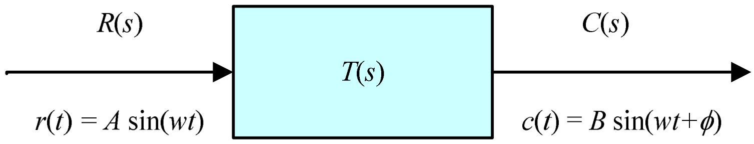
Chapter 8. Frequency Response Methods.

The freq. resp. of a system is defined as the steady-state resp. to sinusoidal input signal as w varies.



$\Rightarrow c(t) = A \cdot \left| T_{ss}(jw) \right| \cdot \sin(wt+\phi)$

proof)

$$T(s) = \frac{m(s)}{\prod_{i=1}^{n}(s+p_i)}, \qquad R(s) = \frac{Aw}{s^2+w^2}$$

$$\Rightarrow C(s) = T(s)R(s) = \frac{k_1}{s+p_1} + \cdots + \frac{k_n}{s+p_n} + \frac{\alpha s+\beta}{s^2+w^2}$$

If the system is stable (at steady-state),

$$\lim_{t\to\infty} \mathcal{L}^{-1}\left\{\frac{k_i}{s+p_i}\right\} = \lim_{t\to\infty} k_i e^{-p_i t} = 0 \quad \leftarrow \text{natural resp.(transient resp.)} = 0$$

In the steady-state $(t \to \infty)$

$$C_{ss}(s) = \frac{\alpha s+\beta}{s^2+w^2} = \frac{\alpha s+\beta}{Aw} \cdot \frac{Aw}{s^2+w^2} = T_{ss}(s) \cdot R(s)$$

where

$$T_{ss}(jw) = T(s)\big|_{s=jw} = \frac{jw\alpha+\beta}{Aw} = \frac{\sqrt{\beta^2+w^2\alpha^2}}{Aw} \measuredangle\left(\tan^{-1}\frac{\alpha w}{\beta}\right)$$

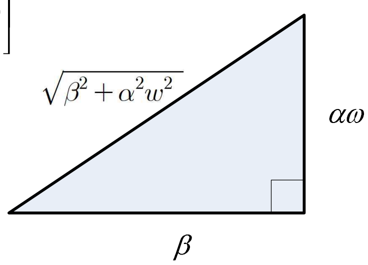
where $\left|T_{ss}(jw)\right| = \dfrac{\sqrt{\beta^2+w^2\alpha^2}}{Aw}$ and $\measuredangle T_{ss}(jw) = \measuredangle\left(\tan^{-1}\dfrac{\alpha w}{\beta}\right)$

$$\therefore)\ c(t) = \mathcal{L}^{-1}\left(\frac{\alpha s+\beta}{s^2+w^2}\right) = \alpha\cos(wt) + \frac{\beta}{w}\sin(wt)$$

$$= \frac{\sqrt{\beta^2+\alpha^2w^2}}{w}\left[\frac{\beta}{\sqrt{\beta^2+\alpha^2w^2}}\sin(wt) + \frac{\alpha w}{\sqrt{\beta^2+\alpha^2w^2}}\cos(wt)\right]$$

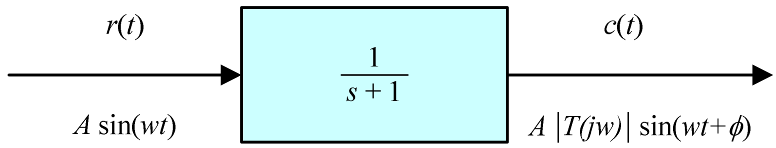
$$= \frac{\sqrt{\beta^2+\alpha^2w^2}}{w}\sin(wt+\phi)$$

$$= A \cdot \left|T_{ss}(jw)\right| \cdot \sin(wt+\phi)$$



$\Rightarrow$ We need some graphical plot for $\left|T_{ss}(jw)\right|$ and $\phi = \measuredangle T_{ss}(jw)$.

Example)



sol) $\quad T(jw) = \dfrac{1}{1+jw}$

$$\Rightarrow \ |T(jw)| = \frac{1}{|1+jw|} = \frac{1}{\sqrt{1+w^2}} \quad \text{and} \quad \phi = \angle T(jw) = -\tan^{-1}\frac{w}{1}$$

If $r(t) = 10\sin(t)$, $(A=10,\ w=1)$

$$\Rightarrow \ |T(jw)| = \frac{1}{\sqrt{2}}, \quad \phi = -\tan^{-1}\frac{1}{1} = -45^\circ$$

$$\Rightarrow \ c(t) = 10\cdot\frac{1}{\sqrt{2}}\sin(t-45^\circ) = \frac{10}{\sqrt{2}}\sin(t-45^\circ)$$

If $r(t) = 10\sin(2t)$, $(A=10,\ w=2)$

$$\Rightarrow \ |T(jw)| = \frac{1}{\sqrt{5}}, \quad \phi = -\tan^{-1}\frac{2}{1} = -63.43^\circ$$

$$\Rightarrow \ c(t) = \frac{10}{\sqrt{5}}\sin(t-63.43^\circ)$$

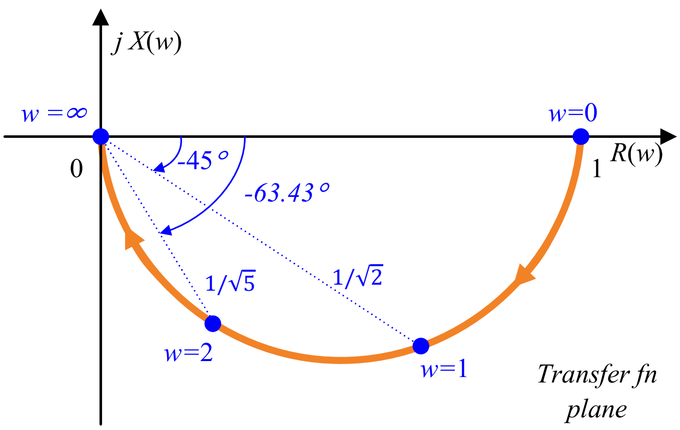
$\vdots$

If $r(t) = 10 \quad \Rightarrow r(t) = 10\sin(0t) \quad (A=10,\ w=0\ :\ \text{dc})$

$$\Rightarrow \ |T(jw)| = \frac{1}{1} = 1, \quad \phi = -\tan^{-1}\frac{0}{1} = 0^\circ$$

$$\Rightarrow \ c(t) = 10\sin(0t+0^\circ) = 10$$

8.2 Freq. resp. plot



<Polar plot>

$$T(jw) = R(w) + jX(w) \qquad \text{v.s}\ \ w = 0 \to \infty$$

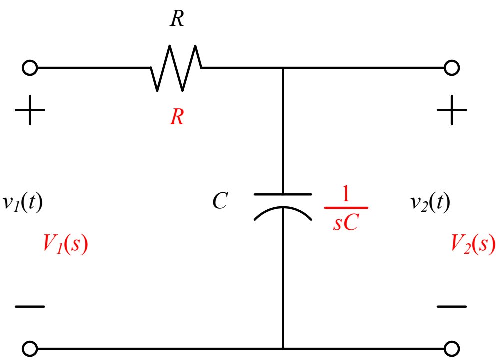
$$= |T(jw)| \measuredangle T(jw)$$

Example) In previous example

w	0	1	2	$\cdots$	∞
$\lvert T(jw)\rvert$	1	$1/\sqrt{2}$	$1/\sqrt{5}$	$\cdots$	0
$\measuredangle T(jw)$	0°	-45°	-63.43	$\cdots$	-90°

Example 8.1) Freq. resp. of RC filter

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{1 + sRC}$$



sinusoidal steady−state TF

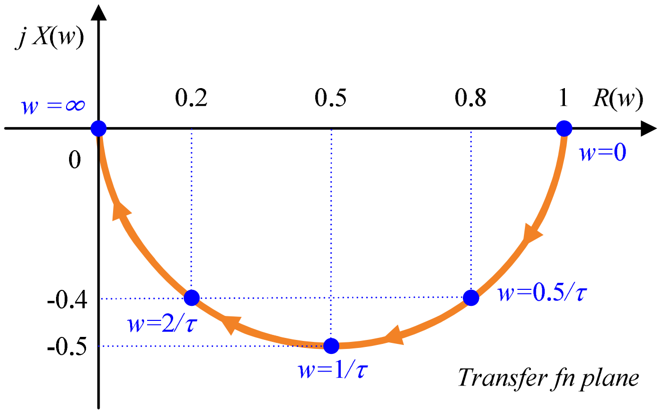
$$T(jw) = \frac{1}{1 + jwRC} \quad \leftarrow \tau = RC > 0$$

$$= \frac{1}{1 + jw\tau} \times \frac{1 - jw\tau}{1 - jw\tau}$$

$$= \frac{1}{1 + (w\tau)^2} + j\frac{-w\tau}{1 + (w\tau)^2}$$

$$= R(w) + jX(w)$$

w	0	$0.5/\tau$	$1/\tau$	$2/\tau$	∞
$R(w)$	1	0.8	0.5	0.2	0
$X(w)$	0	−0.4	−0.5	−0.4	0



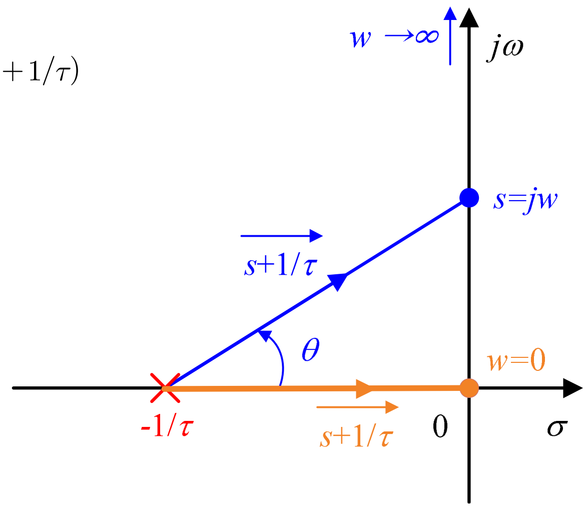
Another solution)

$$T(s) = \frac{1}{\tau s + 1} = \frac{1/\tau}{s + 1/\tau} \quad \leftarrow \ s = jw \ \text{ for sinusoidal input}$$

$$\Rightarrow \ |T(s)| = \frac{1/\tau}{|s + 1/\tau|} \ \text{ and } \ \measuredangle T(s) = -\measuredangle(s + 1/\tau)$$

w	0	$\rightarrow$	∞
$\lvert T(jw)\rvert$	1	$\rightarrow$	0
$\measuredangle T(jw)$	0°	$\rightarrow$	-90°

w	0	$\rightarrow$	∞
$\lvert s+1/\tau\rvert$	$1/\tau$	$\rightarrow$	∞
$\measuredangle(s+1/\tau)$	0°	$\rightarrow$	$+90^\circ$



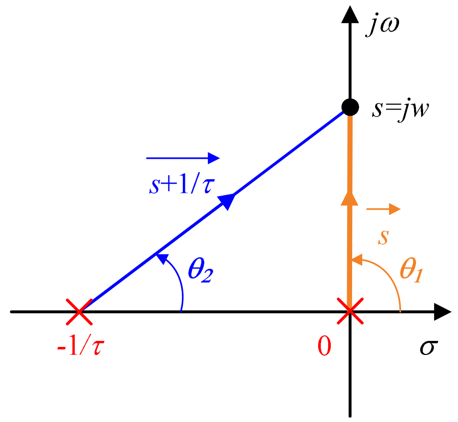
※ $1 + \tau s = \tau(s + 1/\tau)$

$\Rightarrow \ |1 + \tau s| = \tau |s + 1/\tau|$

$\Rightarrow \ \measuredangle(1 + \tau s) = \measuredangle(s + 1/\tau)$

Example 8.2) Polar plot of a TF

$$T(s)=\frac{K}{s(\tau s+1)}=\frac{K/\tau}{s(s+1/\tau)}$$



sol)

$$|T(s)|=\frac{K/\tau}{|s||s+1/\tau|}$$

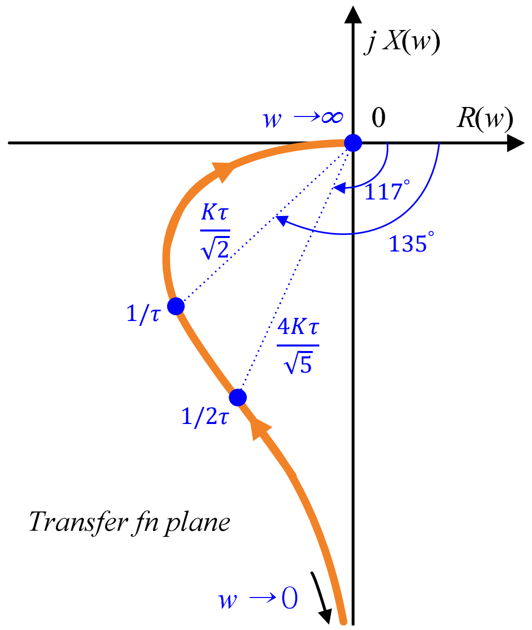
$$\measuredangle T(s)=-[\measuredangle(s)+\measuredangle(s+1/\tau)]=-(\theta_1+\theta_2)$$

w	0^+	$\rightarrow$	∞
$\lvert T(jw)\rvert$	∞	$\rightarrow$	0
$\measuredangle T(jw)$	-90°	$\rightarrow$	-180°

w	0^+	$\rightarrow$	∞
$\lvert s\rvert$	0	$\rightarrow$	∞
$\measuredangle(s)$	$+90^\circ$	$\rightarrow$	$+90^\circ$

always $+90^\circ$

w	0^+	$\rightarrow$	∞
$\lvert s+1/\tau\rvert$	$1/\tau$	$\rightarrow$	∞
$\measuredangle(s+1/\tau)$	0°	$\rightarrow$	$+90^\circ$



At $w=1/\tau$

$$\Rightarrow\ |T(jw)|=\left.\frac{K/\tau}{|s||s+1/\tau|}\right|_{s=jw=j/\tau}$$

$$=\frac{K/\tau}{\frac{1}{\tau}\times\frac{\sqrt{2}}{\tau}}=\frac{K\tau}{\sqrt{2}}$$

$$\Rightarrow\ \measuredangle T(s)=-[\measuredangle(s)+\measuredangle(s+1/\tau)]=-(90^\circ+45^\circ)=-135^\circ$$

At $w=1/2\tau$

$$\Rightarrow\ |T(jw)|=\left.\frac{K/\tau}{|s||s+1/\tau|}\right|_{s=jw=j/2\tau}$$

$$=\frac{K/\tau}{\frac{1}{2\tau}\times\frac{\sqrt{5}}{2\tau}}=\frac{4K\tau}{\sqrt{5}}$$

$$\Rightarrow\ \measuredangle T(s)=-[\measuredangle(s)+\measuredangle(\tau s+1)]=-(90^\circ+27^\circ)=-117^\circ$$

<Bode plot> ~ Logarithmic plot

1) Magnitude plot : $20\log_{10}|T(jw)|$ (dB) versus w

2) Phase plot : $\phi(w)$ versus w

Example 8.3) Bode plot of RC filter ~ Example 8.1)

$$T(s) = \frac{1}{1+sRC} = \frac{1}{1+\tau s} \quad \Rightarrow \quad T(jw) = \frac{1}{1+jw\tau}$$

1) Magnitude plot

$$20\log|T(jw)| = 20\log\frac{1}{|1+jw\tau|}$$

$$= 20\log(1) - 20\log|1+jw\tau|$$

$$= -20\log(1+w^2\tau^2)^{1/2}$$

$$= -10\log(1+w^2\tau^2)$$

2) Phase plot

$$\phi(w) = \measuredangle T(jw) = -\measuredangle(1+jw\tau)$$

$$= -\tan^{-1}\frac{w\tau}{1}$$

w	0	$1/2\tau$	$2/2\tau$	$3/2\tau$	$4/2\tau$	$\cdots$	∞
$\|T(jw)\|$	0	$-10\log(\frac{5}{4})$	$-10\log(2)$	$-10\log(\frac{9}{4})$	$-10\log(5)$		
$\phi(w)$	0°	-27°	-45°		-63°		-90°

⇒ Linear scale of freq. is not judicious choice.

⇒ Logarithmic scale of freq.

For small freq. $w \ll 1/\tau$ $(w\tau \ll 1)$

$$20\log|T(jw)| = -10\log(1+w^2\tau^2) \approx -10\log(1) = 0\,dB$$

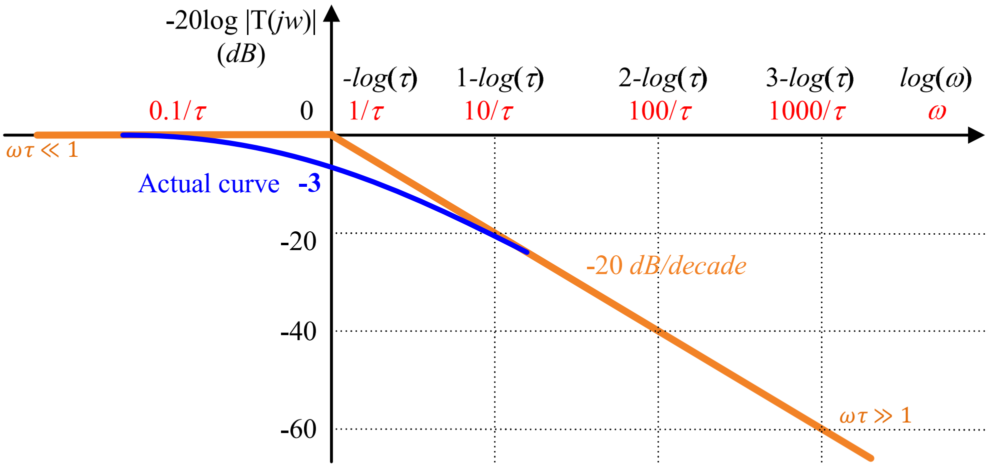
$$\phi(w) = -\tan^{-1}\frac{w\tau}{1} \approx -\tan^{-1}\frac{0}{1} = 0^\circ$$

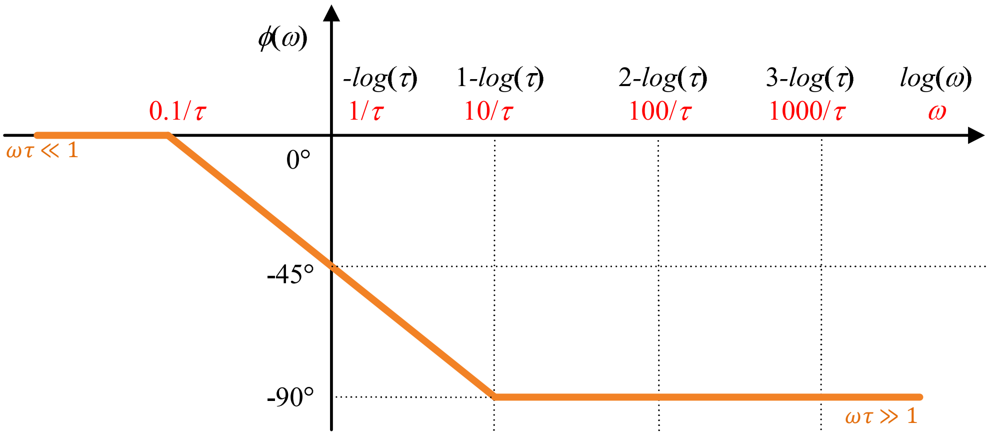
For large freq. $w \gg 1/\tau$ $(w\tau \gg 1)$

$$20\log|T(jw)| = -10\log(1+w^2\tau^2) \approx -10\log(w\tau)^2$$

$$= -20\log(w\tau) = -20\log(w) - 20\log(\tau)$$

$$\phi(w) = -\tan^{-1}\frac{w\tau}{1} \rightarrow -90^\circ$$





※ decade : freq. interval between w_1 and $w_2 = 10w_1$

octave : freq. interval between w_1 and $w_2 = 2w_1$

※ gain difference

1) decade : $w_2 = 10w_1,\ \ w_1 \gg 1/\tau$

$$20\log|T(jw_1)| - 20\log|T(jw_2)| = -20\log(w_1\tau) - [-20\log(w_2\tau)]$$
$$= 20[\log(w_2\tau) - \log(w_1\tau)] = 20\log(\frac{w_2\tau}{w_1\tau})$$
$$= 20\log(10) = 20\ (dB)$$

2) octave : $w_2 = 2w_1,\ \ w_1 \gg 1/\tau$

$$20\log|T(jw_1)| - 20\log|T(jw_2)| = 20\log(\frac{w_2\tau}{w_1\tau})$$
$$= 20\log(2) \approx 6.02\ (dB)$$

※ At $w = 1/\tau$: $-3\,dB$ freq. = cut−off freq.

$$20\log|T(jw)| = -10\log(1+w^2\tau^2) \approx -3\ (dB)$$

$$\phi(w) = -\tan^{-1}\frac{w\tau}{1} = -45^\circ$$

<Generalized TF>

$$T(s) = \frac{G_1(s)\, G_2(s)}{G_3(s)}$$

$$\Rightarrow \; 20\log|T| = 20\log(G_1) + 20\log(G_2) - 20\log(G_3)$$

$$\phi(w) = \measuredangle G_1 + \measuredangle G_2 - \measuredangle G_3$$

1) Constant gain $G(s) = K$

2) poles(zeros) at the origin $G(s) = \frac{1}{s}$, OR $G(s) = s$

3) poles(zeros) on the real axis $G(s) = \frac{1}{1+\tau s}$, OR $G(s) = 1 + \tau s$

4) Complex conjugate poles (zeros) $G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$, OR $G(s) = \frac{s^2 + 2\zeta w_n s + w_n^2}{w_n^2}$

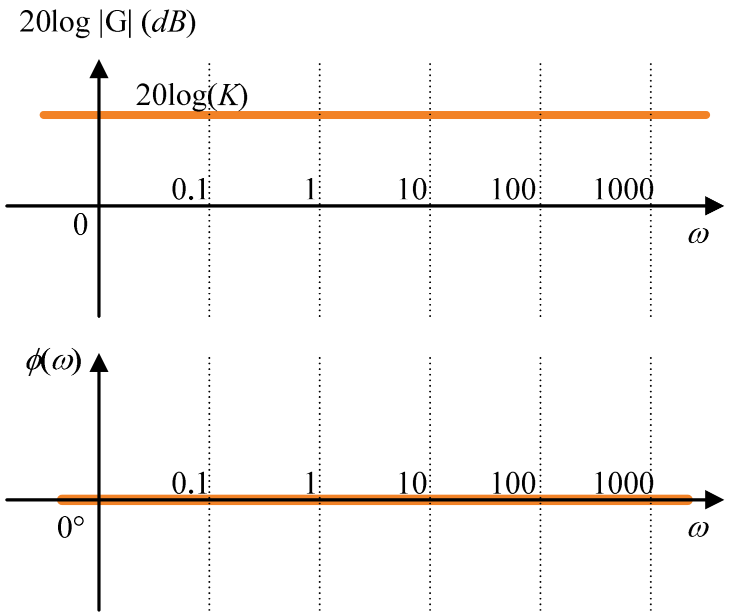
※ Bode 선도 그리기

1) Constant Gain

$G(s) = K$

$$\Rightarrow \; 20\log|G| = 20\log(K) \; : \text{Constant}$$

$$\phi(w) = 0^{\circ}$$



2) Poles at the origin

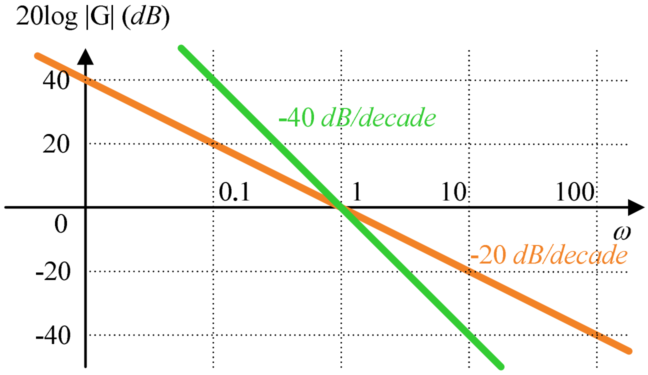
$$G(s) = \frac{1}{s} \quad \rightarrow \quad G(jw) = \frac{1}{jw}$$

$$\Rightarrow 20\log|G(jw)| = -20\log(w) \ (dB)$$

$$\phi(w) = -\tan^{-1}\left(\frac{w}{0}\right) = -90^\circ$$

$$\text{OR} \ \ \phi(w) = \measuredangle G(s)|_{s=jw}$$

$$= \measuredangle(1) - \measuredangle(jw) = -90^\circ$$



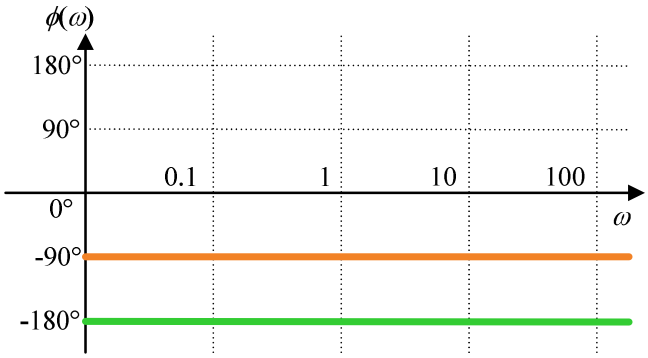
Example)

$$G(s) = \frac{1}{s^2} \quad \rightarrow \quad G(jw) = \frac{1}{(jw)^2}$$

$$\Rightarrow 20\log|G(jw)| = -20\log(w^2)$$
$$= -40\log(w)$$

$$\phi(w) = \measuredangle G(s)|_{s=jw}$$

$$= \measuredangle(1) - 2\measuredangle(jw) = -180^\circ$$



Example)

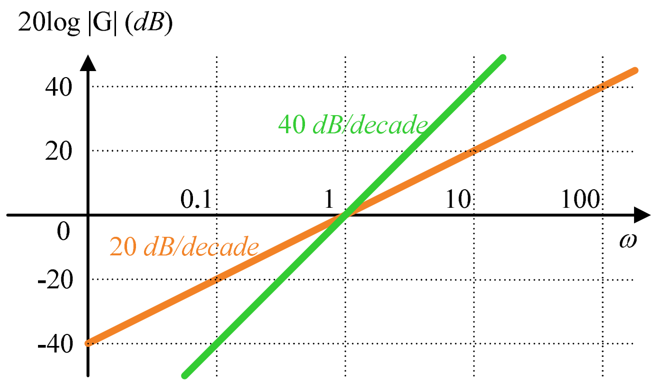
$$G(s) = \frac{1}{s^3}$$

Example) Zeros at the origin

$$G(s) = s \quad \rightarrow \quad G(jw) = jw$$

$$\Rightarrow 20\log|G(jw)| = +20\log(w) \ (dB)$$

$$\phi(w) = \measuredangle(jw) = +90^\circ$$

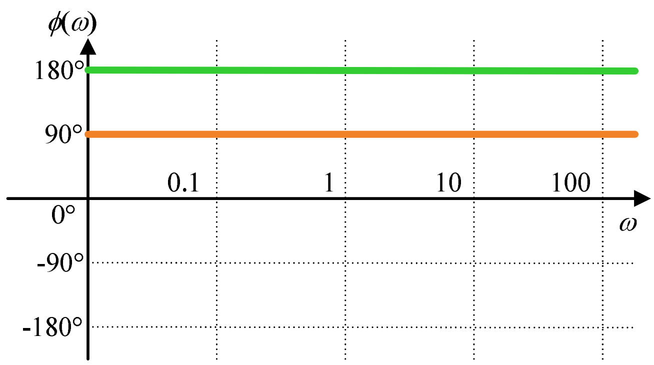


Example)

$$G(s) = s^2 \quad \rightarrow \quad G(jw) = (jw)^2$$

$$\Rightarrow 20\log|G(jw)| = +40\log(w) \ (dB)$$

$$\phi(w) = 2\measuredangle(jw) = +180^\circ$$



Example)

$$G(s) = s^3$$

3) Poles on the real axis

$G(s) = \dfrac{1}{1+\tau s} \quad \rightarrow \quad G(jw) = \dfrac{1}{1+jw\tau}$

$$\Rightarrow \; 20\log|G(jw)| = -20\log(1+w^2\tau^2)^{-1/2} = -10\log(1+w^2\tau^2)$$

$$\phi(w) = -\tan^{-1}\left(\frac{w\tau}{1}\right)$$

If $w \ll 1/\tau$ $(w\tau \ll 1)$,

then $20\log|G(jw)| \approx 0$ and $\phi(w) \approx 0$

If $w \gg 1/\tau$ $(w\tau \gg 1)$,

then $20\log|G(jw)| \approx -10\log(w\tau)^2 = -20\log(w\tau)$

$$\phi(w) \approx -90$$

※ At $w = 1/\tau$: $-3\,dB$ freq.

: lower $3dB$ freq.

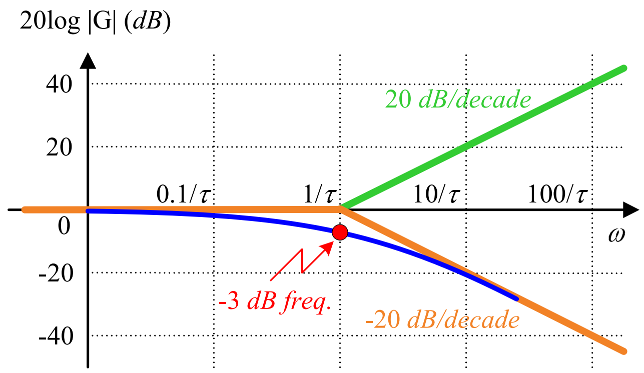
: cut−off freq.

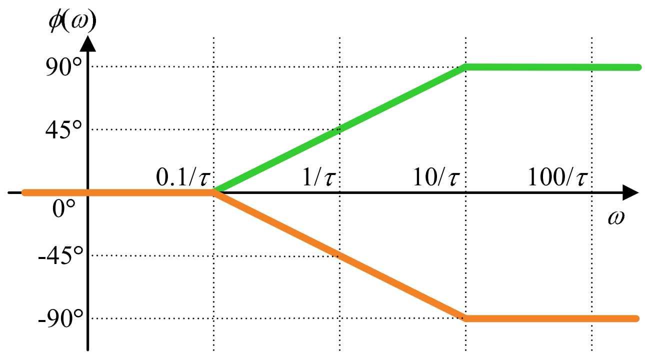
: coner freq.

: break freq.

$$20\log|G(jw)| = -10\log(1+w^2\tau^2) \approx -3\ (dB)$$

$$\phi(w) = -\tan^{-1}\frac{w\tau}{1} = -45^\circ$$





Example) Zeros on the real axis

$G(s) = 1+\tau s$

$$\Rightarrow \; 20\log|G(jw)| = 20\log(1+w^2\tau^2)^{1/2} = 10\log(1+w^2\tau^2)$$

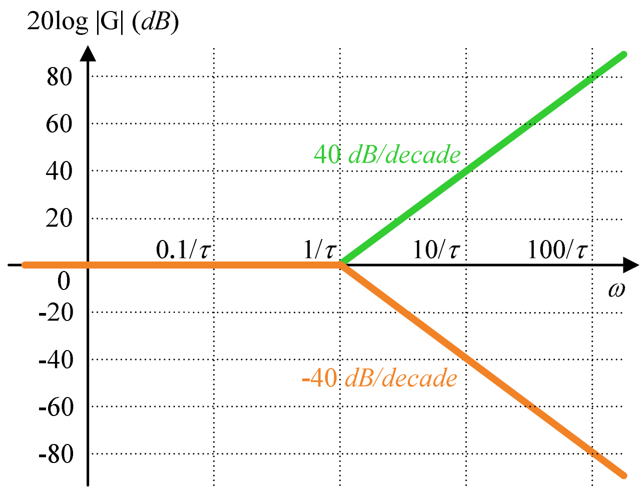
$$\phi(w) = \tan^{-1}\left(\frac{w\tau}{1}\right)$$

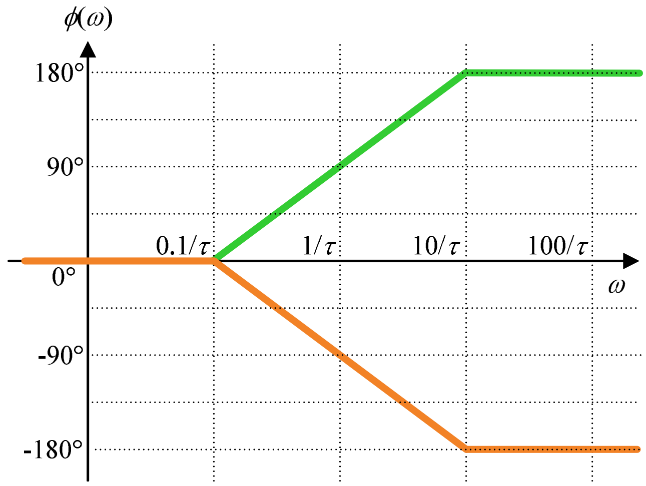
Example) Double poles on the real axis

Double zeros on the real axis

$$G(s) = \frac{1}{(1+\tau s)^2}$$

$$G(s) = (1+\tau s)^2$$





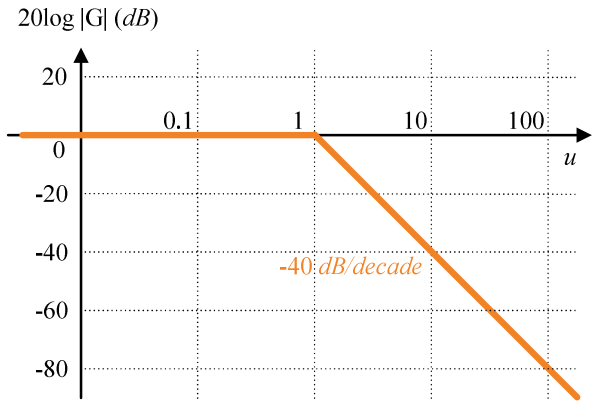
4) Complex conjugate poles

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2} \quad \rightarrow \quad G(jw) = \frac{w_n^2}{(jw)^2 + 2\zeta w_n (jw) + w_n^2} = \frac{1}{\left(\frac{jw}{w_n}\right)^2 + j\left(\frac{2\zeta w}{w_n}\right) + 1}$$

$$= \frac{1}{1 - u^2 + j2\zeta u}, \quad \text{where } u \equiv \frac{w}{w_n} \ : \text{ normalized freq.}$$

$$20\log|G(jw)| = -20\log\left[(1-u^2)^2 + 4\zeta^2 u^2\right]^{-1/2}$$
$$= -10\log\left[(1-u^2)^2 + 4\zeta^2 u^2\right]$$

$$\phi(w) = -\tan^{-1}\left(\frac{2\zeta u}{1-u^2}\right)$$



If $u \ll 1$,

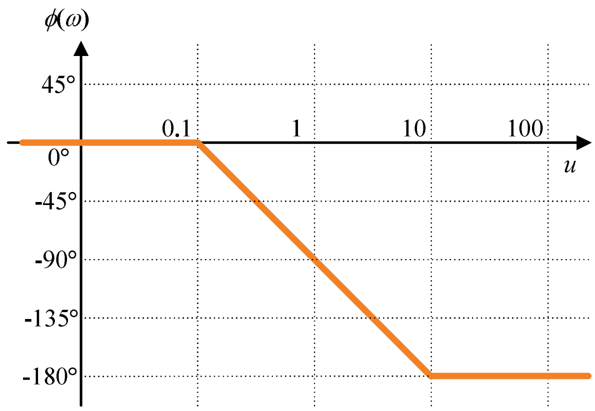
then $20\log|G(jw)| \cong -10\log(1) = 0\ dB$

$$\phi(w) \cong -\tan^{-1}\left(\frac{0}{1}\right) = 0^\circ$$

If $u \gg 1$,

then

$$20\log|G(jw)| \cong -10\log(u^4) = -40\log(u)$$
$$\phi(w) \cong -\tan^{-1}\left(\frac{2\zeta u}{-u^2}\right) = -180^\circ$$



※ At $u = 1$

$$20\log|G(jw)| = -10\log(4\zeta^2) = -20\log(2\zeta)$$
$$\phi(w) \cong -\tan^{-1}\left(\frac{2\zeta}{0}\right) = -90^\circ$$

If $\zeta = 1$,

then $20\log|G| = -20\log(2) \cong -6\ dB$

If $\zeta = 1/\sqrt{2}$,

then $20\log|G| = -20\log(2/\sqrt{2}) \cong -3\ dB$

If $2\zeta = 1$,

then $20\log|G| = -20\log(1) = 0\ dB$

If $2\zeta = 10^{-1}$,

then $20\log|G| = -20\log(10^{-1}) = 20\ dB$

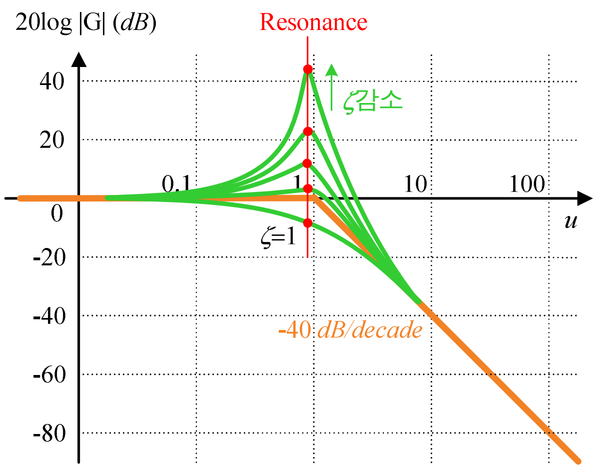
If $2\zeta = 10^{-2}$,

then $20\log|G| = -20\log(10^{-2}) = 40\ dB$

If $2\zeta = 10^{-3}$,

then $20\log|G| = -20\log(10^{-3}) = 60\ dB$

$\vdots \qquad\qquad \vdots$



※ Resonant freq.

$$|G(jw)| = \frac{1}{\sqrt{(1-u^2)^2+4\zeta^2u^2}} \quad \rightarrow \quad \frac{d\,|G|}{du}=0$$

$$\frac{d\,|G|}{du}=\frac{-\frac{1}{2}\left[(1-u^2)^2+4\zeta^2u^2\right]^{-1/2}\cdot\left[(1-u^2)^2+4\zeta^2u^2\right]'}{(1-u^2)^2+4\zeta^2u^2}=0$$

$$(1-u^2)^2+4\zeta^2u^2 \neq 0$$

Hence, $\left[(1-u^2)^2+4\zeta^2u^2\right]' = 2(1-u^2)(-2u)+8\zeta^2u = 0$

$$4u(u^2-1+2\zeta^2)=0$$

$$\Rightarrow\ u=0 \ \text{ or } \ u=\pm\sqrt{1-2\zeta^2}$$

$\therefore)$ $u=+\sqrt{1-2\zeta^2} \equiv \dfrac{w}{w_n}$

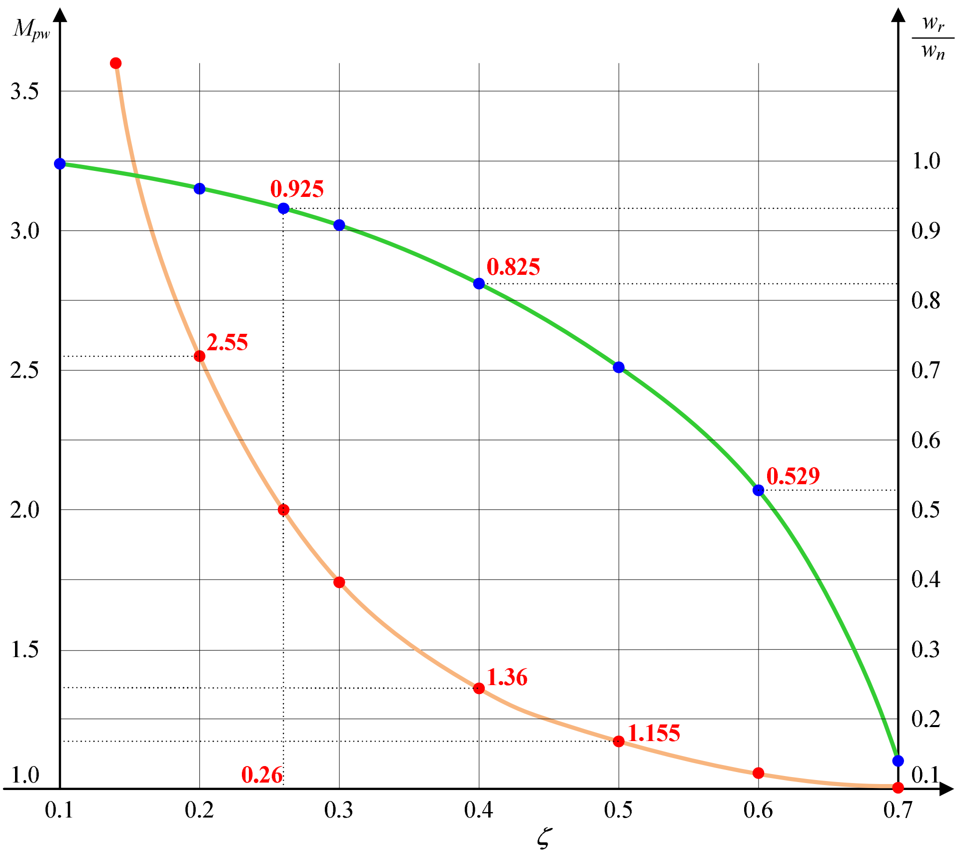
※ Resonant freq.

$$w_r = w_n\sqrt{1-2\zeta^2} \qquad (8-36)$$

※ Maximum magnitude

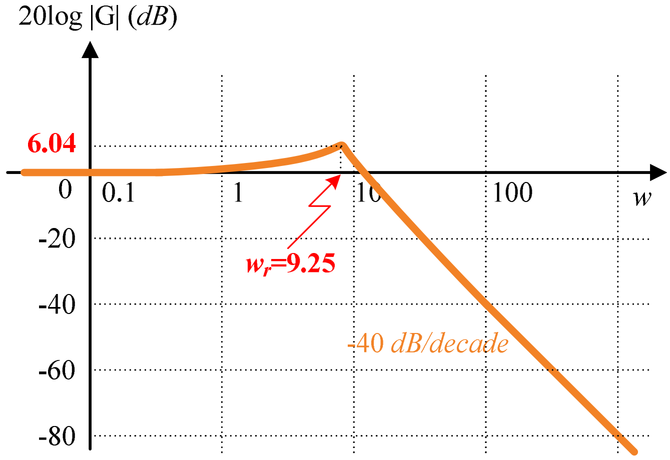
$$M_{pw} = |G|_{\max} = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad (8-37)$$

proof) $|G(jw)| = \left.\dfrac{1}{\sqrt{(1-u^2)^2+4\zeta^2u^2}}\right|_{u=\sqrt{1-2\zeta^2}}$



Example) Given Bode plot of 2nd order system

$$G(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$



sol)

$20\log(M_{pw}) = 6.04\ dB$

$\Rightarrow\ M_{pw} = 2$

From Fig. 8.11

$\Rightarrow\quad \zeta = 0.26$

$\frac{w_r}{w_n} = 0.925$

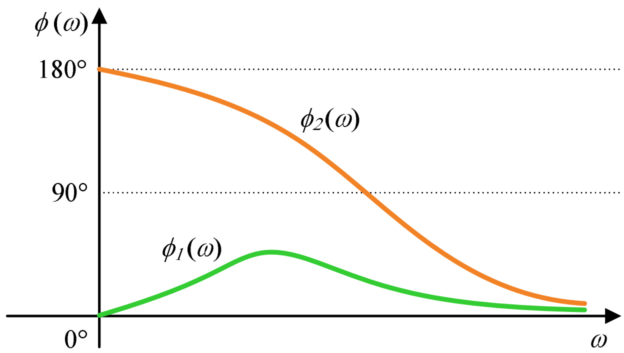
$\Rightarrow\ w_n = \frac{w_r}{0.925} \approx 10$

Hence, $\quad G(s) = \frac{10^2}{s^2 + 2 \cdot 0.26 \cdot 10\, s + 10^2} = \frac{100}{s^2 + 5.12\, s + 100}$

※ "Minimum phase" Transfer function if all its zeros lie in the lhp.
"Non−minimum phase" Transfer function if it has zeros lie in the rhp.

Example)

$G_1(s) = \frac{s+1}{s+2}$ and $G_1(s) = \frac{s-1}{s+2}$



1) Magnitude 특성 : $|G_1| = |G_2|$: same

$$|G_1| = \left.\frac{|s+1|}{|s+2|}\right|_{s=jw} = \frac{|1+jw|}{|2+jw|} = \frac{\sqrt{1+w^2}}{\sqrt{4+w^2}}$$

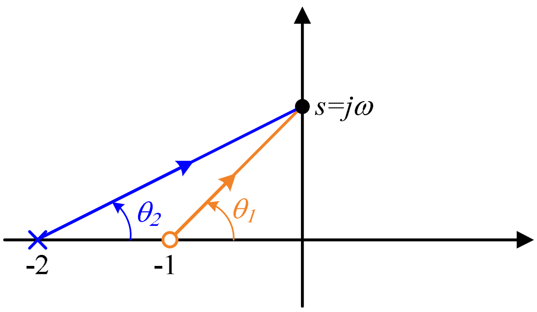
$$|G_2| = \left.\frac{|s-1|}{|s+2|}\right|_{s=jw} = \frac{|-1+jw|}{|2+jw|} = \frac{\sqrt{1+w^2}}{\sqrt{4+w^2}}$$

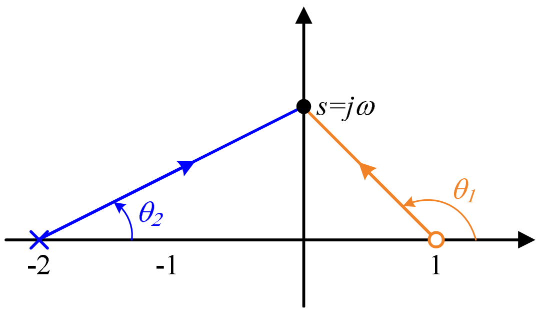
2) Phase 특성

$\measuredangle G_1 = \measuredangle(s+1) - \measuredangle(s+2) = \theta_1 - \theta_2 \leqq 90^\circ$

$\measuredangle G_2 = \measuredangle(s-1) - \measuredangle(s+2) = \theta_1 - \theta_2 \leqq 180^\circ$

※ Net phase shift over all freq. range $\Rightarrow$ $\phi_1 \ll \phi_2$





Example)

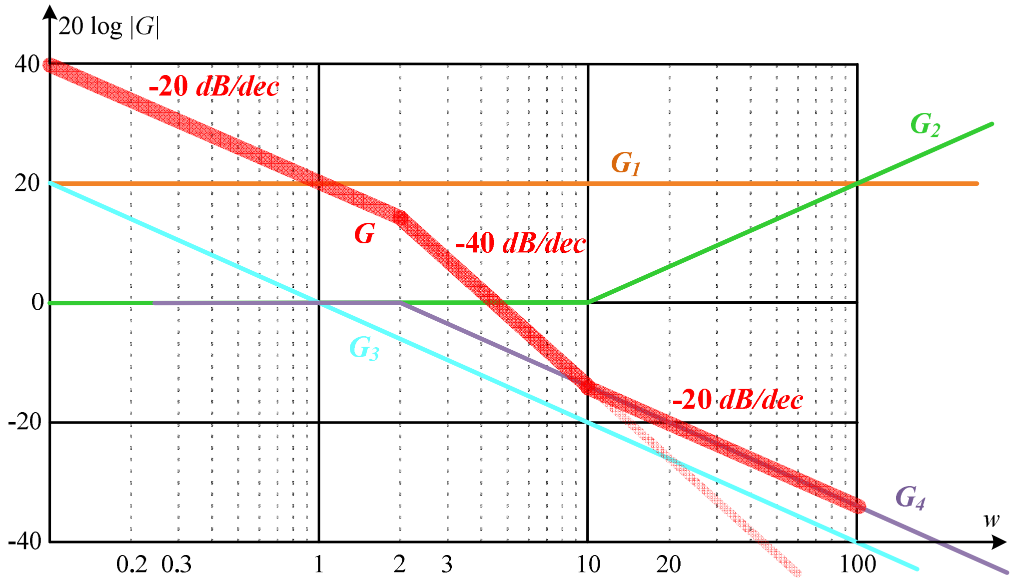
$$G(s) = \frac{10(1+s/10)}{s(1+s/2)}$$

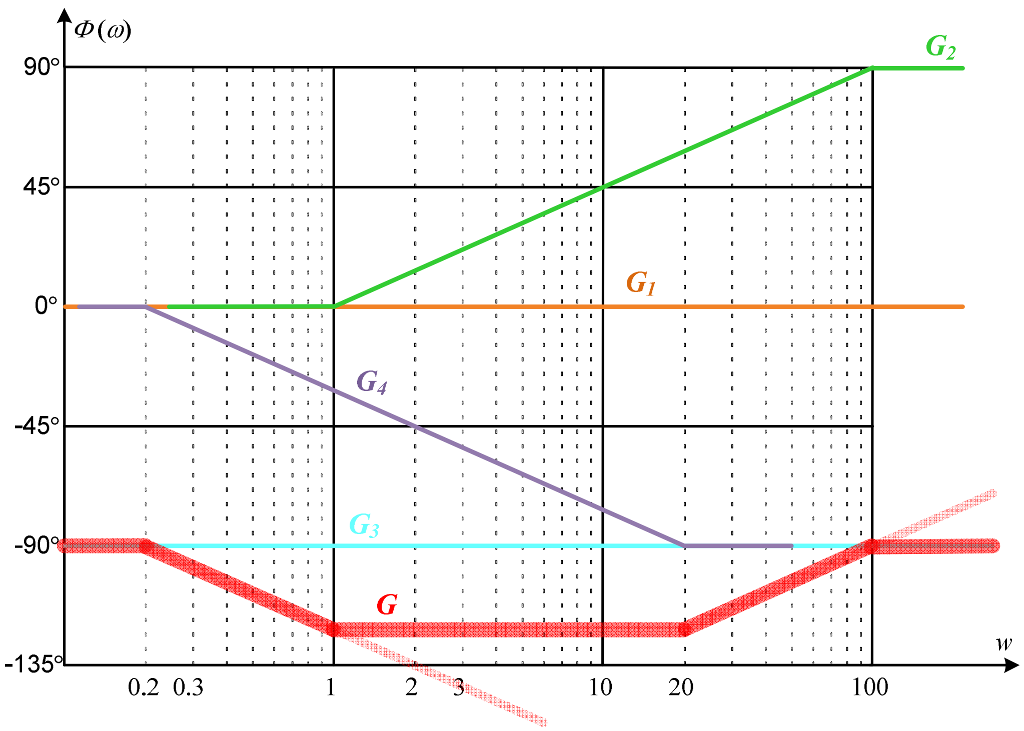
$$= 10 \times (1+s/10) \times \frac{1}{s} \times \frac{1}{(1+s/2)}$$

$$= G_1 \times \quad G_2 \quad \times G_3 \times \quad G_4$$

1) $20\log|G| = 20\log|G_1| + 20\log|G_2| + 20\log|G_3| + 20\log|G_4|$

2) $\measuredangle G = \measuredangle G_1 + \measuredangle G_2 + \measuredangle G_3 + \measuredangle G_4$





Example) Given Bode plot

sol)

1) $G_1(s) = \dfrac{K}{s}$

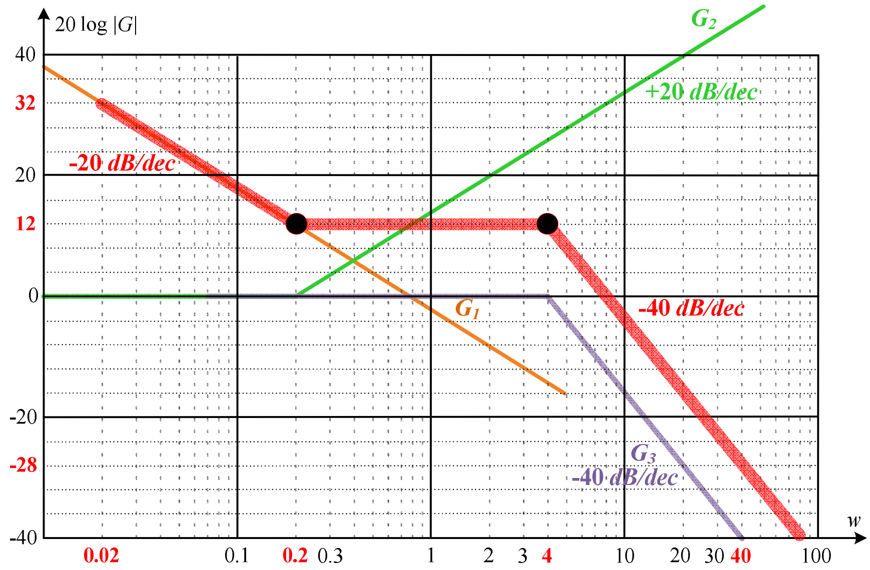
At $w = 0.2$

$\Rightarrow\ 20\log\left|G_1\right| = 12\ dB$

$\Rightarrow\ 20\log\left(\dfrac{K}{0.2}\right) = 12$

$\Rightarrow\ 5K = 10^{0.6} = 0.8$

$\Rightarrow\ K = 0.13$



2) $G_2(s) = 1 + s/0.2$

$= 5s + 1$

3) $G_3(s) = \dfrac{1}{(1+s/4)^2} = \dfrac{16}{(s+4)^2}$

Hence, $G(s) = G_1(s)\cdot G_2(s)\cdot G_3(s) = \dfrac{0.13}{s}\cdot(5s+1)\cdot\dfrac{16}{(s+4)^2}$

$= \dfrac{2.08(5s+1)}{s(s+4)^2}$

Example) Given Bode plot

sol)

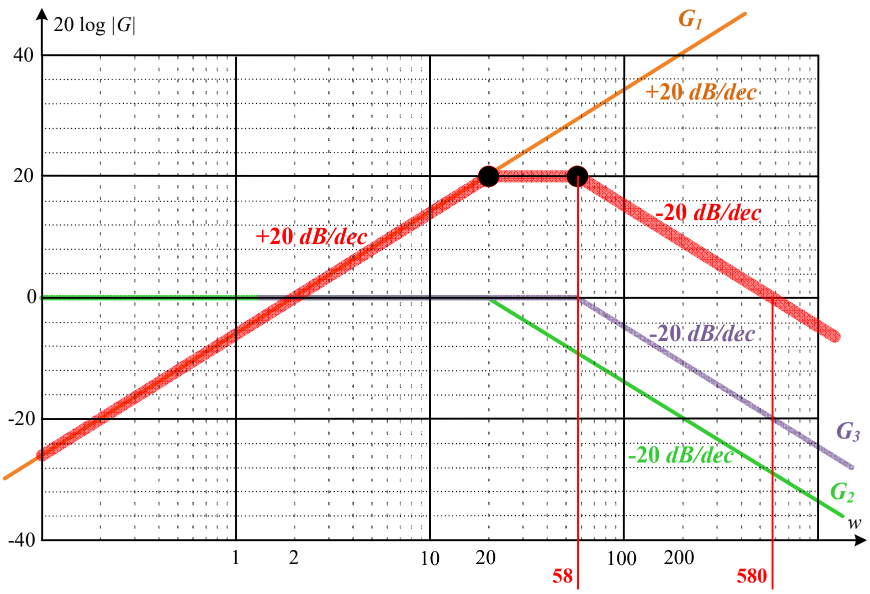
1) $G_1(s) = Ks$

At $w = 2$

$\Rightarrow\ 20\log\left|G_1\right| = 0\ dB$

$\Rightarrow\ 20\log(2K) = 0$

$\Rightarrow\ K = 0.5$



2) $G_2(s) = \dfrac{1}{1+s/20}$

$= \dfrac{20}{s+20}$

3) $G_3(s) = \dfrac{1}{1+s/58} = \dfrac{58}{s+58}$

Hence, $G(s) = G_1(s)\cdot G_2(s)\cdot G_3(s) = 0.5s\cdot\dfrac{20}{s+20}\cdot\dfrac{58}{s+58}$

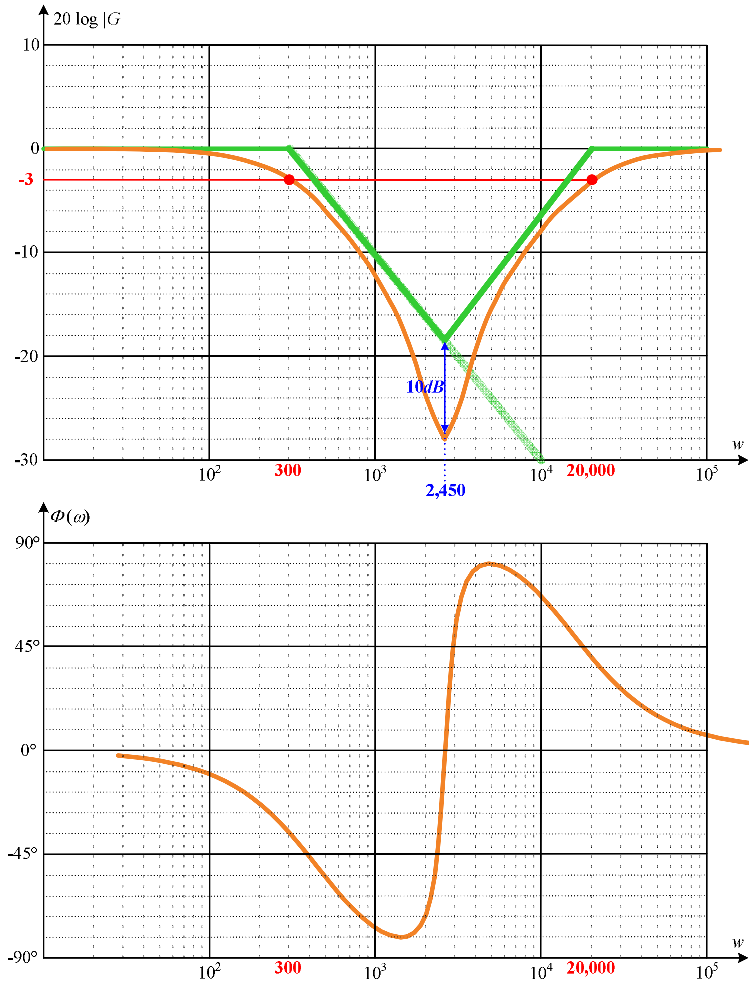
$= \dfrac{580s}{(s+20)(s+58)}$

8.4 Freq. response measurements

Bode plot (freq. resp. for unknown system) from Signal Analyzer

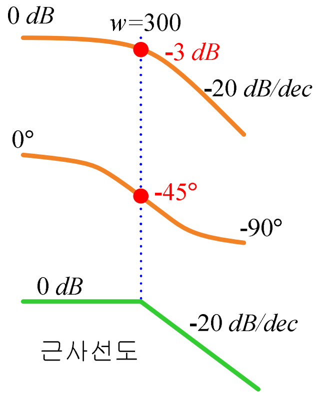
$\Rightarrow$ Transfer fn.

Example) Given Bode plot



sol)

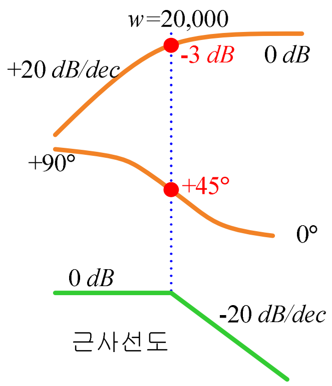
1) 1st section



$\Rightarrow$ real pole with $w_1 = 300$

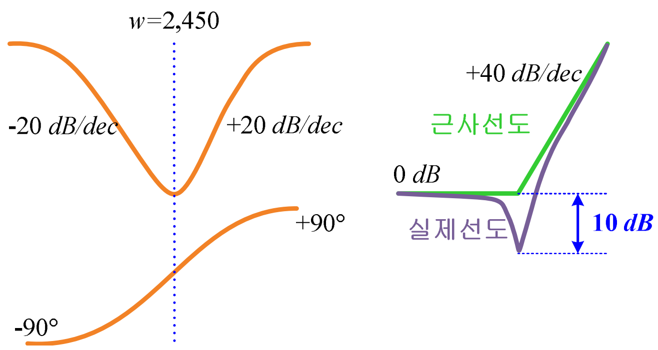
$$G_1(s) = \frac{1}{1 + s/300}$$

2) 3rd section



$\Rightarrow$ real pole with $w_1 = 300$ $\qquad G_3(s) = \dfrac{1}{1 + s/20{,}000}$

3) 2nd section



$$G_2(s) = \frac{s^2 + 2\zeta w_n s + w_n^2}{w_n^2}$$

$\Rightarrow$ complex zeros with $w_r = 2{,}450$

※ 복소영점의 실제선도와 근사선도의 차이

$20\log(M_{pw}) = 10\,dB$

$\Rightarrow\ M_{pw} = \sqrt{10} = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}}$ or from Fig. 8.11

$\Rightarrow\ 10 = \dfrac{1}{4\zeta^2(1-\zeta^2)}$ $\quad\Rightarrow\ \zeta^4 - \zeta^2 + \dfrac{1}{40} = 0$ $\quad\Rightarrow\ \zeta^2 = 0.974$ or 0.0256

$\Rightarrow\ \zeta = 0.988$ or 0.16 $\quad\leftarrow\ \zeta \leqq 0.707$

Hence, $\zeta = 0.16$

$w_r = w_n\sqrt{1-2\zeta^2} = 2{,}450$ or from Fig. 8.11

$\Rightarrow\ w_n = \dfrac{2{,}450}{\sqrt{1-2\zeta^2}} \approx 2{,}515$

Hence, $G_2(s) = \dfrac{s^2 + 2\zeta w_n s + w_n^2}{w_n^2} = \dfrac{s^2 + 2\cdot 0.16\cdot 2{,}515\,s + 2{,}515^2}{2{,}515^2}$

$\approx \dfrac{s^2 + 805\,s + 6{,}325{,}225}{6{,}325{,}225}$

Total transfer fn

$T(s) = G_1 \cdot G_2 \cdot G_3$

$= \dfrac{1}{1+s/300}\ \dfrac{s^2 + 805\,s + 6{,}325{,}225}{6{,}325{,}225}\ \dfrac{1}{1+s/20{,}000}$

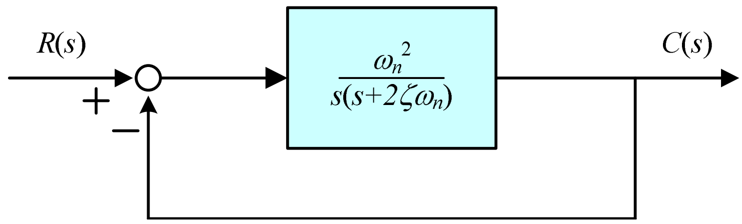
$= \dfrac{0.9486\,(s^2 + 805\,s + 6{,}325{,}225)}{(s+300)(s+20{,}000)}$

8.5. Performance spec. in the freq. domain

Freq. resp. (freq. domain) $\Leftrightarrow$ Transient resp. (time domain)

(Bode plot) $(P.O.,\ T_r,\ T_s,\ \zeta$ 등)

Example) 2nd order closed-loop system ~ Fig. 8.24



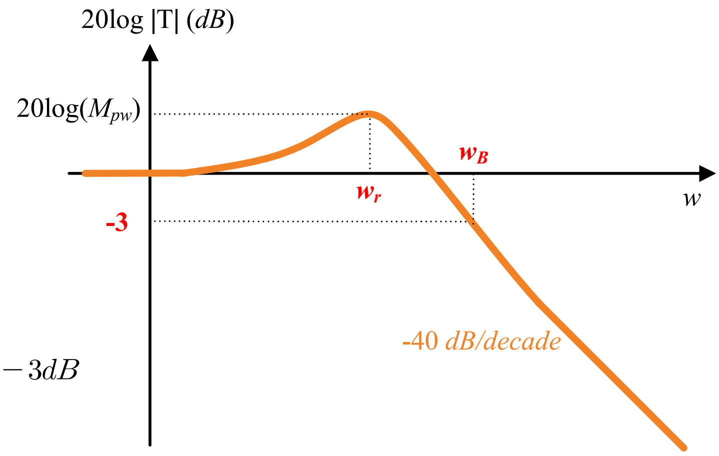
$$T(s)=\frac{w_n^2}{s^2+2\zeta w_n s+w_n^2}$$

Resonant freq.

$$w_r = w_n\sqrt{1-2\zeta^2}$$

Maximum magnitude

$$M_{pw}=\frac{1}{2\zeta\sqrt{1-\zeta^2}}$$



※ Bandwidth : $w_B = -3\,dB$ freq.

$$20\log|T| = -10\log\left[(1-u^2)+2\zeta^2u^2\right] = -3dB$$

$$(1-u^2)+2\zeta^2u^2 = 10^{-3} = 1.9955$$

$\therefore)\ u \approx -1.1961\zeta + 1.8508$ ~ Fig. 8.26

$$=\frac{w_B}{w_n}$$

$$w_B = w_n(-1.1961\zeta + 1.8508)$$

for $0.3 \leqq \zeta \leqq 0.8$

※ w_B 증가 (w_n고정) $\rightarrow$ ζ 감소 $\rightarrow$ T_r 감소 : $T_r=\dfrac{2.16\zeta+0.6}{w_n}$

M_{pw} 증가 $\rightarrow$ ζ 감소 $\rightarrow$ $P.O.$ 증가 : $P.O. = 100\exp\left(\dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right)$

w_n 증가 (ζ고정) $\rightarrow$ T_s 감소 : $T_s=\dfrac{4}{\zeta w_n}$

※ Desirable freq. domain spec.

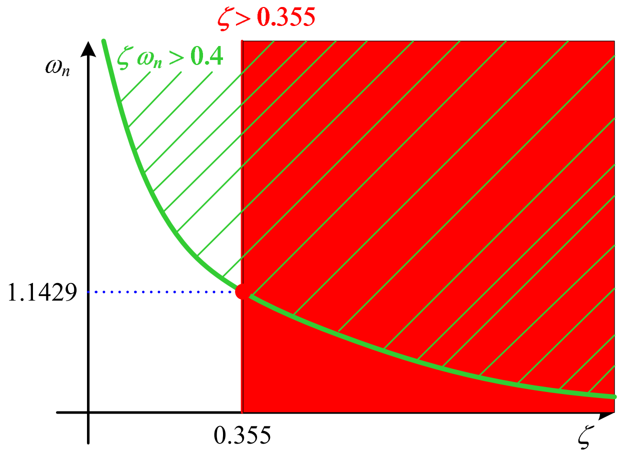
1) Relatively small M_{pw}

2) Relatively large w_B

Example) Find ζ and w_n to satisfy the following design spec.

1) $M_{pw} \le 1.5$

2) $T_s \le 10$ sec.



sol)

1) $M_{pw} \le 1.5$

$\rightarrow$ $\zeta \ge 0.355$ from Fig. 8.11

2) $T_s = \dfrac{4}{\zeta w_n} \le 10$ sec.

$\rightarrow$ $\zeta w_n \ge 0.4$

※ Steady-state error spec.

$$T(s) = \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

$$E(s) = R(s) - C(s) = [1 - T(s)]\,R(s)$$

1) unit step input, $R(s) = \dfrac{1}{s}$

$$e_{ss} = \lim_{s \to 0} sE(s) = 1 - T(0) = 1 - 1 = 0$$

2) unit ramp input, $R(s) = \dfrac{1}{s^2}$

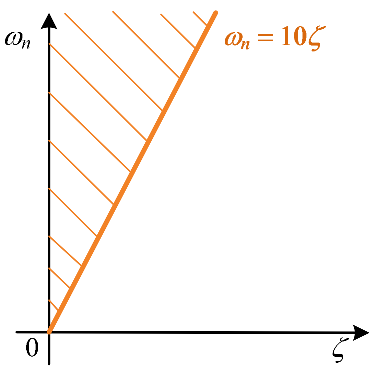
$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{1 - T(s)}{s} = \lim_{s \to 0} \frac{s + 2\zeta w_n}{s^2 + 2\zeta w_n s + w_n^2} = \frac{2\zeta}{w_n}$$

Example) $e_{ss} < 0.2$ for unit ramp input

sol)

$$e_{ss} = \frac{2\zeta}{w_n} < 0.2$$

$\Rightarrow$ $w_n > 10\zeta$

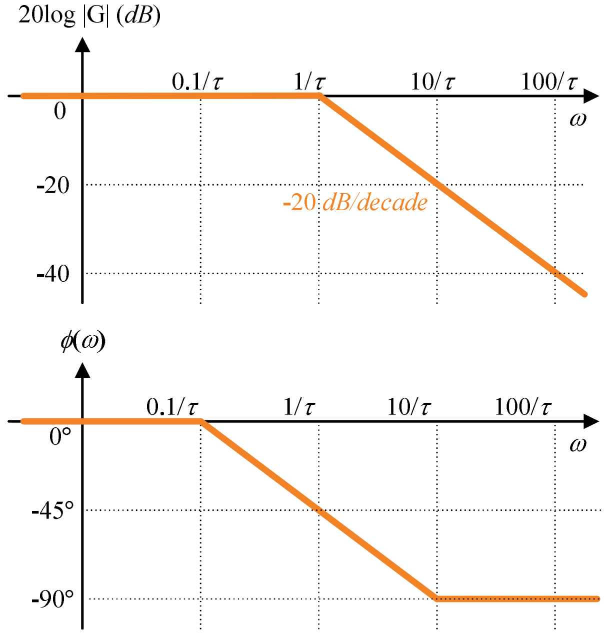


8.6 Log Magnitude and Phase diagram

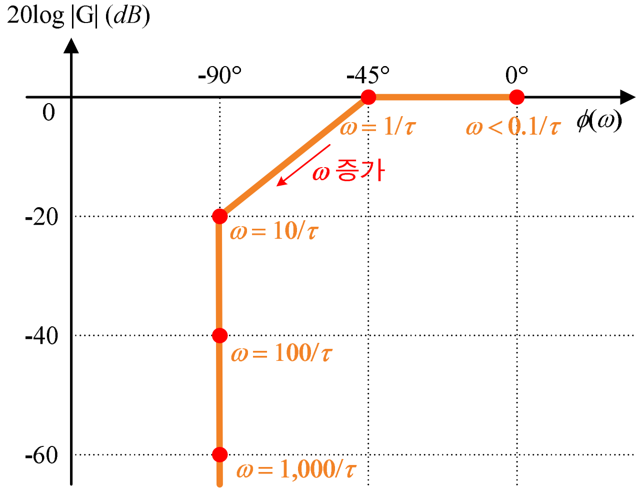
Example)

$$G(s) = \frac{1}{1 + \tau s}$$

sol)



<Bode plot>



<Log Mag. and Phase diagram>

Example) Find transfer fn.

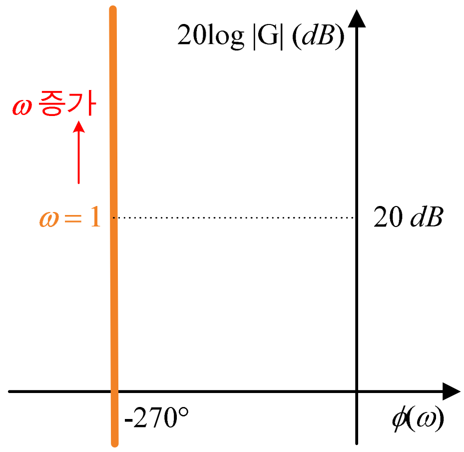
sol)

$$G(s) = \frac{K}{s^3}$$

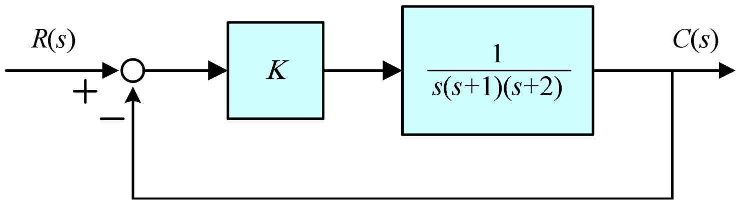
At $w = 1$, $\quad 20\log|G| = 20\ dB$

$$\Rightarrow\ 20\log\left(\frac{K}{w^3}\right)_{w=1} = 20\ dB$$

Hence, $K = 10$

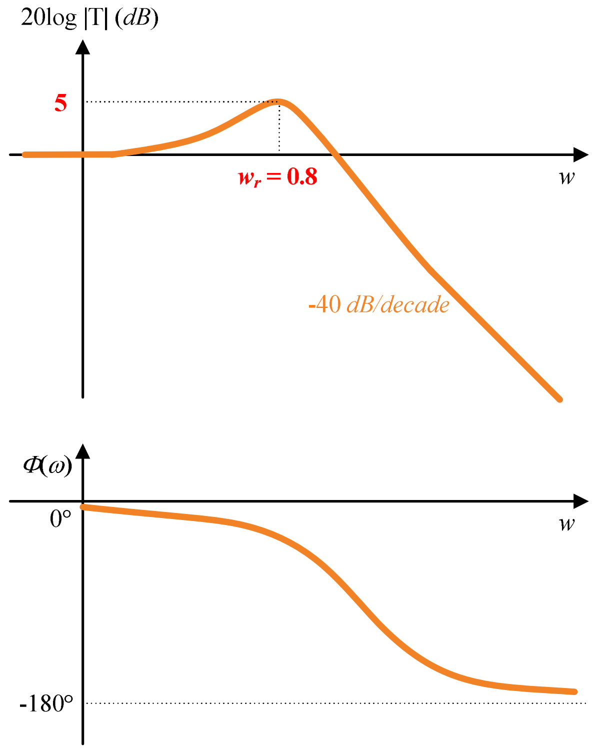


8.7 Design Example : Engraving machine control system



Given Bode plot

sol)



$$T(s) = \frac{K}{s^3 + 3s^2 + 2s + K} \approx \frac{w_n^2}{s^2 + 2\zeta w_n s + w_n^2}$$

: $s_{1,2,3} = -2.5214,\ -0.2393 \pm j0.8579$

: approximation to 2nd order system

$s_{1,2} = -0.2393 \pm j0.8579$

$20\log(M_{pw}) = 5\ dB$ at $w_r = 0.8$

$\Rightarrow M_{pw} = 1.7783$

$\Rightarrow \zeta = 0.285$ and $\dfrac{w_r}{w_n} = 0.91$ from Fig. 8.11

$$w_n = \frac{0.91}{w_r} = 0.88$$

Hence,

$$T(s) \approx \frac{0.774}{s^2 + 0.5s + 0.774}$$

$P.O. = 37\%$ from Fig. 5.8

$$T_s = \frac{4}{\zeta w_n} = 15.7\text{sec.}$$