A}$.
To analyse the right-hand circuit, we first find the Thévenin equivalent to the left of the wire marked $i_{\mathrm{B}}{ }^{\prime}$, noting that the $33-\mathrm{k} \Omega$ and $17-\mathrm{k} \Omega$ resistors are now in parallel. We find that $\mathrm{V}_{\mathrm{TH}}=16.85 \cos 6 t \mathrm{~V}$ by voltage division, and $\mathrm{R}_{\mathrm{TH}}=100 \| 17 \mathrm{k}| | 33 \mathrm{k}=$ $99.12 \Omega$. We may now proceed:

$$
\begin{gather*}
20 i_{\mathrm{B}}{ }^{\prime}=v_{\mathrm{x}}{ }^{\prime} / 1000+\left(v_{\mathrm{x}}^{\prime}-16.85 \cos 6 t\right) / 99.12  \tag{1}\\
20 i_{\mathrm{B}}{ }^{\prime}+i_{\mathrm{B}}{ }^{\prime \prime}=v_{\mathrm{x}}^{\prime} / 1000 \tag{2}
\end{gather*}
$$

Solving, we find that $i_{B}{ }^{\prime}=798.6 \cos 6 t \mathrm{~mA}$. Thus, adding our two results, we find the complete current is

$$
i_{\mathrm{B}}=i_{\mathrm{B}}{ }^{\prime}+\mathrm{I}_{\mathrm{B}}=-31.04+798.6 \cos 6 \mathrm{t} \mu \mathrm{~A} .
$$

17. 



We first consider the effect of the 2-A source separately, using the left circuit:

$$
\mathrm{V}_{\mathrm{x}}^{\prime}=5\left[2 \frac{3}{3+14}\right]=1.765 \mathrm{~V}
$$

Next we consider the effect of the 6-A source on its own using the right circuit:

$$
\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=5\left[6 \frac{9}{9+8}\right]=15.88 \mathrm{~V}
$$

Thus, $\mathrm{V}_{\mathrm{x}}=\mathrm{V}_{\mathrm{x}}{ }^{\prime}+\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=17.65 \mathrm{~V}$.
(b) PSpice verification (DC Sweep)



18.

(a) Beginning with the circuit on the left, we find the contribution of the $2-\mathrm{V}$ source to $\mathrm{V}_{\mathrm{x}}$ :

$$
-4 \mathrm{~V}_{\mathrm{x}}^{\prime}=\frac{\mathrm{V}_{\mathrm{x}}^{\prime}}{100}+\frac{\mathrm{V}_{\mathrm{x}}^{\prime}-2}{50}
$$

which leads to $\mathrm{V}_{\mathrm{x}}{ }^{\prime}=9.926 \mathrm{mV}$.
The circuit on the right yields the contribution of the 6-A source to Vx:

$$
-4 \mathrm{~V}_{\mathrm{x}}^{\prime \prime}=\frac{\mathrm{V}_{\mathrm{x}}^{\prime \prime}}{100}+\frac{\mathrm{V}_{\mathrm{x}}^{\prime \prime}}{50}
$$

which leads to $\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=0$.
Thus, $\mathrm{V}_{\mathrm{x}}=\mathrm{V}_{\mathrm{x}}{ }^{\prime}+\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=9.926 \mathrm{mV}$.
(b) PSpice verification.


As can be seen from the two separate PSpice simulations, our hand calculations are correct; the pV -scale voltage in the second simulation is a result of numerical inaccuracy.

19.


$$
\begin{aligned}
& \frac{\mathrm{V}_{\mathrm{x}}^{\prime}-6}{1}+\frac{\mathrm{V}_{\mathrm{x}}^{\prime}}{3}+\frac{\mathrm{V}_{\mathrm{x}}^{\prime}+10}{2}=0 \\
& \text { so } \mathrm{V}_{\mathrm{x}}^{\prime}=0.5455 \mathrm{~V}
\end{aligned}
$$


$\frac{\mathrm{V}_{\mathrm{x}}^{\prime \prime}-6}{1}+\frac{\mathrm{V}_{\mathrm{x}}^{\prime \prime}}{3}+\frac{\mathrm{V}_{\mathrm{x}}^{\prime \prime}+5}{2}=0$
so $\mathrm{V}_{\mathrm{x}}^{\prime \prime}=1.909 \mathrm{~V}$

Adding, we find that $\mathrm{V}_{\mathrm{x}}{ }^{\prime}+\mathrm{V}_{\mathrm{x}}{ }^{\prime \prime}=2.455 \mathrm{~V}=\mathrm{V}_{\mathrm{x}}$ as promised.
20. (a) We first recognise that the two current sources are in parallel, and hence may be replaced by a single -7 A source (arrow directed downward). This source is in parallel with a $10 \mathrm{k} \Omega$ resistor. A simple source transformation therefore yields a $10 \mathrm{k} \Omega$ resistor in series with a $(-7)(10,000)=-70,000 \mathrm{~V}$ source ( + reference on top):

(b) This circuit requires several source transformations. First, we convert the 8 V source and $3 \Omega$ resistor to an $8 / 3$ A current source in parallel with $3 \Omega$. This yields a circuit with a $3 \Omega$ and $10 \Omega$ parallel combination, which may be replaced with a $2.308 \Omega$ resistor. We may now convert the $8 / 3$ A current source and $2.308 \Omega$ resistor to a $(8 / 3)(2.308)=6.155 \mathrm{~V}$ voltage source in series with a $2.308 \Omega$ resistor. This modified circuit contains a series combination of $2.308 \Omega$ and $5 \Omega$; performing a source transformation yet again, we obtain a current source with value (6.155)/(2.308 $+5)=0.8422 \mathrm{~A}$ in parallel with $7.308 \Omega$ and in parallel with the remaining $5 \Omega$ resistor. Since $7.308 \Omega \| 5 \Omega=2.969 \Omega$, our solution is:

21. (a) First we note the three current sources are in parallel, and may be replaced by a single current source having value $5-1+3=7 \mathrm{~A}$, arrow pointing upwards. This source is in parallel with the $10 \Omega$ resistor and the $6 \Omega$ resistor. Performing a source transformation on the current source and $6 \Omega$ resistor, we obtain a voltage source (7)(6) $=42 \mathrm{~V}$ in series with a $6 \Omega$ resistor and in series with the $10 \Omega$ resistor:

(b) By voltage division, $v=42(10) / 16=26.25 \mathrm{~V}$.
(c) Once the $10 \Omega$ resistor is involved in a source transformation, it disappears, only to be replaced by a resistor having the same value - but whose current and voltage can be different. Since the quantity $v$ appearing across this resistor is of interest, we cannot involve the resistor in a transformation.
22. (a) $[120 \cos 400 t] / 60=2 \cos 400 t$ A. $60 \| 120=40 \Omega$.
$[2 \cos 400 t](40)=80 \cos 400 t \mathrm{~V} .40+10=50 \Omega$.
[ $80 \cos 400 t] / 50=1.6 \cos 400 t \mathrm{~A} .50 \| 50=25 \Omega$.

(b) $2 \mathrm{k}\|3 \mathrm{k}+6 \mathrm{k}=7.2 \mathrm{k} \Omega . \quad 7.2 \mathrm{k}\| 12 \mathrm{k}=4.5 \mathrm{k} \Omega$

$(20)(4.5)=90 \mathrm{~V}$.

23. We can ignore the $1-\mathrm{k} \Omega$ resistor, at least when performing a source transformation on this circuit, as the $1-\mathrm{mA}$ source will pump 1 mA through whatever value resistor we place there. So, we need only combine the 1 and 2 mA sources (which are in parallel once we replace the $1-\mathrm{k} \Omega$ resistor with a $0-\Omega$ resistor). The current through the 5.8$\mathrm{k} \Omega$ resistor is then simply given by voltage division:

$$
i=3 \times 10^{-3} \frac{4.7}{4.7+5.8}=1.343 \mathrm{~mA}
$$

The power dissipated by the $5.8-\mathrm{k} \Omega$ resistor is then $\mathrm{i}^{2} \cdot 5.8 \times 10^{3}=10.46 \mathrm{~mW}$.
(Note that we did not "transform" either source, but rather drew on the relevant discussion to understand why the $1-\mathrm{k} \Omega$ resistor could be omitted.)
24. We may ignore the $10-\mathrm{k} \Omega$ and $9.7-\mathrm{k} \Omega$ resistors, as $3-\mathrm{V}$ will appear across them regardless of their value. Performing a quick source transformation on the $10-\mathrm{k} \Omega$ resistor/ 4-mA current source combination, we replace them with a $40-\mathrm{V}$ source in series with a $10-\mathrm{k} \Omega$ resistor:

$\mathrm{I}=43 / 15.8 \mathrm{~mA}=2.722 \mathrm{~mA}$. Therefore, $\mathrm{P}_{5.8 \Omega}=\mathrm{I}^{2} .5 .8 \times 10^{3}=42.97 \mathrm{~mW}$.
25. $(100 \mathrm{k} \Omega)(6 \mathrm{~mA})=0.6 \mathrm{~V}$

$470 \mathrm{k}|\mid 300 \mathrm{k}=183.1 \mathrm{k} \Omega$
$(-3-0.6) / 300 \times 10^{3}=-12 \mu \mathrm{~A}$
$(183.1 \mathrm{k} \Omega)(-12 \mu \mathrm{~A})=-2.197 \mathrm{~V}$


Solving, $9+1183.1 \times 10^{3} \mathrm{I}-2.197=0$, so $\mathrm{I}=-5.750 \mu \mathrm{~A}$. Thus,

$$
\mathrm{P}_{1 \mathrm{M} \Omega}=\mathrm{I}^{2} \cdot 10^{6}=33.06 \mu \mathrm{~W} .
$$

26. $(1)(47)=47$ V. $(20)(10)=200 \mathrm{~V}$. Each voltage source " + " corresponds to its corresponding current source's arrow head.


Using KVL on the simplified circuit above,

$$
47+47 \times 10^{3} \mathrm{I}_{1}-4 \mathrm{I}_{1}+13.3 \times 10^{3} \mathrm{I}_{1}+200=0
$$

Solving, we find that $\mathrm{I}_{1}=-247 /\left(60.3 \times 10^{3}-4\right)=-4.096 \mathrm{~mA}$.
27. (a) $\left(2 \mathrm{~V}_{1}\right)(17)=34 \mathrm{~V}_{1}$


Analysing the simplified circuit above,

$$
\begin{equation*}
34 \mathrm{~V}_{1}-0.6+7 \mathrm{I}+2 \mathrm{I}+17 \mathrm{I}=0 \quad[1] \quad \text { and } \quad \mathrm{V}_{1}=2 \mathrm{I} \tag{2}
\end{equation*}
$$

Substituting, we find that $\mathrm{I}=0.6 /(68+7+2+17)=6.383 \mathrm{~mA}$. Thus,

$$
\mathrm{V}_{1}=2 \mathrm{I}=12.77 \mathrm{mV}
$$

(b)

28. (a) $12 / 9000=1.333 \mathrm{~mA} .9 \mathrm{k} \| 7 \mathrm{k}=3.938 \mathrm{k} \Omega . \rightarrow(1.333 \mathrm{~mA})(3.938 \mathrm{k} \Omega)=5.249$ V.

$5.249 / 473.938 \times 10^{3}=11.08 \mu \mathrm{~A}$

$473.9 \mathrm{k} \| \mid 0 \mathrm{k}=9.793 \mathrm{k} \Omega .(11.08 \mathrm{~mA})(9.793 \mathrm{k} \Omega)=0.1085 \mathrm{~V}$

$\mathrm{I}_{\mathrm{x}}=0.1085 / 28.793 \times 10^{3}=3.768 \mu \mathrm{~A}$.
(b)

29. First, $(-7 \mu \mathrm{~A})(2 \mathrm{M} \Omega)=-14 \mathrm{~V}$, " + " reference down. $2 \mathrm{M} \Omega+4 \mathrm{M} \Omega=6 \mathrm{M} \Omega$. $+14 \mathrm{~V} / 6 \mathrm{M} \Omega=2.333 \mu \mathrm{~A}$, arrow pointing up; $6 \mathrm{M} \| 10 \mathrm{M}=3.75 \mathrm{M} \Omega$.

$(2.333)(3.75)=8.749 \mathrm{~V} . \mathrm{R}_{\mathrm{eq}}=6.75 \mathrm{M} \Omega$
$\therefore \mathrm{I}_{\mathrm{x}}=8.749 /(6.75+4.7) \mu \mathrm{A}=764.1 \mathrm{nA}$.
30. To begin, note that $(1 \mathrm{~mA})(9 \Omega)=9 \mathrm{mV}$, and $5 \| 4=2.222 \Omega$.


The above circuit may not be further simplified using only source transformation techniques.
31. Label the "-" terminal of the $9-V$ source node $\mathbf{x}$ and the other terminal node $\mathbf{x}$ '. The $9-\mathrm{V}$ source will force the voltage across these two terminals to be -9 V regardless of the value of the current source and resistor to its left. These two components may therefore be neglected from the perspective of terminals $\mathbf{a} \& \mathbf{b}$. Thus, we may draw:

32. Beware of the temptation to employ superposition to compute the dissipated power- it won't work!

Instead, define a current I flowing into the bottom terminal of the $1-\mathrm{M} \Omega$ resistor. Using superposition to compute this current,

$$
\mathrm{I}=1.8 / 1.840+0+0 \mu \mathrm{~A}=978.3 \mathrm{nA} .
$$

Thus,

$$
\mathrm{P}_{1 \mathrm{M} \Omega}=\left(978.3 \times 10^{-9}\right)^{2}\left(10^{6}\right)=957.1 \mathrm{nW} .
$$

33. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:


## Least-squares fit results:

| Voltage (V) | Current (mA) |
| :--- | :--- |
| 1.567 | 1.6681 |
| 1.563 | 6.599 |
| 1.558 | 12.763 |

We see from the figure that we cannot draw a very good line through all data points representing currents from 1 mA to 20 mA . We have therefore chosen to perform a linear fit for the three lower voltages only, as shown. Our model will not be as accurate at 1 mA ; there is no way to know if our model will be accurate at 20 mA , since that is beyond the range of the experimental data.

Modeling this system as an ideal voltage source in series with a resistance (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$
\begin{aligned}
& 1.567=\mathrm{V}_{\text {src }}-\mathrm{R}_{\mathrm{S}}\left(1.6681 \times 10^{-3}\right) \\
& 1.558=\mathrm{V}_{\text {src }}-\mathrm{R}_{\mathrm{s}}\left(12.763 \times 10^{-3}\right)
\end{aligned}
$$

Solving, $\mathrm{V}_{\text {src }}=1.568 \mathrm{~V}$ and $\mathrm{R}_{\mathrm{s}}=811.2 \mathrm{~m} \Omega$. It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.
34. Let's begin by plotting the experimental results, along with a least-squares fit to part of the data:


## Least-squares fit results:

| Voltage (V) | Current (mA) |
| :--- | :--- |
| 1.567 | 1.6681 |
| 1.563 | 6.599 |
| 1.558 | 12.763 |

We see from the figure that we cannot draw a very good line through all data points representing currents from 1 mA to 20 mA . We have therefore chosen to perform a linear fit for the three lower voltages only, as shown. Our model will not be as accurate at 1 mA ; there is no way to know if our model will be accurate at 20 mA , since that is beyond the range of the experimental data.

Modeling this system as an ideal current source in parallel with a resistance $R_{p}$ (representing the internal resistance of the battery) and a varying load resistance, we may write the following two equations based on the linear fit to the data:

$$
\begin{aligned}
& 1.6681 \times 10^{-3}=\mathrm{I}_{\text {src }}-1.567 / \mathrm{R}_{\mathrm{p}} \\
& 12.763 \times 10^{-3}=\mathrm{I}_{\text {src }}-1.558 / \mathrm{R}_{\mathrm{p}}
\end{aligned}
$$

Solving $\mathrm{I}_{\mathrm{src}}=1.933 \mathrm{~A}$ and $\mathrm{R}_{\mathrm{s}}=811.2 \mathrm{~m} \mathrm{\Omega}$. It should be noted that depending on the line fit to the experimental data, these values can change somewhat, particularly the series resistance value.
35. Working from left to right,
$2 \mu \mathrm{~A}-1.8 \mu \mathrm{~A}=200 \mathrm{nA}$, arrow up.
$1.4 \mathrm{M} \Omega+2.7 \mathrm{M} \Omega=4.1 \mathrm{M} \Omega$
A transformation to a voltage source yields $(200 \mathrm{nA})(4.1 \mathrm{M} \Omega)=0.82 \mathrm{~V}$ in series with $4.1 \mathrm{M} \Omega+2 \mathrm{M} \Omega=6.1 \mathrm{M} \Omega$, as shown below:


Then, $0.82 \mathrm{~V} / 6.1 \mathrm{M} \Omega=134.4 \mathrm{nA}$, arrow up. $6.1 \mathrm{M} \Omega \| 3 \mathrm{M} \Omega=2.011 \mathrm{M} \Omega$ $4.1 \mu \mathrm{~A}+134.4 \mathrm{nA}=4.234 \mathrm{~mA}$, arrow up. $(4.234 \mu \mathrm{~A})(2.011 \mathrm{M} \Omega)=8.515 \mathrm{~V}$.

The final circuit is an 8.515 V voltage source in series with a $2.011 \mathrm{M} \Omega$ resistor, as shown:

36. To begin, we note that the $5-\mathrm{V}$ and $2-\mathrm{V}$ sources are in series:


Next, noting that $3 \mathrm{~V} / 1 \Omega=3 \mathrm{~A}$, and $4 \mathrm{~A}-3 \mathrm{~A}=+1 \mathrm{~A}$ (arrow down), we obtain:


The left-hand resistor and the current source are easily transformed into a $1-\mathrm{V}$ source in series with a $1-\Omega$ resistor:

By voltage division, the voltage across the $5-\Omega$ resistor in the circuit to the right is:
(-1) $\frac{2 \| 5}{2 \| 5+2}=-0.4167 \mathrm{~V}$.
$-1 \mathrm{~V}$


Thus, the power dissipated by the $5-\Omega$ resistor is $(-0.4167)^{2} / 5=34.73 \mathrm{~mW}$.
37. (a) We may omit the $10 \Omega$ resistor from the circuit, as it does not affect the voltage or current associated with $R_{L}$ since it is in parallel with the voltage source. We are thus left with an 8 V source in series with a $5 \Omega$ resistor. These may be transformed to an $8 / 5$ A current source in parallel with $5 \Omega$, in parallel with $R_{L}$.

(b)


We see from simulating both circuits simultaneously that the voltage across $R_{L}$ is the same ( 4 V ).
38. (a) We may begin by omitting the $7 \Omega$ and $1 \Omega$ resistors. Performing the indicated source transformations, we find a $6 / 4 \mathrm{~A}$ source in parallel with $4 \Omega$, and a $5 / 10 \mathrm{~A}$ source in parallel with $10 \Omega$. These are both in parallel with the series combination of the two $5 \Omega$ resistors. Since $4 \Omega \| 10 \Omega=2.857 \Omega$, and $6 / 4+5 / 10=2$ A, we may further simplify the circuit to a single current source ( 2 A ) in parallel with $2.857 \Omega$ and the series combination of two $5 \Omega$ resistors. Simple current division yields the current flowing through the $5 \Omega$ resistors:
$\mathrm{I}=\frac{2(2.857)}{2.857+10}=0.4444 \mathrm{~A}$
The power dissipated in either of the $5 \Omega$ resistors is then $\mathrm{I}^{2} \mathrm{R}=987.6 \mathrm{~mW}$.
(b) We note that PSpice will NOT allow the $7 \Omega$ resistor to be left floating! For both circuits simulated, we observe 987.6 mW of power dissipated for the $5 \Omega$ resistor, confirming our analytic solution.

(c) Neither does. No current flows through the $7 \Omega$ resistor; the $1 \Omega$ resistor is in parallel with a voltage source and hence cannot affect any other part of the circuit.
39. We obtain a $5 \mathrm{v}_{3} / 4$ A current source in parallel with $4 \Omega$, and a 3 A current source in parallel with $2 \Omega$. We now have two dependent current sources in parallel, which may be combined to yield a single $-0.75 \mathrm{v}_{3}$ current source (arrow pointing upwards) in parallel with $4 \Omega$. Selecting the bottom node as a reference terminal, and naming the top left node $V_{x}$ and the top right node $V_{y}$, we write the following equations:

$$
\begin{aligned}
& -0.75 \mathrm{v}_{3}=\mathrm{V}_{\mathrm{x}} / 4+\left(\mathrm{V}_{\mathrm{x}}-\mathrm{V}_{\mathrm{y}}\right) / 3 \\
& 3=\mathrm{V}_{\mathrm{y}} / 2+\left(\mathrm{V}_{\mathrm{y}}-\mathrm{V}_{\mathrm{x}}\right) / 3 \\
& \mathrm{v}_{3}=\mathrm{V}_{\mathrm{y}}-\mathrm{V}_{\mathrm{x}}
\end{aligned}
$$

Solving, we find that $\mathrm{v}_{3}=-2 \mathrm{~V}$.
40.
(a) $\mathrm{R}_{\mathrm{TH}}=25\|(10+15)=25\| 25=12.5 \Omega$.
$\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{ab}}=50\left(\frac{25}{10+15+25}\right)+100\left(\frac{15+10}{15+10+25}\right)=75 \mathrm{~V}$.
(b) If $\mathrm{R}_{\mathrm{ab}}=50 \Omega$,
$\mathrm{P}_{50 \Omega}=\left[75\left(\frac{50}{50+12.5}\right)\right]^{2}\left(\frac{1}{50}\right)=72 \mathrm{~W}$
(c) If $\mathrm{R}_{\mathrm{ab}}=12.5 \Omega$,
$\mathrm{P}_{12.5 \Omega}=\left[75\left(\frac{12.5}{12.5+12.5}\right)\right]^{2}\left(\frac{1}{12.5}\right)=112.5 \mathrm{~W}$
41. (a) Shorting the 14 V source, we find that $\mathrm{R}_{\mathrm{TH}}=10 \| 20+10=16.67 \Omega$.

Next, we find $\mathrm{V}_{\text {TH }}$ by determining $\mathrm{V}_{\text {OC }}$ (recognising that the right-most $10 \Omega$ resistor carries no current, hence we have a simple voltage divider):

$$
\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{OC}}=14 \frac{10+10}{10+10+10}=9.333 \mathrm{~V}
$$

Thus, our Thevenin equivalent is a 9.333 V source in series with a $16.67 \Omega$ resistor, which is in series with the $5 \Omega$ resistor of interest.
(b) $\mathrm{I}_{5 \Omega}=9.333 /(5+16.67)=0.4307 \mathrm{~A}$. Thus,

$$
\mathrm{P}_{5 \Omega}=(0.4307)^{2} \cdot 5=927.5 \mathrm{~mW}
$$

(c) We see from the PSpice simulation that keeping four significant digits in calculating the Thévenin equivalent yields at least 3 digits' agreement in the results.

42. (a) Replacing the $7 \Omega$ resistor with a short circuit, we find

$$
\mathrm{I}_{\mathrm{SC}}=15(8) / 10=12 \mathrm{~A} .
$$

Removing the short circuit, and open-circuiting the 15 A source, we see that
$\mathrm{R}_{\mathrm{TH}}=2+8=10 \Omega$.
Thus, $\mathrm{V}_{\mathrm{TH}}=\mathrm{I}_{\mathrm{SC}} \mathrm{R}_{\mathrm{TH}}=(12)(10)=120 \mathrm{~V}$.
Our Thévenin equivalent is therefore a 120 V source in series with $10 \Omega$.
(b) As found above, $\mathrm{I}_{\mathrm{N}}=\mathrm{I}_{\mathrm{SC}}=12 \mathrm{~A}$, and $\mathrm{R}_{\mathrm{TH}}=10 \Omega$.
(c) Using the Thévenin equivalent circuit, we may find $v_{1}$ using voltage division:

$$
v_{1}=120(7) / 17=49.41 \mathrm{~V} .
$$

Using the Norton equivalent circuit and a combination of current division and Ohm's law, we find

$$
v_{1}=7\left(12 \frac{10}{17}\right)=49.41 \mathrm{~V}
$$

As expected, the results are equal.
(d) Employing the more convenient Thévenin equivalent model,

$$
v_{1}=120(1) / 17=7.059 \mathrm{~V} .
$$

43. 

(a) $\mathrm{R}_{\mathrm{TH}}=10 \mathrm{mV} / 400 \mu \mathrm{~A}=25 \Omega$
(b) $\mathrm{R}_{\mathrm{TH}}=110 \mathrm{~V} / 363.6 \times 10^{-3} \mathrm{~A}=302.5 \Omega$
(c) Increased current leads to increased filament temperature, which results in a higher resistance (as measured). This means the Thévenin equivalent must apply to the specific current of a particular circuit - one model is not suitable for all operating conditions (the light bulb is nonlinear).
44. (a) We begin by shorting both voltage sources, and removing the $1 \Omega$ resistor of interest. Looking into the terminals where the $1 \Omega$ resistor had been connected, we see that the $9 \Omega$ resistor is shorted out, so that

$$
\mathrm{R}_{\mathrm{TH}}=(5+10) \| 10+10=16 \Omega .
$$

To continue, we return to the original circuit and replace the $1 \Omega$ resistor with a short circuit. We define three clockwise mesh currents: $i_{1}$ in the left-most mesh, $i_{2}$ in the top-right mesh, and isc in the bottom right mesh. Writing our three mesh equations,

$$
\begin{aligned}
-4+9 i_{1}-9 i_{2}+3 & =0 \\
-9 i_{1}+34 i_{2}-10 i_{\text {sc }} & =0 \\
-3-10 i_{2}+20 i_{\text {sc }} & =0
\end{aligned}
$$

Solving using MATLAB:
$\gg \mathrm{e} 1={ }^{\prime}-4+9 * \mathrm{i} 1-9 * \mathrm{i} 2+3=0$ ';
$\gg \mathrm{e} 2={ }^{\prime}-9 * \mathrm{i} 1+34 * \mathrm{i} 2-10 * \mathrm{isc}=0$ ';
$\gg$ e3 $=$ '-3 $+20 *$ isc $-10 *$ i2 $=0$ ';
$\gg \mathrm{a}=$ solve(e1,e2,e3,'i1','i2','isc');
we find $i_{\text {sc }}=0.2125 \mathrm{~A}$, so $\mathrm{I}_{\mathrm{N}}=212.5 \mathrm{~mA}$ and $\mathrm{V}_{\mathrm{TH}}=\mathrm{I}_{\mathrm{N}} \mathrm{R}_{\mathrm{TH}}=(0.2125)(16)=3.4 \mathrm{~V}$.
(b) Working with the Thévenin equivalent circuit, $\mathrm{I}_{1 \Omega}=\mathrm{V}_{\mathrm{TH}} /\left(\mathrm{R}_{\mathrm{TH}}+1\right)=200 \mathrm{~mA}$. Thus, $\mathrm{P}_{1 \Omega}=(0.2) 2.1=40 \mathrm{~mW}$.

Switching to the Norton equivalent, we find $\mathrm{I} 1 \Omega$ by current division:
$\mathrm{I}_{1 \Omega}=(0.2125)(16) /(16+1)=200 \mathrm{~mA}$. Once again, $\mathrm{P}_{1 \Omega}=40 \mathrm{~mW}$ (as expected).
(c) As we can see from simulating the original circuit simultaneously with its Thevenin and Norton equivalents, the $1 \Omega$ resistor does in fact dissipate 40 mW , and either equivalent is equally applicable. Note all three SOURCES provide a different amount of power in total.

45. (a) Removing terminal $\mathbf{c}$, we need write only one nodal equation:

$0.1=\frac{\mathrm{V}_{\mathrm{b}}-2}{12}+\frac{\mathrm{V}_{\mathrm{b}}-5}{15}$, which may be solved to yield $\mathrm{V}_{\mathrm{b}}=4 \mathrm{~V}$. Therefore, $\mathrm{V}_{\mathrm{ab}}=\mathrm{V}_{\mathrm{TH}}=2-4$ $=-2 \mathrm{~V}$.
$\mathrm{R}_{\mathrm{TH}}=12 \| 15=6.667 \Omega$. We may then calculate $\mathrm{I}_{\mathrm{N}}$ as $\mathrm{I}_{\mathrm{N}}=\mathrm{V}_{\mathrm{TH}} / \mathrm{R}_{\mathrm{TH}}$
$=-300 \mathrm{~mA}$ (arrow pointing upwards).
(b) Removing terminal a, we again find $\mathrm{R}_{\mathrm{TH}}=6.667 \Omega$, and only need write a single nodal equation; in fact, it is identical to that written for the circuit above, and we once again find that $\mathrm{V}_{\mathrm{b}}=4 \mathrm{~V}$. In this case, $\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{bc}}=4-5=-1 \mathrm{~V}$, so $\mathrm{I}_{\mathrm{N}}=-1 / 6.667$
$=-150 \mathrm{~mA}$ (arrow pointing upwards).
46. (a) Shorting out the 88 -V source and open-circuiting the 1 -A source, we see looking into the terminals x and $\mathrm{x}^{\prime}$ a $50-\Omega$ resistor in parallel with $10 \Omega$ in parallel with ( $20 \Omega+40 \Omega$ ), so

$$
\mathrm{R}_{\mathrm{TH}}=50\|10\|(20+40)=7.317 \Omega
$$

Using superposition to determine the voltage $\mathrm{V}_{\mathrm{xx}}$ across the $50-\Omega$ resistor, we find

$$
\begin{aligned}
\mathrm{V}_{\mathrm{xx}}{ }^{\prime}= & \mathrm{V}_{\mathrm{TH}}=\left[88 \frac{50 \|(20+40)}{10+[50 \|(20+40)]}\right]+(1)(50 \| 10)\left[\frac{40}{40+20+(50 \| 10)}\right] \\
& =\left[88 \frac{27.27}{37.27}\right]+(1)(8.333)\left[\frac{40}{40+20+8.333}\right]=69.27 \mathrm{~V}
\end{aligned}
$$

(b) Shorting out the $88-\mathrm{V}$ source and open-circuiting the $1-\mathrm{A}$ source, we see looking into the terminals y and y' a $40-\Omega$ resistor in parallel with $[20 \Omega+(10 \Omega \| 50 \Omega)]$ :

$$
\mathrm{R}_{\mathrm{TH}}=40 \|[20+(10| | 50)]=16.59 \Omega
$$

Using superposition to determine the voltage $\mathrm{V}_{\mathrm{yy}}$ across the 1-A source, we find

$$
\mathrm{V}_{\mathrm{yy}}{ }^{\prime}=\mathrm{V}_{\mathrm{TH}}=(1)\left(\mathrm{R}_{\mathrm{TH}}\right)+\left[88 \frac{27.27}{10+27.27}\right]\left(\frac{40}{20+40}\right)
$$

47. (a) Select terminal $\mathbf{b}$ as the reference terminal, and define a nodal voltage $V_{1}$ at the top of the $200-\Omega$ resistor. Then,

$$
\begin{align*}
& 0=\frac{\mathrm{V}_{1}-20}{40}+\frac{\mathrm{V}_{1}-\mathrm{V}_{\mathrm{TH}}}{100}+\frac{\mathrm{V}_{1}}{200}  \tag{1}\\
& 1.5 i_{1}=\left(\mathrm{V}_{\mathrm{TH}}-\mathrm{V}_{1}\right) / 100 \tag{2}
\end{align*}
$$

where $i_{1}=\mathrm{V}_{1} / 200$, so Eq. [2] becomes
$150 \mathrm{~V}_{1} / 200+\mathrm{V}_{1}-\mathrm{V}_{\mathrm{TH}}=0$
Simplifying and collecting terms, these equations may be re-written as:

$$
\begin{array}{r}
(0.25+0.1+0.05) \mathrm{V}_{1}-0.1 \mathrm{~V}_{\mathrm{TH}}=5 \\
(1+15 / 20) \mathrm{V}_{1}-\mathrm{V}_{\mathrm{TH}}=0 \tag{2}
\end{array}
$$

Solving, we find that $\mathrm{V}_{\mathrm{TH}}=38.89 \mathrm{~V}$. To find $\mathrm{R}_{\mathrm{TH}}$, we short the voltage source and inject 1 A into the port:


$$
\begin{align*}
& 0=\frac{\mathrm{V}_{1}-\mathrm{V}_{\text {in }}}{100}+\frac{\mathrm{V}_{1}}{40}+\frac{\mathrm{V}_{1}}{200}  \tag{1}\\
& 1.5 i_{1}+1=\frac{\mathrm{V}_{\text {in }}-\mathrm{V}_{1}}{100}  \tag{2}\\
& i_{1}=\mathrm{V}_{1} / 200 \tag{3}
\end{align*}
$$

Combining Eqs. [2] and [3] yields

$$
\begin{equation*}
1.75 \mathrm{~V}_{1}-\mathrm{V}_{\mathrm{in}}=-100 \tag{4}
\end{equation*}
$$

Solving Eqs. [1] \& [4] then results in $\mathrm{V}_{\text {in }}=177.8 \mathrm{~V}$, so that $\mathrm{R}_{\mathrm{TH}}=\mathrm{V}_{\text {in }} / 1 \mathrm{~A}=177.8 \Omega$.
(b) Adding a $100-\Omega$ load to the original circuit or our Thévenin equivalent, the voltage across the load is
$\mathrm{V}_{100 \Omega}=\mathrm{V}_{\mathrm{TH}}\left(\frac{100}{100+177.8}\right)=14.00 \mathrm{~V}$, and so $\mathrm{P}_{100 \Omega}=\left(\mathrm{V}_{100 \Omega}\right)^{2} / 100=1.96 \mathrm{~W}$.
48. We inject a current of 1 A into the port (arrow pointing up), select the bottom terminal as our reference terminal, and define the nodal voltage $V_{x}$ across the $200-\Omega$ resistor.

Then,

$$
\begin{array}{ll}
1=\mathrm{V}_{1} / 100+\left(\mathrm{V}_{1}-\mathrm{V}_{\mathrm{x}}\right) / 50 & {[1]} \\
-0.1 \mathrm{~V}_{1}=\mathrm{V}_{\mathrm{x}} / 200+\left(\mathrm{V}_{\mathrm{x}}-\mathrm{V}_{1}\right) / 50
\end{array}
$$

which may be simplified to

$$
\begin{array}{rr}
3 \mathrm{~V}_{1}-2 \mathrm{~V}_{\mathrm{x}}=100 \\
16 \mathrm{~V}_{1}+5 \mathrm{~V}_{\mathrm{x}}= & {[1]}
\end{array}
$$

Solving, we find that $\mathrm{V}_{1}=10.64 \mathrm{~V}$, so $\mathrm{R}_{\mathrm{TH}}=\mathrm{V}_{1} /(1 \mathrm{~A})=10.64 \Omega$.
Since there are no independent sources present in the original network, $\mathrm{I}_{\mathrm{N}}=0$.
49. With no independent sources present, $\mathrm{V}_{\mathrm{TH}}=0$.

We decide to inject a 1-A current into the port:


Node ' x ': $\quad 0.01 v_{\mathrm{ab}}=v_{\mathrm{x}} / 200+\left(v_{\mathrm{x}}-v_{\mathrm{f}}\right) / 50 \quad$ [1]
Supernode: $1=v_{\mathrm{ab}} / 100+\left(v_{\mathrm{f}}-v_{\mathrm{x}}\right)$ [2]
and: $\quad v_{\mathrm{ab}}-v_{\mathrm{f}}=0.2 v_{\mathrm{ab}}$
Rearranging and collecting terms,

$$
\begin{array}{ccc}
-2 v_{\mathrm{ab}}+5 v_{\mathrm{x}}-4 v_{\mathrm{f}}=0 & {[1]} \\
v_{\mathrm{ab}}-2 v_{\mathrm{x}}+2 v_{\mathrm{f}}=100 & {[2]} \\
0.8 v_{\mathrm{ab}} & -v_{\mathrm{f}}=0 & {[3]}
\end{array}
$$

Solving, we find that $v_{\mathrm{ab}}=192.3 \mathrm{~V}$, so $\mathrm{R}_{\mathrm{TH}}=v_{\mathrm{ab}} /(1 \mathrm{~A})=192.3 \Omega$.
50. We first find $\mathrm{R}_{T H}$ by shorting out the voltage source and open-circuiting the current source.


Looking into the terminals $\mathbf{a} \& \mathbf{b}$, we see $\mathrm{R}_{\text {TH }}=10 \|[47+(100| | 12)]$
$=8.523 \Omega$.

Returning to the original circuit, we decide to perform nodal analysis to obtain $\mathrm{V}_{\mathrm{TH}}$ :

$$
\begin{align*}
& -12 \times 10^{3}=\left(\mathrm{V}_{1}-12\right) / 100 \times 10^{3}+\mathrm{V}_{1} / 12 \times 10^{3}+\left(\mathrm{V}_{1}-\mathrm{V}_{\mathrm{TH}}\right) / 47 \times 10^{3}  \tag{1}\\
& 12 \times 10^{3}=\mathrm{V}_{\mathrm{TH}} / 10 \times 10^{3}+\left(\mathrm{V}_{\mathrm{TH}}-\mathrm{V}_{1}\right) / 47 \times 10^{3}
\end{align*}
$$

Rearranging and collecting terms,

$$
\begin{aligned}
& 0.1146 \mathrm{~V}_{1}-0.02128 \mathrm{~V}_{\mathrm{TH}}=-11.88 \\
& -0.02128 \mathrm{~V}_{1}+0.02128 \mathrm{~V}_{\mathrm{TH}}=12
\end{aligned}
$$

Solving, we find that $\mathrm{V}_{\mathrm{TH}}=83.48 \mathrm{~V}$.
51. (a) $\mathrm{R}_{\mathrm{TH}}=4+2 \| 2+10=15 \Omega$.
(b) same as above: $15 \Omega$.
52. For Fig. 5.78a, $\mathrm{I}_{\mathrm{N}}=12 / \sim 0 \rightarrow \infty \mathrm{~A}$ in parallel with $\sim 0 \Omega$.

For Fig. 5.78b, $\mathrm{V}_{\mathrm{TH}}=(2)(\sim \infty) \rightarrow \infty \mathrm{V}$ in series with $\sim \infty \Omega$.
53. With no independent sources present, $\mathrm{V}_{\mathrm{TH}}=0$.

Connecting a $1-\mathrm{V}$ source to the port and measuring the current that flows as a result,

$\mathrm{I}=0.5 \mathrm{~V}_{\mathrm{x}}+0.25 \mathrm{~V}_{\mathrm{x}}=0.5+0.25=0.75 \mathrm{~A}$.
$\mathrm{R}_{\mathrm{TH}}=1 / \mathrm{I}=1.333 \Omega$.
The Norton equivalent is 0 A in parallel with $1.333 \Omega$.
54. Performing nodal analysis to determine $\mathrm{V}_{\mathrm{TH}}$,

$$
\begin{equation*}
100 \times 10^{-3}=\mathrm{V}_{\mathrm{x}} / 250+\mathrm{V}_{\mathrm{oc}} 7.5 \times 10^{3} \tag{1}
\end{equation*}
$$

and $\quad V_{x}-V_{o c}=5 i_{x}$
where $i_{\mathrm{x}}=\mathrm{V}_{\mathrm{x}} / 250$. Thus, we may write the second equation as

$$
\begin{equation*}
0.98 \mathrm{~V}_{\mathrm{x}}-\mathrm{V}_{\mathrm{oc}}=0 \tag{2}
\end{equation*}
$$

Solving, we find that $\mathrm{V}_{\mathrm{oc}}=\mathrm{V}_{\mathrm{TH}}=23.72 \mathrm{~V}$.
In order to determine $\mathrm{R}_{T H}$, we inject 1 A into the port:


Solving, we find that $V_{a b}=237.2 \mathrm{~V}$. Since $\mathrm{R}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{ab}} /(1 \mathrm{~A}), \mathrm{R}_{\mathrm{TH}}=237.2 \Omega$.
Finally, $\mathrm{I}_{\mathrm{N}}=\mathrm{V}_{\mathrm{TH}} / \mathrm{R}_{\mathrm{TH}}=100 \mathrm{~mA}$.
55. We first note that $\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{x}}$, so performing nodal analysis,

$$
-5 \mathrm{~V}_{\mathrm{x}}=\mathrm{V}_{\mathrm{x}} / 19 \quad \text { which has the solution } \mathrm{V}_{\mathrm{x}}=0 \mathrm{~V} .
$$

Thus, $\mathrm{V}_{\mathrm{TH}}\left(\right.$ and hence $\left.\mathrm{I}_{\mathrm{N}}\right)=0$. (Assuming $\mathrm{R}_{\mathrm{TH}} \neq 0$ )
To find $\mathrm{R}_{\mathrm{TH}}$, we inject 1 A into the port, noting that $\mathrm{R}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{x}} / 1 \mathrm{~A}$ :

$$
-5 \mathrm{~V}_{\mathrm{x}}+1=\mathrm{V}_{\mathrm{x}} / 19
$$

Solving, we find that $\mathrm{V}_{\mathrm{x}}=197.9 \mathrm{mV}$, so that $\mathrm{R}_{\mathrm{TH}}=\mathrm{R}_{\mathrm{N}}=197.9 \mathrm{~m} \Omega$.
56. Shorting out the voltage source, we redraw the circuit with a 1-A source in place of the $2-k \Omega$ resistor:


Noting that $300 \Omega \| 2 \mathrm{M} \Omega \approx 300 \Omega$,

$$
\begin{gather*}
0=\left(v_{\mathrm{gs}}-\mathrm{V}\right) / 300  \tag{1}\\
1-0.02 v_{\mathrm{gs}}=\mathrm{V} / 1000+\left(\mathrm{V}-v_{\mathrm{gs}}\right) / 300 \tag{2}
\end{gather*}
$$

Simplifying \& collecting terms,

$$
\begin{array}{r}
v_{\mathrm{gs}} \quad-\mathrm{V}=0 \\
0.01667 v_{\mathrm{gs}}+0.00433 \mathrm{~V}=1 \tag{2}
\end{array}
$$

Solving, we find that $v_{\mathrm{gs}}=\mathrm{V}=47.62 \mathrm{~V}$. Hence, $\mathrm{R}_{\mathrm{TH}}=\mathrm{V} / 1 \mathrm{~A}=47.62 \Omega$.
57. We replace the source $v_{\mathrm{s}}$ and the $300-\Omega$ resistor with a $1-\mathrm{A}$ source and seek its voltage:


By nodal analysis, $\quad 1=\mathrm{V}_{1} / 2 \times 10^{6} \quad$ so $\mathrm{V}_{1}=2 \times 10^{6} \mathrm{~V}$.
Since $V=V_{1}$, we have $R_{\text {in }}=V / 1 A=2 \mathrm{M} \Omega$.
58. Removing the voltage source and the $300-\Omega$ resistor, we replace them with a $1-\mathrm{A}$ source and seek the voltage that develops across its terminals:


We select the bottom node as our reference terminal, and define nodal voltages $\mathrm{V}_{1}$ and $V_{2}$. Then,

$$
\begin{gather*}
1=\mathrm{V}_{1} / 2 \times 10^{6}+\left(\mathrm{V}_{1}-\mathrm{V}_{2}\right) / \mathrm{r}_{\pi}  \tag{1}\\
0.02 v_{\pi}=\left(\mathrm{V}_{2}-\mathrm{V}_{1}\right) / \mathrm{r}_{\pi}+\mathrm{V}_{2} / 1000+\mathrm{V}_{2} / 2000 \tag{2}
\end{gather*}
$$

where $v_{\pi}=\mathrm{V}_{1}-\mathrm{V}_{2}$
Simplifying \& collecting terms,
$\left(2 \times 10^{6}+\mathrm{r}_{\pi}\right) \mathrm{V}_{1}-2 \times 10^{6} \mathrm{~V}_{2}=2 \times 10^{6} \mathrm{r}_{\pi}$
$-\left(2000+40 r_{\pi}\right) V_{1}+\left(2000+43 r_{\pi}\right) V_{2}=0$ [2]
Solving, we find that $\mathrm{V}_{1}=\mathrm{V}=2 \times 10^{6}\left(\frac{666.7+14.33 \mathrm{r}_{\pi}}{2 \times 10^{6}+666.7+14.33 \mathrm{r}_{\pi}}\right)$.
Thus,

$$
\mathrm{R}_{\mathrm{TH}}=2 \times 10^{6} \|\left(666.7+14.33 \mathrm{r}_{\pi}\right) \Omega
$$

59. (a) We first determine $v_{\text {out }}$ in terms of $v_{\text {in }}$ and the resistor values only; in this case, $\mathrm{V}_{\mathrm{TH}}=v_{\text {out }}$. Performing nodal analysis, we write two equations:

$$
\begin{equation*}
0=\frac{-v_{d}}{R_{i}}+\frac{\left(-v_{d}-v_{i n}\right)}{R_{1}}+\frac{\left(-v_{d}-v_{o}\right)}{R_{f}} \quad[1] \quad \text { and } \quad 0=\frac{\left(v_{o}+v_{d}\right)}{R_{f}}+\frac{\left(v_{o}-A v_{d}\right)}{R_{o}} \tag{2}
\end{equation*}
$$

Solving using MATLAB, we obtain:
$\gg \mathrm{e} 1=\mathrm{vd} / \mathrm{Ri}+(\mathrm{vd}+\mathrm{vin}) / \mathrm{R} 1+(\mathrm{vd}+\mathrm{vo}) / \mathrm{Rf}=0$ ';
$\gg \mathrm{e} 2=$ ' $(\mathrm{vo}+\mathrm{vd}) / \mathrm{Rf}+\left(\mathrm{vo}-\mathrm{A}^{*} \mathrm{vd}\right) / \mathrm{Ro}=0^{\prime} ;$
>> a = solve(e1,e2,'vo','vd');
>> pretty(a.vo)

$$
\begin{aligned}
& \text { Ri vin (-Ro + Rf A) } \\
& \text { R1 Ro + Ri Ro + R1 Rf + Ri Rf + R1 Ri + A R1 Ri }
\end{aligned}
$$

Thus, $\mathrm{V}_{\mathrm{TH}}=\frac{v_{i n} R_{i}\left(R_{o}-A R_{f}\right)}{R_{1} R_{o}+R_{i} R_{o}+R_{1} R_{f}+R_{i} R_{f}+R_{1} R_{i}+A R_{1} R_{i}}$, which in the limit of $\mathrm{A} \rightarrow \infty$, approaches $-\mathrm{R}_{\mathrm{f}} / \mathrm{R}_{1}$.

To find $\mathrm{R}_{\mathrm{TH}}$, we short out the independent source $v_{\mathrm{in}}$, and squirt 1 A into the terminal marked $v_{\text {out }}$, renamed $\mathrm{V}_{\mathrm{T}}$. Analyzing the resulting circuit, we write two nodal equations:
$0=\frac{-v_{d}}{R_{i}}-\frac{v_{d}}{R_{1}}+\frac{\left(-v_{d}-v_{T}\right)}{R_{f}} \quad[1] \quad$ and $\quad 1=\frac{\left(v_{T}+v_{d}\right)}{R_{f}}+\frac{\left(v_{T}-A v_{d}\right)}{R_{o}}$
Solving using MATLAB:
>> e1 = 'vd/R1 + vd/Ri + (vd + VT)/Rf=0';
$\gg \mathrm{e} 2=11=(\mathrm{VT}+\mathrm{vd}) / \mathrm{Rf}+(\mathrm{VT}-\mathrm{A} * \mathrm{vd}) / \mathrm{Ro}$ ';
$\gg \mathrm{a}=$ solve(e1,e2,'vd','VT');
$\gg$ pretty (a.VT)
$\square$
Since $V_{T} / 1=V_{T}$, this is our Thévenin equivalent resistance $\left(R_{T H}\right)$.


Such a scheme probably would lead to maximum or at least near-maximum power transfer to our home. Since we pay the utility company based on the power we use, however, this might not be such a hot idea..
61. We need to find the Thévenin equivalent resistance of the circuit connected to $R_{L}$, so we short the $20-\mathrm{V}$ source and open-circuit the $2-\mathrm{A}$ source; by inspection, then

$$
\mathrm{R}_{\mathrm{TH}}=12 \| 8+5+6=15.8 \Omega
$$

Analyzing the original circuit to obtain $V_{1}$ and $V_{2}$ with $R_{L}$ removed:

$\mathrm{V}_{1}=208 / 20=8 \mathrm{~V} ; \quad \mathrm{V}_{2}=-2(6)=-12 \mathrm{~V}$.
We define $\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{1}-\mathrm{V}_{2}=8+12=20 \mathrm{~V}$. Then,
$\left.\mathrm{P}_{\mathrm{R}_{\mathrm{L}}}\right|_{\text {max }}=\frac{\mathrm{V}_{\mathrm{TH}}^{2}}{4 \mathrm{R}_{\mathrm{L}}}=\frac{400}{4(15.8)}=6.329 \mathrm{~W}$
62.
(a) $\mathrm{R}_{\mathrm{TH}}=25 \|(10+15)=12.5 \Omega$

Using superposition, $\mathrm{V}_{\mathrm{ab}}=\mathrm{V}_{\mathrm{TH}}=50 \frac{25}{15+10+25}+100 \frac{15+10}{50}=75 \mathrm{~V}$.
(b) Connecting a $50-\Omega$ resistor,
$\mathrm{P}_{\text {load }}=\frac{\mathrm{V}_{\mathrm{TH}}^{2}}{\mathrm{R}_{\mathrm{TH}}+\mathrm{R}_{\text {load }}}=\frac{75^{2}}{12.5+50}=90 \mathrm{~W}$
(c) Connecting a $12.5-\Omega$ resistor,
$\mathrm{P}_{\text {load }}=\frac{\mathrm{V}_{\mathrm{TH}}^{2}}{4 \mathrm{R}_{\mathrm{TH}}}=\frac{75^{2}}{4(12.5)}=112.5 \mathrm{~W}$
63. (a) By inspection, we see that $i_{10}=5 \mathrm{~A}$, so
$\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{ab}}=2(0)+3 i_{10}+10 i_{10}=13 i_{10}=13(5)=65 \mathrm{~V}$.
To find $\mathrm{R}_{\mathrm{TH}}$, we open-circuit the 5-A source, and connect a 1-A source between terminals $\mathbf{a} \& \mathbf{b}$ :


A simple KVL equation yields $\mathrm{V}_{\mathrm{x}}=2(1)+3 i_{10}+10 i_{10}$.
Since $i_{10}=1 \mathrm{~A}$ in this circuit, $\mathrm{V}_{\mathrm{x}}=15 \mathrm{~V}$.
We thus find the Thevenin equivalent resistance is $15 / 1=15 \Omega$.
(b) $\mathrm{P}_{\max }=\frac{\mathrm{V}_{\mathrm{TH}}^{2}}{4 \mathrm{R}_{\mathrm{TH}}}=\frac{65^{2}}{4(15)}=70.42 \mathrm{~W}$
64.
(a) Replacing the resistor $\mathrm{R}_{\mathrm{L}}$ with a 1-A source, we seek the voltage that develops across its terminals with the independent voltage source shorted:


$$
\begin{array}{ll}
\quad-10 i_{1}+20 i_{x}+40 i_{1}=0 & {[1] \Rightarrow 30 i_{1}+20 i_{x}=0} \\
\text { and } i_{1}-i_{x}=1 & {[2] \Rightarrow i_{1}-i_{x}=1}
\end{array}
$$

Solving, $i_{1}=400 \mathrm{~mA}$
So $V=40 i_{1}=16 \mathrm{~V}$ and $R_{T H}=\frac{V}{1 \mathrm{~A}}=16 \Omega$
(b) Removing the resistor $\mathrm{R}_{\mathrm{L}}$ from the original circuit, we seek the resulting open-circuit voltage:

$0=\frac{V_{T H}-10 i_{1}}{20}+\frac{V_{T H}-50}{40}$
where $i_{1}=\frac{V_{T H}-50}{40}$
so [1] becomes $0=\frac{V_{T H}}{20}-\frac{1}{2}\left(\frac{V_{T H}-50}{40}\right)+\left(\frac{V_{T H}-50}{40}\right)$

$$
\begin{aligned}
& 0=\frac{V_{T H}}{20}+\frac{V_{T H}-50}{80} \\
& 0=4 V_{T H}+V_{T H}-50 \\
& 5 V_{T H}=50 \\
& \text { or } V_{T H}=10 \mathrm{~V}
\end{aligned}
$$

Thus, if

$$
\begin{aligned}
& R_{L}=R_{T H}=16 \Omega, \\
& V_{R_{L}}=V_{T H} \frac{R_{L}}{R_{L}+R_{T H}}=\frac{V_{T H}}{2}=5 \mathrm{~V}
\end{aligned}
$$

65. 

(a) $\quad I_{N}=2.5 \mathrm{~A}$


$$
\begin{aligned}
& 20 i^{2}=80 \\
& i=2 \mathrm{~A}
\end{aligned}
$$

By current division,
$2=2.5 \frac{R_{N}}{R_{N}+20}$
Solving, $R_{N}=R_{T H}=80 \Omega$
Thus, $V_{T H}=V_{O C}=2.5 \times 80=200 \mathrm{~V}$
(b) $\quad P_{\text {max }}=\frac{V_{T H}{ }^{2}}{4 R_{T H}}=\frac{200^{2}}{4 \times 80}=125 \mathrm{~W}$
(c) $R_{L}=R_{T H}=80 \Omega$
66.


10 W to $250 \Omega$ corresp to 200 mA .
20 W to $80 \Omega$ corresp to 500 mA .

By Voltage $\div, \quad I_{R}=I_{N} \frac{R_{N}}{R+R_{N}}$

$$
\text { So } \begin{align*}
0.2 & =I_{N} \frac{R_{N}}{250+R_{N}}  \tag{1}\\
0.5 & =I_{N} \frac{R_{N}}{80+R_{N}} \tag{2}
\end{align*}
$$

Solving, $I_{N}=1.7 \mathrm{~A}$ and $R_{N}=33.33 \Omega$
(a) If $v_{L} i_{L}$ is a maximum,
$R_{L}=R_{N}=33.33 \Omega$
$i_{L}=1.7 \times \frac{33.33}{33.33+33.33}=850 \mathrm{~mA}$
$v_{L}=33.33 i_{L}=28.33 \mathrm{~V}$
(b) If $v_{L}$ is a maximum
$V_{L}=I_{N}\left(R_{N} \| R_{L}\right)$
So $v_{L}$ is a maximum when $R_{N} \| R_{L}$ is a maximum, which occurs at $R_{L}=\infty$.
Then $i_{L}=0$ and $v_{L}=1.7 \times R_{N}=56.66 \mathrm{~V}$
(c) If $i_{L}$ is a maximum

$$
\begin{aligned}
& i_{L}=i_{N} \frac{R_{N}}{R_{N}+R_{L}} ; \text { max when } R_{L}=0 \Omega \\
& \text { So } i_{L}=1.7 \mathrm{~A} \\
& v_{L}=0 \mathrm{~V}
\end{aligned}
$$

67. There is no conflict with our derivation concerning maximum power. While a dead short across the battery terminals will indeed result in maximum current draw from the battery, and power is indeed proportional to $i^{2}$, the power delivered to the load is $i^{2} R_{\text {LOAD }}=i^{2}(0)=0$ watts. This is the minimum, not the maximum, power that the battery can deliver to a load.
68. Remove $R_{E}: R_{T H}=R_{E} \| R_{\text {in }}$
bottom node: $1-3 \times 10^{-3} v_{\pi}=\frac{V-v_{\pi}}{300}+\frac{V-v_{\pi}}{70 \times 10^{3}}$
at other node: $\quad 0=\frac{v_{\pi}}{10 \times 10^{3}}+\frac{v_{\pi}-V}{300}+\frac{v_{\pi}-V}{70 \times 10^{3}}$


Simplifying and collecting terms,
$210 \times 10^{5}=70 \times 10^{3} V+300 V+63000 v_{\pi}-70 \times 10^{3} v_{\pi}-300 v_{\pi}$
or $70.3 \times 10^{3} V-7300 v=210 \times 10^{5}$
or $70.3 \times 10^{3} V-7300 v_{\pi}=210 \times 10^{5} \quad$ [1]
$0=2100 v_{\pi}+70 \times 10^{3} v_{\pi}-70 \times 10^{3} V+300 v_{\pi}-300 V$
or $-69.7 \times 10^{3} V+72.4 \times 10^{3} v_{\pi}=0$
solving, $V=331.9 \mathrm{~V}$ So $R_{T H}=R_{E} \| 331.9 \Omega$
Next, we determine $v_{\text {TH }}$ using mesh analysis:

$$
\begin{align*}
& -v_{s}+70.3 \times 10^{3} i_{1}-70 \times 10^{3} i_{2}=0  \tag{1}\\
& 80 \times 10^{3} i_{2}-70 \times 10^{3} i_{1}+R_{E} i_{3}=0 \tag{2}
\end{align*}
$$


and: $\quad i_{3}-i_{2}=3 \times 10^{-3} v_{\pi}$
or $i_{3}-i_{2}=3 \times 10^{-3}\left(10 \times 10^{3}\right) i_{2}$
or $i_{3}-i_{2}=30 i_{2}$
or

$$
\begin{equation*}
-31 i_{2}+i_{3}=0 \tag{3}
\end{equation*}
$$

Solving : $\left[\begin{array}{ccc}70.3 \times 10^{3} & -70 \times 10^{3} & 0 \\ -70 \times 10^{3} & 80 \times 10^{3} & R_{E} \\ 0 & -31 & 1\end{array}\right]\left[\begin{array}{l}i_{1} \\ i_{2} \\ i_{3}\end{array}\right]=\left[\begin{array}{c}v_{s} \\ 0 \\ 0\end{array}\right]$
We seek $i_{3}$ :

$$
i_{3}=\frac{-21.7 \times 10^{3} v_{s}}{7.24 \times 10^{6}+21.79 \times 10^{3} R_{E}}
$$

So $\quad V_{O C}=V_{T H}=R_{E} i_{3}=\frac{-21.7 \times 10^{3} R_{E}}{7.24 \times 10^{6}+21.79 \times 10^{3} R_{E}} v_{S}$

$$
\begin{aligned}
P_{8 \Omega}=8\left[\frac{V_{T H}}{R_{T H}+8}\right]^{2} & =\left[\frac{-21.7 \times 10^{3} R_{E}}{7.24 \times 10^{6}+21.79 \times 10^{3} R_{E}}\right]^{2} \frac{8 \mathrm{vs}^{2}}{\left[\frac{331.9 R_{E}}{331.9+R_{E}}\right]^{2}} \\
& =\frac{11.35 \times 10^{6}\left(331.9+R_{E}\right)^{2}}{\left(7.24 \times 10^{6}+21.79 \times 10^{3} R_{E}\right)^{2}} \mathrm{vs}^{2}
\end{aligned}
$$

This is maximized by setting $R_{E}=\infty$.
69. Thévenize the left-hand network, assigning the nodal voltage $V_{x}$ at the free end of right-most $1-\mathrm{k} \Omega$ resistor.
A single nodal equation: $40 \times 10^{-3}=\frac{\left.V_{\chi}\right|_{o c}}{7 \times 10^{3}}$
So $V_{T H}=\left.V_{x}\right|_{o c}=280 \mathrm{~V}$
$\mathrm{R}_{\mathrm{TH}}=1 \mathrm{k}+7 \mathrm{k}=8 \mathrm{k} \Omega$
Select $\mathrm{R}_{1}=\mathrm{R}_{\mathrm{TH}}=8 \mathrm{k} \Omega$.
70.

$D=R_{A}+R_{B}+R_{C}=1+850+0.1=851.1 \times 10^{6}$
$R_{1}=\frac{R_{A} R_{B}}{D}=\frac{10^{6} \times 10^{5}}{D}=117.5 \Omega$
$R_{2}=\frac{R_{B} R_{C}}{D}=\frac{10^{5} \times 850 \times 10^{6}}{851.1 \times 10^{6}}=99.87 \mathrm{k} \Omega$
$R_{3}=\frac{R_{C} R_{A}}{D}=\frac{850 \times 10^{6} \times 10^{6}}{851.1 \times 10^{6}}=998.7 \mathrm{k} \Omega$
71.

$$
\begin{aligned}
\\
\begin{aligned}
N & =R_{1} R_{2}+R_{2} R_{3}+R_{3} R_{1} \\
& =0.1 \times 0.4+0.4 \times 0.9+0.9 \times 0.1 \\
& =0.49 \Omega \\
R_{A} & =\frac{N}{R_{2}}=1.225 \Omega \\
R_{B} & =\frac{N}{R_{3}}=544.4 \mathrm{~m} \Omega \\
R_{C} & =\frac{N}{R_{1}}=4.9 \Omega
\end{aligned} \\
=\underbrace{0.4 \Omega}_{R_{2}}
\end{aligned}
$$


72.
$\Delta_{1}: 1+6+3=10 \Omega$

$$
\frac{6 \times 1}{10}=0.6, \frac{6 \times 3}{10}=1.8, \frac{3 \times 1}{10}=0.3
$$

$\Delta_{2}: 5+1+4=10 \Omega$
$\frac{5 \times 1}{10}=0.5, \frac{1 \times 4}{10}=0.4, \frac{5 \times 4}{10}=2$
$1.8+2+0.5=4.3 \Omega$
$0.3+0.6+0.4=1.3 \Omega$
$1.3 \| 4.3=0.9982 \Omega$

$0.9982+0.6+2=3.598 \Omega$
$3.598 \| 6=2.249 \Omega$
73.
$6 \times 2+2 \times 3+3 \times 6=36 \Omega^{2}$
$\frac{36}{6}=6 \Omega, \frac{36}{2}=18 \Omega, \frac{36}{3}=12 \Omega$ Rin $\rightarrow$
$12\|4=3 \Omega, 6\| 12 \Omega=4 \Omega$

$4+3+18=25 \Omega$
$3 \times \frac{18}{25}=2.16 \Omega$
$4 \times \frac{18}{25}=2.88 \Omega$
$4 \times \frac{3}{25}=0.48 \Omega$

$9.48 \times 2.16+9.48 \times 2.88+2.88 \times 2.16=54 \Omega^{2}$
$\frac{54}{2.88}=18.75 \Omega \quad \frac{54}{9.48}=5.696 \Omega$
$\frac{54}{2.16}=25 \Omega$
$75 \| 18.75=15 \Omega$

$100 \| 25=20 \Omega$
$(15+20) \| 5.696=4.899 \Omega$
$\therefore R_{\text {in }}=5+4.899=9.899 \Omega$
74. We begin by converting the $\Delta$-connected network consisting of the $4-, 6-$, and $3-\Omega$ resistors to an equivalent Y -connected network:

$D=6+4+3=13 \Omega$
$R_{1}=\frac{R_{A} R_{B}}{D}=\frac{6 \times 4}{13}=1.846 \Omega$
$R_{2}=\frac{R_{B} R_{C}}{D}=\frac{4 \times 3}{13}=923.1 \mathrm{~m} \Omega$
$R_{3}=\frac{R_{C} R_{A}}{D}=\frac{3 \times 6}{13}=1.385 \Omega$
Then network becomes:


Then we may write
$R_{i n}=12 \|[13.846+(19.385 \| 6.9231)]$
$=7.347 \Omega$
75.


$$
\begin{aligned}
& 1+1+2=4 \Omega \\
& R_{1}=\frac{1 \times 2}{4}=\frac{1}{2} \Omega \\
& R_{2}=\frac{2 \times 1}{4}=\frac{1}{2} \Omega \\
& R_{3}=\frac{1 \times 1}{4}=0.25 \Omega
\end{aligned}
$$

Next, we convert the Y-connected network on the left to a $\Delta$-connected network:
$1 \times 0.5+0.5 \times 2+2 \times 1=3.5 \Omega^{2}$
$R_{A}=\frac{3.5}{0.5}=7 \Omega$
$R_{B}=\frac{3.5}{2}=1.75 \Omega$
$R_{C}=\frac{3.5}{1}=3.5 \Omega$
After this procedure, we have a $3.5-\Omega$ resistor in parallel with the $2.5-\Omega$ resistor. Replacing them with a $1.458-\Omega$ resistor, we may redraw the circuit:


This circuit may be easily analysed to find:

$$
\begin{aligned}
V_{o c} & =\frac{12 \times 1.458}{1.75+1.458}=5.454 \mathrm{~V} \\
R_{T H} & =0.25+1.458 \| 1.75 \\
& =1.045 \Omega
\end{aligned}
$$

76. We begin by converting the Y-network to a $\Delta$-connected network:
$N=1.1+1.1+1.1=3 \Omega^{2}$
$R_{A}=\frac{3}{1}=3 \Omega$
$R_{B}=\frac{3}{1}=3 \Omega$
$R_{C}=\frac{3}{1}=3 \Omega$


Next, we note that $1 \| 3=0.75 \Omega$, and hence have a simple $\Delta$-network. This is easily converted to a Y-connected network:
$0.75+3+3=6.75 \Omega$
$R_{1}=\frac{0.75 \times 3}{6.75}=0.3333 \Omega$
$R_{2}=\frac{3 \times 3}{6.75}=1.333 \Omega$

$R_{3}=\frac{3 \times 0.75}{6.75}=0.3333 \Omega$

Analysing this final circuit,

$$
\begin{aligned}
R_{N} & =1.333+0.3333 \\
& =1.667 \Omega \\
I_{N} & =I_{S C}=1 \times \frac{1 / 3}{1 / 3+1+1 / 3} \\
& =\frac{1}{1+3+1}=\frac{1}{5} \\
& =0.2 \mathrm{~A} \\
& =200 \mathrm{~mA}
\end{aligned}
$$

77. Since 1 V appears across the resistor associated with $\mathrm{I}_{1}$, we know that $\mathrm{I}_{1}=1 \mathrm{~V} / 10 \Omega$ $=100 \mathrm{~mA}$. From the perspective of the open terminals, the $10-\Omega$ resistor in parallel with the voltage source has no influence if we replace the "dependent" source with a fixed $0.5-\mathrm{A}$ source:


Then, we may write:

$$
-1+(10+10+10) i_{\mathrm{a}}-10(0.5)=0
$$

so that $i_{\mathrm{a}}=200 \mathrm{~mA}$.
We next find that $\mathrm{V}_{\mathrm{TH}}=\mathrm{V}_{\mathrm{ab}}=10(-0.5)+10\left(i_{\mathrm{a}}-0.5\right)+10(-0.5)=-13 \mathrm{~V}$.
To determine $\mathrm{R}_{\mathrm{TH}}$, we first recognise that with the $1-\mathrm{V}$ source shorted, $\mathrm{I}_{1}=0$ and hence the dependent current source is dead. Thus, we may write $\mathrm{R}_{\mathrm{TH}}$ from inspection:

$$
\mathrm{R}_{\mathrm{TH}}=10+10+10 \| 20=26.67 \Omega
$$

78. (a) We begin by splitting the $1-\mathrm{k} \Omega$ resistor into two $500-\Omega$ resistors in series. We then have two related Y-connected networks, each with a $500-\Omega$ resistor as a leg. Converting those networks into $\Delta$-connected networks,

$$
\begin{gathered}
\Sigma=(17)(10)+(1)(4)+(4)(17)=89 \times 106 \Omega^{2} \\
89 / 0.5=178 \mathrm{k} \Omega ; \quad 89 / 17=5.236 \mathrm{k} \Omega ; \quad 89 / 4=22.25 \mathrm{k} \Omega
\end{gathered}
$$

Following this conversion, we find that we have two $5.235 \mathrm{k} \Omega$ resistors in parallel, and a $178-\mathrm{k} \Omega$ resistor in parallel with the $4-\mathrm{k} \Omega$ resistor. Noting that $5.235 \mathrm{k} \| 5.235 \mathrm{k}$ $=2.618 \mathrm{k} \Omega$ and $178 \mathrm{k} \| 4 \mathrm{k}=3.912 \mathrm{k} \Omega$, we may draw the circuit as:


We next attack the Y-connected network in the center:

$$
\begin{gathered}
\Sigma=(22.25)(22.25)+(22.25)(2.618)+(2.618)(22.25)=611.6 \times 106 \Omega^{2} \\
611.6 / 22.25=27.49 \mathrm{k} \Omega ; 611.6 / 2.618=233.6 \mathrm{k} \Omega
\end{gathered}
$$

Noting that $178 \mathrm{k}|\mid 27.49 \mathrm{k}=23.81 \mathrm{k} \Omega$ and 27.49$| \mid 3.912=3.425 \mathrm{k} \Omega$, we are left with a simple $\Delta$-connected network. To convert this to the requested Y-network,
$\Sigma=23.81+233.6+3.425=260.8 \mathrm{k} \Omega$
(23.81)(233.6)/260.8 $=21.33 \mathrm{k} \Omega$
$(233.6)(3.425) / 260.8=3.068 \mathrm{k} \Omega$
$(3.425)(23.81) / 260.8=312.6 \Omega$

79. (a) Although this network may be simplified, it is not possible to replace it with a three-resistor equivalent.
(b) See (a).
80. First, replace network to left of the $0.7-\mathrm{V}$ source with its Thévenin equivalent:
$V_{T H}=20 \times \frac{15}{100+15}=2.609 \mathrm{~V}$
$R_{T H}=100 \mathrm{k} \| 15 \mathrm{k}=13.04 \mathrm{k} \Omega$
Redraw:


Analysing the new circuit to find $I_{\mathrm{B}}$, we note that $I_{\mathrm{C}}=250 I_{\mathrm{B}}$ :

$$
\begin{aligned}
&-2.609+13.04 \times 10^{3} I_{B}+0.7+5000\left(I_{B}+250 I_{B}\right)=0 \\
& I_{B}=\frac{2.609-0.7}{13.04 \times 10^{3}+251 \times 5000}=1.505 \mu \mathrm{~A} \\
& I_{C}=250 I_{B} \\
&=3.764 \times 10^{-4} \mathrm{~A} \\
&=376.4 \mu \mathrm{~A}
\end{aligned}
$$

81. (a) Define a nodal voltage $\mathrm{V}_{1}$ at the top of the current source $\mathrm{I}_{\mathrm{S}}$, and a nodal voltage $V_{2}$ at the top of the load resistor $R_{L}$. Since the load resistor can safely dissipate 1 W , and we know that

$$
\mathrm{P}_{\mathrm{R}_{\mathrm{L}}}=\frac{\mathrm{V}_{2}^{2}}{1000}
$$

then $\left.\mathrm{V}_{2}\right|_{\text {max }}=31.62 \mathrm{~V}$. This corresponds to a load resistor (and hence lamp) current of 32.62 mA , so we may treat the lamp as a $10.6-\Omega$ resistor.

Proceeding with nodal analysis, we may write:

$$
\begin{align*}
& \mathrm{I}_{\mathrm{S}}=\mathrm{V}_{1} / 200+\left(\mathrm{V}_{1}-5 \mathrm{~V}_{\mathrm{x}}\right) / 200  \tag{1}\\
& 0=\mathrm{V}_{2} / 1000+\left(\mathrm{V}_{2}-5 \mathrm{~V}_{\mathrm{x}}\right) / 10.6  \tag{2}\\
& \mathrm{~V}_{\mathrm{x}}=\mathrm{V}_{1}-5 \mathrm{~V}_{\mathrm{x}} \quad \text { or } \mathrm{V}_{\mathrm{x}}=\mathrm{V}_{1} / 6 \tag{3}
\end{align*}
$$

Substituting Eq. [3] into Eqs. [1] and [2], we find that

$$
\begin{align*}
& 7 \mathrm{~V}_{1}=1200 \mathrm{I}_{\mathrm{S}}  \tag{1}\\
& -5000 \mathrm{~V}_{1}+6063.6 \mathrm{~V}_{2}=0 \tag{2}
\end{align*}
$$

Substituting $\left.\mathrm{V}_{2}\right|_{\max }=31.62 \mathrm{~V}$ into Eq. [2] then yields $\mathrm{V}_{1}=38.35 \mathrm{~V}$, so that
$\left.\mathrm{I}_{\mathrm{S}}\right|_{\max }=(7)(38.35) / 1200=223.7 \mathrm{~mA}$.
(b) PSpice verification.


The lamp current does not exceed 36 mA in the range of operation allowed (i.e. a load power of $<1 \mathrm{~W}$.) The simulation result shows that the load will dissipate slightly more than 1 W for a source current magnitude of 224 mA , as predicted by hand analysis.
82. Short out all but the source operating at $10^{4} \mathrm{rad} / \mathrm{s}$, and define three clockwise mesh currents $i_{1}, i_{2}$, and $i_{3}$ starting with the left-most mesh. Then

$$
\begin{gather*}
608 i_{1}-300 i_{2}=3.5 \cos 10^{4} t  \tag{1}\\
-300 i_{1}+316 i_{2}-8 i_{3}=0  \tag{2}\\
-8 i_{2}+322 i_{3}=0 \tag{3}
\end{gather*}
$$

Solving, we find that $i_{1}(t)=10.84 \cos 10^{4} t \mathrm{~mA}$

$$
\begin{aligned}
& i_{2}(t)=10.29 \cos 10^{4} t \mathrm{~mA} \\
& i_{3}(t)=255.7 \cos 10^{4} t \mu \mathrm{~A}
\end{aligned}
$$

Next, short out all but the $7 \sin 200 t \mathrm{~V}$ source, and and define three clockwise mesh currents $i_{\mathrm{a}}, i_{\mathrm{b}}$, and $i_{\mathrm{c}}$ starting with the left-most mesh. Then

$$
\begin{array}{cl}
608 i_{\mathrm{a}}-300 i_{\mathrm{b}}=-7 \sin 200 t & {[1]} \\
-300 i_{\mathrm{a}}+316 i_{\mathrm{b}}-8 i_{\mathrm{c}}=7 \sin 200 t \\
-8 i_{\mathrm{b}}+322 i_{\mathrm{c}}=0 \tag{3}
\end{array}
$$

Solving, we find that $i_{\mathrm{a}}(t)=-1.084 \sin 200 t \mathrm{~mA}$

$$
\begin{aligned}
& i_{\mathrm{b}}(t)=21.14 \sin 200 t \mathrm{~mA} \\
& i_{\mathrm{c}}(t)=525.1 \sin 200 t \mu \mathrm{~A}
\end{aligned}
$$

Next, short out all but the source operating at $10^{3} \mathrm{rad} / \mathrm{s}$, and define three clockwise mesh currents $i_{\mathrm{A}}, i_{\mathrm{B}}$, and $i_{\mathrm{C}}$ starting with the left-most mesh. Then

$$
\begin{aligned}
608 i_{\mathrm{A}}-300 i_{\mathrm{B}} & = \\
-300 i_{\mathrm{A}}+316 i_{\mathrm{B}}-8 i_{\mathrm{C}} & =0
\end{aligned}
$$

Solving, we find that $i_{\mathrm{A}}(t)=-584.5 \cos 10^{3} t \mu \mathrm{~A}$

$$
\begin{aligned}
& i_{\mathrm{B}}(t)=-1.185 \cos 10^{3} t \mathrm{~mA} \\
& i_{\mathrm{C}}(t)=-24.87 \cos 10^{3} t \mathrm{~mA}
\end{aligned}
$$

We may now compute the power delivered to each of the three $8-\Omega$ speakers:

$$
\begin{aligned}
& p_{1}=8\left[i_{1}+i_{\mathrm{a}}+i_{\mathrm{A}}\right]^{2}=8\left[10.84 \times 10^{-3} \cos 10^{4} t-1.084 \times 10^{-3} \sin 200 t-584.5 \times 10^{-6} \cos 10^{3} t\right]^{2} \\
& p_{2}=8\left[i_{2}+i_{\mathrm{b}}+i_{\mathrm{B}}\right]^{2}=8\left[10.29 \times 10^{-3} \cos 10^{4} t+21.14 \times 10^{-3} \sin 200 t-1.185 \times 10^{-3} \cos 10^{3} t\right]^{2} \\
& p_{3}=8\left[i_{3}+i_{\mathrm{c}}+i_{\mathrm{C}}\right]^{2}=8\left[255.7 \times 10^{-6} \cos 10^{4} t+525.1 \times 10^{-6} \sin 200 t-24.87 \times 10^{-3} \cos 10^{3} t\right]^{2}
\end{aligned}
$$

83. Replacing the DMM with a possible Norton equivalent (a 1-M $\Omega$ resistor in parallel with a 1-A source):


We begin by noting that $33 \Omega \| 1 \mathrm{M} \Omega \approx 33 \Omega$. Then,

$$
\begin{equation*}
0=\left(\mathrm{V}_{1}-\mathrm{V}_{\text {in }}\right) / 33+\mathrm{V}_{1} / 275 \times 10^{3} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
1-0.7 \mathrm{~V}_{1}=\mathrm{V}_{\text {in }} / 10^{6}+\mathrm{V}_{\text {in }} / 33 \times 10^{3}+\left(\mathrm{V}_{\text {in }}-\mathrm{V}_{1}\right) / 33 \tag{2}
\end{equation*}
$$

Simplifying and collecting terms,

$$
\begin{array}{r}
\left(275 \times 10^{3}+33\right) \mathrm{V}_{1}-275 \times 10^{3} \mathrm{~V}_{\text {in }}=0 \\
22.1 \mathrm{~V}_{1}+1.001 \mathrm{~V}_{\text {in }}=33 \tag{2}
\end{array}
$$

Solving, we find that $\mathrm{V}_{\mathrm{in}}=1.429 \mathrm{~V}$; in other words, the DMM sees 1.429 V across its terminals in response to the known current of 1 A it's supplying. It therefore thinks that it is connected to a resistance of $1.429 \Omega$.
84. We know that the resistor R is absorbing maximum power. We might be tempted to say that the resistance of the cylinder is therefore $10 \Omega$, but this is wrong: The larger we make the cylinder resistance, the small the power delivery to R:

$$
\mathrm{P}_{\mathrm{R}}=10 i^{2}=10\left[\frac{120}{R_{\text {cylinder }}+10}\right]^{2}
$$

Thus, if we are in fact delivering the maximum possible power to the resistor from the $120-\mathrm{V}$ source, the resistance of the cylinder must be zero.

This corresponds to a temperature of absolute zero using the equation given.
85. We note that the buzzer draws 15 mA at 6 V , so that it may be modeled as a $400-\Omega$ resistor. One possible solution of many, then, is:


Note: construct the $18-\mathrm{V}$ source from $121.5-\mathrm{V}$ batteries in series, and the two $400-\Omega$ resistors can be fabricated by soldering $4001-\Omega$ resistors in series, although there's probably a much better alternative...
86. To solve this problem, we need to assume that " 45 W " is a designation that applies when 120 Vac is applied directly to a particular lamp. This corresponds to a current draw of 375 mA , or a light bulb resistance of $120 / 0.375=320 \Omega$.


Original wiring scheme


New wiring scheme

In the original wiring scheme, Lamps $1 \& 2$ draw (40)2/320 = 5 W of power each, and Lamp 3 draws (80)2/320 $=20 \mathrm{~W}$ of power. Therefore, none of the lamps is running at its maximum rating of 45 W . We require a circuit which will deliver the same intensity after the lamps are reconnected in a $\Delta$ configuration. Thus, we need a total of 30 W from the new network of lamps.

There are several ways to accomplish this, but the simplest may be to just use one 120 -Vac source connected to the left port in series with a resistor whose value is chosen to obtain 30 W delivered to the three lamps.


In other words,

$$
\frac{\left[120 \frac{213.3}{\mathrm{Rs}+213.3}\right]^{2}}{320}+2 \frac{\left[60 \frac{213.3}{\mathrm{Rs}+213.3}\right]^{2}}{320}=30
$$

Solving, we find that we require Rs $=106.65 \Omega$ as confirmed by the PSpice simulation below, which shows that both wiring configurations lead to one lamp with $80-\mathrm{V}$ across it, and two lamps with 40 V across each.

87.

- Maximum current rating for the LED is 35 mA .
- Its resistance can vary between 47 and $117 \Omega$.
- A 9-V battery must be used as a power source.
- Only standard resistance values may be used.

One possible current-limiting scheme is to connect a $9-\mathrm{V}$ battery in series with a resistor $\mathrm{R}_{\text {limiting }}$ and in series with the LED.
From KVL,

$$
I_{\text {LED }}=\frac{9}{R_{\text {limiting }}+R_{\text {LED }}}
$$

The maximum value of this current will occur at the minimum LED resistance, $47 \Omega$. Thus, we solve

$$
35 \times 10^{-3}=\frac{9}{\mathrm{R}_{\text {limiting }}+47}
$$

to obtain $\mathrm{R}_{\text {limiting }} \geq 210.1 \Omega$ to ensure an LED current of less than 35 mA . This is not a standard resistor value, however, so we select

$$
\mathrm{R}_{\text {limiting }}=220 \Omega \text {. }
$$

