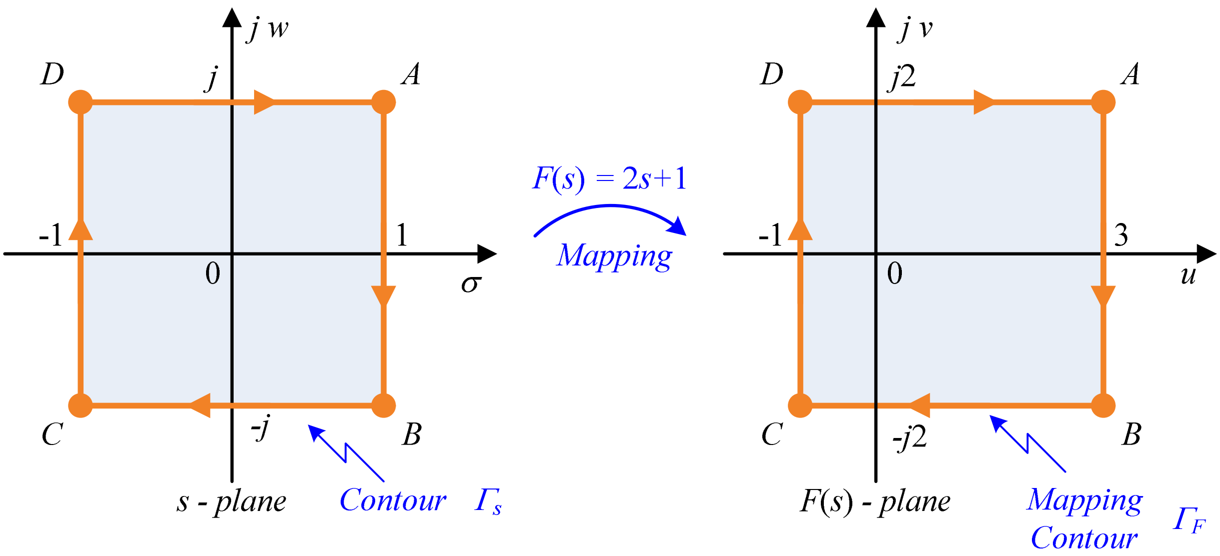
Chapter 9. Stability in the freq. domain

- Nyquist stability criterion based on Cauchy's theorem

9.2 Mapping Contour in the s-domain

Example) $F(s) = 2s + 1$



1) point A $\rightarrow$ $F(s)|_{s=1+j} = 2(1+j)+1 = 3+j2$

2) point B $\rightarrow$ $F(s)|_{s=1-j} = 2(1-j)+1 = 3-j2$

3) point C $\rightarrow$ $F(s)|_{s=-1-j} = 2(-1-j)+1 = -1-j2$

4) point D $\rightarrow$ $F(s)|_{s=-1+j} = 2(-1+j)+1 = -1+j2$

Example) $F(s) = \dfrac{s}{s+2}$

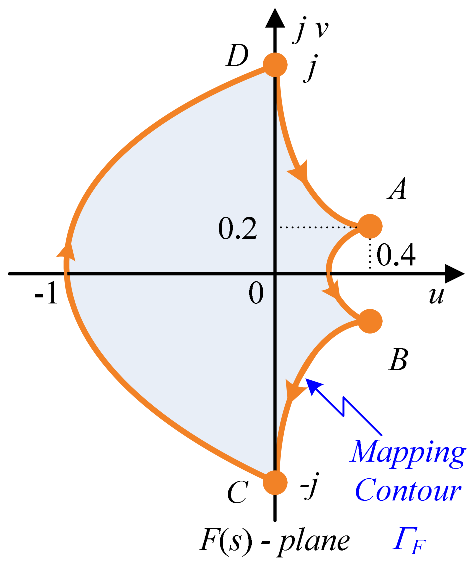
1) $F(s)|_{s=1+j} = \dfrac{1+j}{3+j} = \dfrac{1+j}{3+j}\,\dfrac{3-j}{3-j} = \dfrac{4+j2}{10}$

2) $F(s)|_{s=1-j} = \dfrac{1-j}{3-j} = \dfrac{1-j}{3-j}\,\dfrac{3+j}{3+j} = \dfrac{4-j2}{10}$

3) $F(s)|_{s=-1-j} = \dfrac{-1-j}{1-j} = \dfrac{-1-j}{1-j}\,\dfrac{1+j}{1+j} = -j$

4) $F(s)|_{s=-1+j} = \dfrac{-1+j}{1+j} = \dfrac{-1+j}{1+j}\,\dfrac{1-j}{1-j} = j$

※ $s = -1$ $\rightarrow$ $F(s)|_{s=-1} = \dfrac{-1}{1} = -1$



※ Enclosed Area

~ Contour area to the right of traversal of the contour

~ "clockwise and eyes right" rule

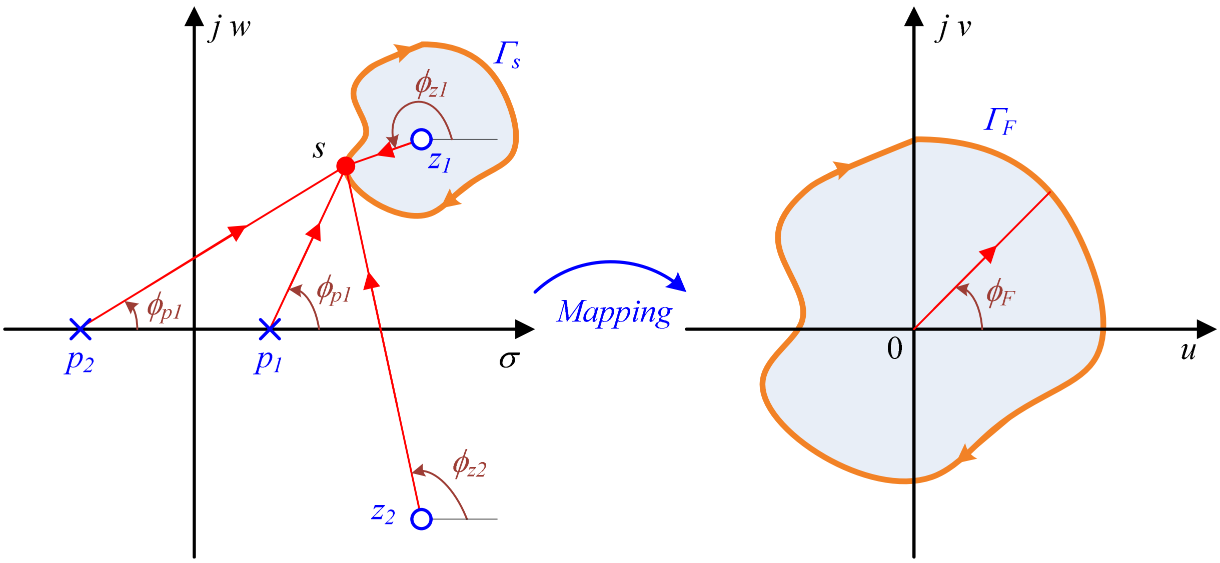
※ **Cauchy's thm** : argument principal

If a contour Γ_s enclose Z zeros and P poles of $F(s)$,

the mapping contour Γ_F enclose the origin of the $F(s)$-plane $N = Z - P$ times.

Example) $F(s) = \dfrac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)}$

$= |F(s)| \measuredangle \left(\phi_{z_1} + \phi_{z_2} - \phi_{p_1} - \phi_{p_2}\right)$



The net angle change for unenclosed poles and zeros as s traverses along Γ_s is 0°.

" for enclosed " is 360°.

$\Rightarrow \Phi_F = \Phi_Z - \Phi_P$

where Φ_F : net angle changes of $F(s)$

Φ_Z : net angle changes of enclosed zeros

Φ_P : net angle changes of enclosed poles

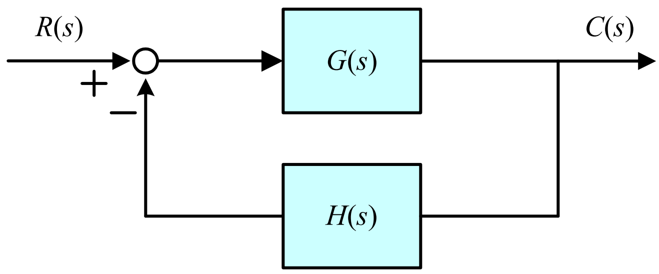
$\Rightarrow 2\pi \cdot N = 2\pi \cdot Z - 2\pi \cdot P$

Hence, $N = Z - P$

Example) Above figure

$Z = 1,\ P = 0 \quad \Rightarrow \ N = 1$

9.3 Nyquist Criterion



$$T(s) = \frac{G(s)}{1 + GH(s)}$$

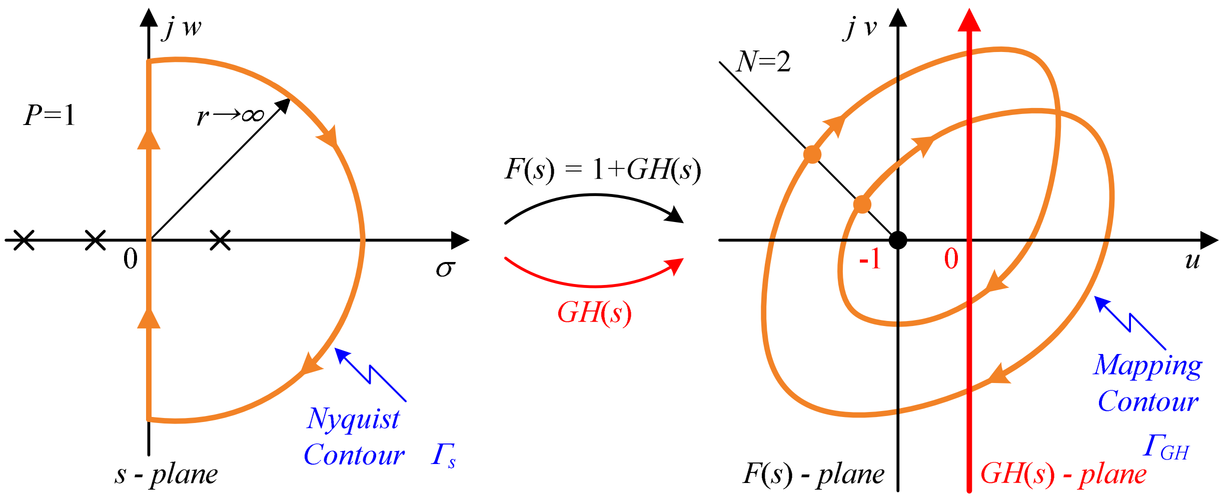
$$F(s) \equiv 1 + GH(s)$$

where $F(s)$의 zeros = roots of char. eq.

$F(s)$의 poles = $GH(s)$의 poles

※ Note : $F(s)$의 zeros $\neq$ $GH(s)$의 zeros

<Nyquist Contour> Fig. 9.8



# of clockwise encirclements of the origin in the $F(s)$ plane

= # of clockwise encirclements of the $(-1,0)$ point in the $GH(s)$ plane

Example) Above case

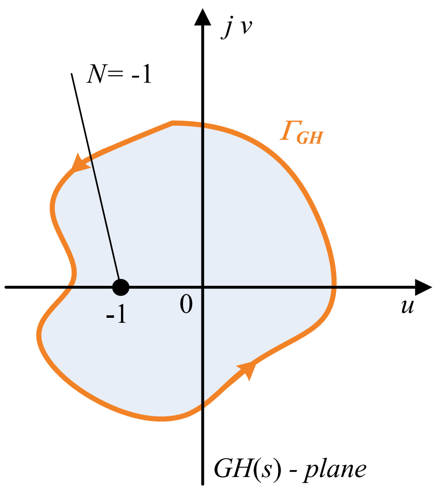
$N = Z - P$

$\Rightarrow\ Z = N + P = 2 + 1 = 3$

# of zeros of $F(s)$ in the rhp.

(불안정한 근의 개수)

Hence, "Unstable"



Example) Right figure

$N = -1$

$\Rightarrow\ Z = N + P = -1 + 1 = 0$ Hence, "Stable"

※ **Nyquist stability criterion**

If $Z = N + P = 0$, then the system is stable.

Otherwise, unstable.

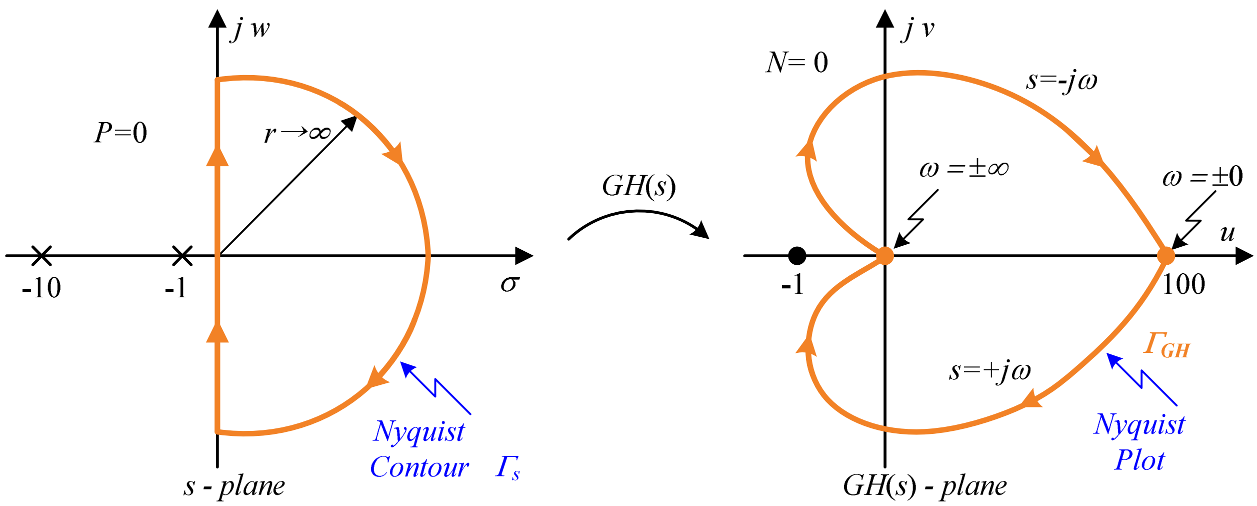
Example 9.1) System with 2 real poles

$$GH(s) = \frac{1{,}000}{(s+1)(s+10)} \qquad \text{cf) char. eq. } 1 + GH(s) = 0$$

sol)

$$|GH(s)| = \frac{1{,}000}{|s+1|\,|s+10|}$$

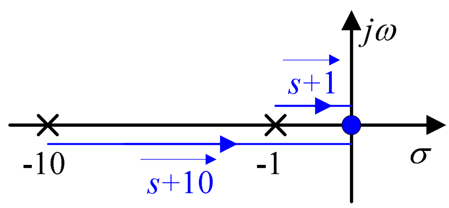
$$\measuredangle GH(s) = -\measuredangle(s+1) - \measuredangle(s+10) = -(\theta_1 + \theta_2)$$



1) Origin $(s = 0)$

$$|GH| = \frac{1{,}000}{1 \cdot 10} = 100$$

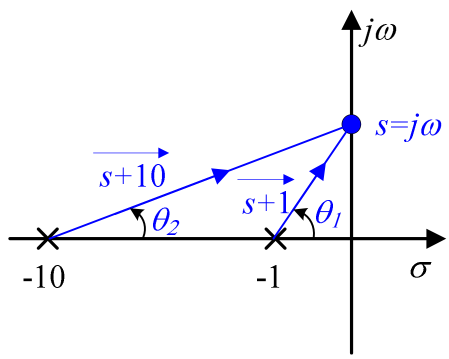
$$\measuredangle GH(s) = (0^\circ + 0^\circ) = 0^\circ$$



2) $+jw$ axis $(s = jw,\ w = 0^+ \to \infty)$

$$|GH| = 100 \to 0$$

$$\measuredangle GH(s) = -0^\circ \to -180^\circ$$



3) $-jw$ axis $(s = -jw,\ w = 0^- \leftarrow \infty)$

$$|GH| = 100 \leftarrow 0$$

$$\measuredangle GH(s) = +0^\circ \leftarrow +180^\circ$$

※ $s = jw$와 대칭

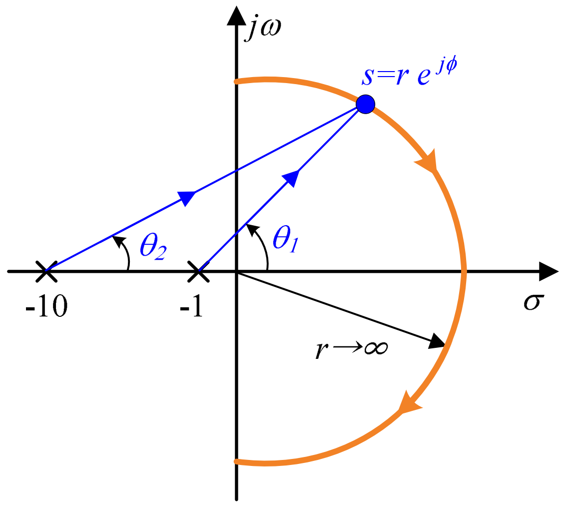
4) infinite semi-circle

$(s = r\,e^{j\phi},\quad r \to \infty,\quad \phi = +90^\circ \to -90^\circ)$

$$|GH| = \frac{1{,}000}{\infty \cdot \infty} = 0$$

$$\measuredangle\, GH(s) = -180^\circ \to +180^\circ$$

※ 제자리 360° 회전



Nyquist plot does not enclose the (-1,0) point.

$\Rightarrow N = 0$

$\Rightarrow Z = N + P = 0 + 0 = 0$

Hence, the system is "stable".

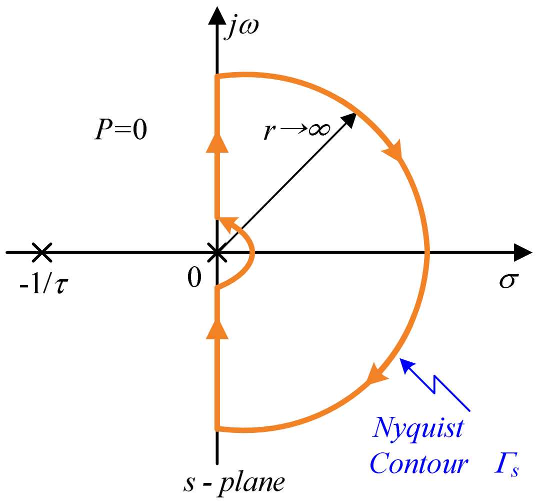
Example 9.2) System with a pole at the origin

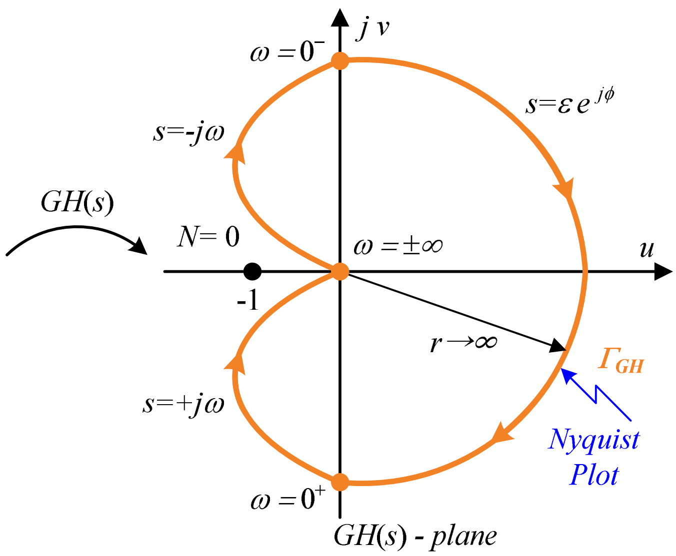
$$GH(s) = \frac{K}{s(\tau s + 1)} \quad \text{where } K > 0$$

sol)

$$|GH(s)| = \frac{K}{|s||\tau s + 1|}$$

$$\measuredangle\, GH(s) = -\measuredangle(s) - \measuredangle(\tau s + 1) = -(\theta_1 + \theta_2)$$



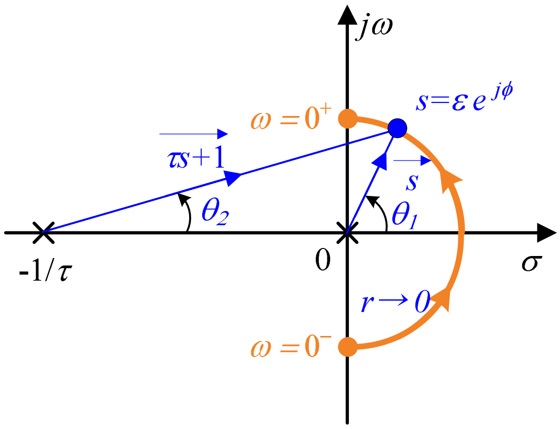


1) small semi-circle at the origin

$(s = \varepsilon\, e^{j\phi},\quad \varepsilon \to 0,\quad \phi = -90^\circ \to +90^\circ)$

$$|GH| = \frac{K}{0 \cdot 1/\tau} = \infty$$

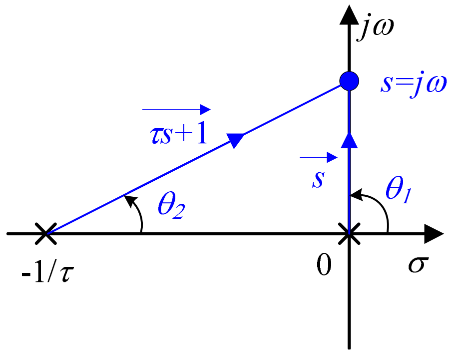
$$\measuredangle\, GH(s) = +90^\circ \to -90^\circ$$



2) $+jw$ axis $(s = jw,\ w = 0^+ \to \infty)$

$$|GH| = \infty \to 0$$

$$\measuredangle\, GH(s) = -90^\circ \to -180^\circ$$



3) $-jw$ axis $(s = -jw,\ w = 0^- \leftarrow \infty)$

※ $s = jw$와 대칭

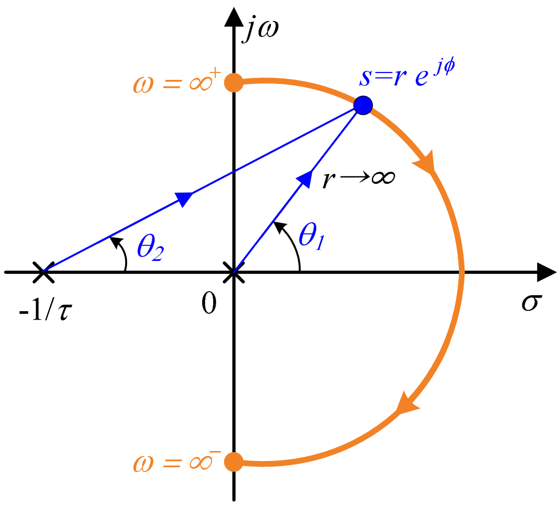
4) infinite semi-circle

$(s = r\, e^{j\phi},\quad r \to \infty,\quad \phi = +90^\circ \to -90^\circ)$

$$|GH| = \frac{K}{\infty \cdot \infty} = 0$$

$$\measuredangle\, GH(s) = -180^\circ \to +180^\circ$$

※ 제자리 360° 회전



Nyquist plot does not enclose the (-1,0) point.

$\Rightarrow\ N = 0$

$\Rightarrow\ Z = N + P = 0 + 0 = 0$

Hence, the system is “stable”.

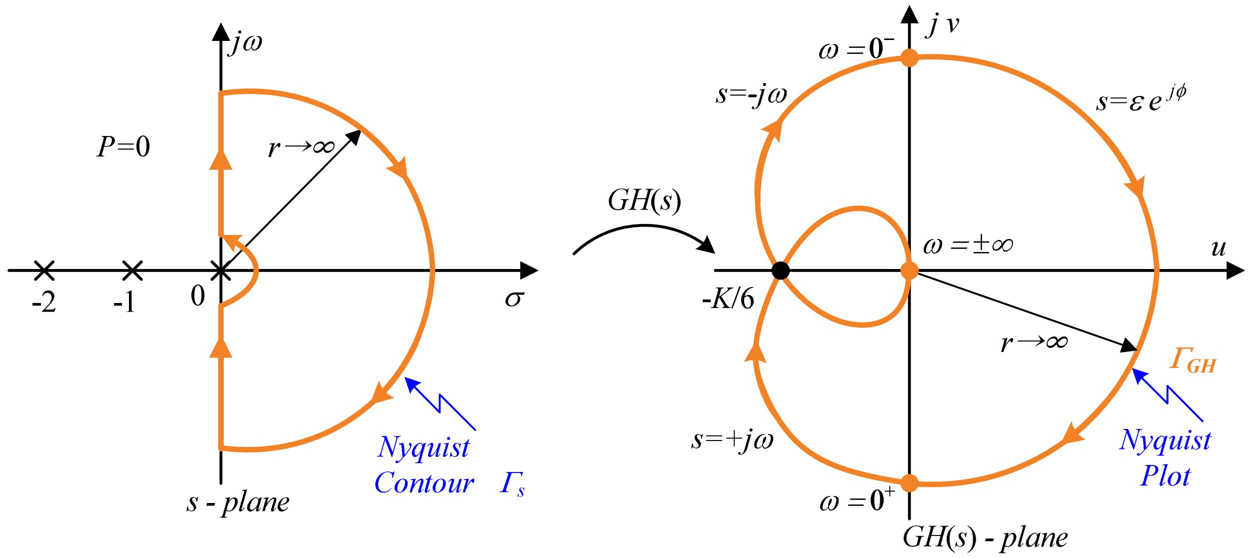
Example 9.3) System with 3 poles

$$GH(s) = \frac{K}{s(s+1)(s+2)}$$

sol)

$$|GH(s)| = \frac{K}{|s||s+1||s+2|}$$

$$\measuredangle GH(s) = -\measuredangle(s) - \measuredangle(s+1) - \measuredangle(s+2) = -(\theta_1 + \theta_2 + \theta_3)$$

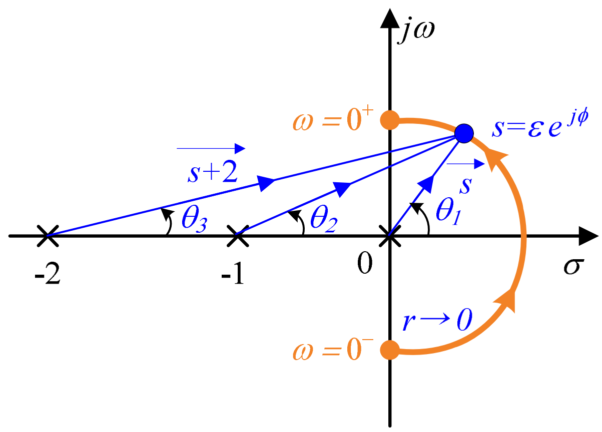


1) small semi-circle at the origin

$(s = \varepsilon e^{j\phi},\quad \varepsilon \to 0,\quad \phi = -90^\circ \to +90^\circ)$

$$|GH| = \frac{K}{0 \cdot 1 \cdot 2} = \infty$$

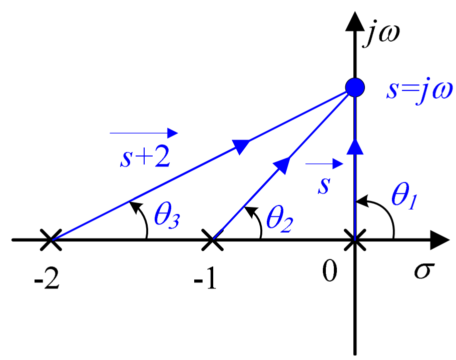
$$\measuredangle GH(s) = +90^\circ \to -90^\circ$$



2) $+jw$ axis $(s = jw,\ w = 0^+ \to \infty)$

$$|GH| = \infty \to 0$$

$$\measuredangle GH(s) = -90^\circ \to -270^\circ$$



3) $-jw$ axis $(s = -jw,\ w = 0^- \leftarrow \infty)$

※ $s = jw$와 대칭

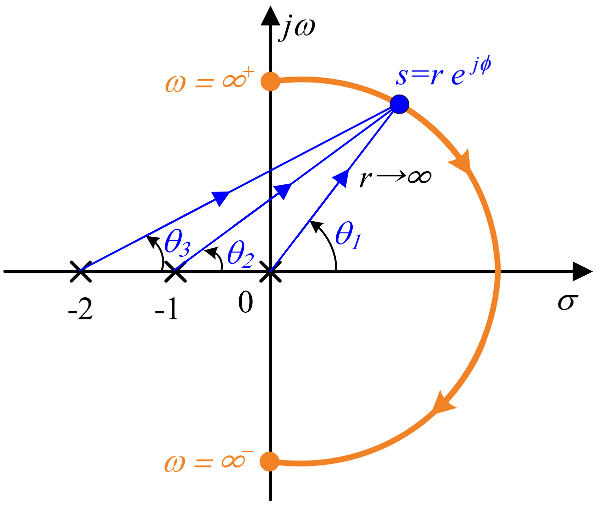
4) infinite semi-circle

$(s = r\,e^{j\phi},\quad r \to \infty,\quad \phi = +90^\circ \to -90^\circ)$

$$|GH| = \frac{K}{\infty \cdot \infty \cdot \infty} = 0$$

$$\measuredangle GH(s) = -270^\circ \;\to\; +270^\circ$$

※ 제자리 540° 회전



※ Intersection with real axis

⇒ Imaginary part of $GH(s)|_{s=jw} = 0$

$$GH(jw) = \frac{K}{jw(1+jw)(2+jw)} = \frac{jK(1-jw)(2-jw)}{-w(1+w^2)(4+w^2)}$$

$$= \frac{3Kw}{-w(1+w^2)(4+w^2)} + j\frac{K(2-w^2)}{-w(1+w^2)(4+w^2)}$$

⇒ $Im(GH) = 0$ at $w^2 = 2$

⇒ $GH(jw)\Big|_{w^2=2} = \dfrac{3K}{-3 \cdot 6} = -\dfrac{K}{6}$

1) If $K < 6$, then $-\dfrac{K}{6} > -1$

⇒ $N = 0$ $\quad Z = N + P = 0$ $\quad$ “stable“

2) If $K > 6$, then $-\dfrac{K}{6} < -1$

⇒ $N = 2$ $\quad Z = N + P = 2$ $\quad$ “unstable“

※ If $K = 6$ ⇒ “marginally stable“

char. eq.

$$s^3 + 3s^2 + 2s + K = 0$$

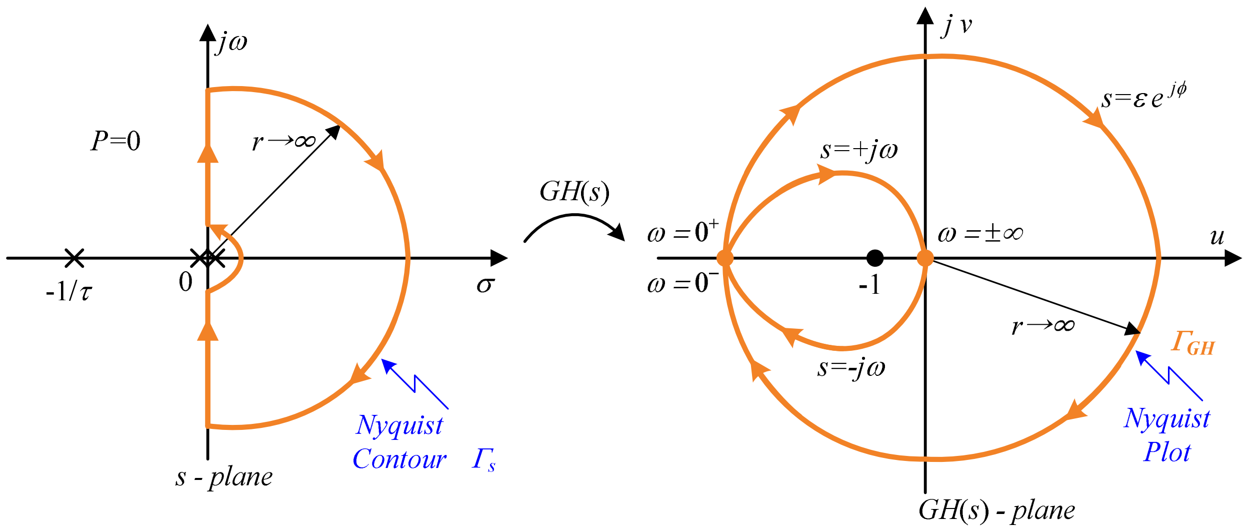
$$s_{1,2,3} = -3,\ \pm j\sqrt{2}$$

Example 9.4) System with 2 poles at the origin

$$GH(s)=\frac{K}{s^2(\tau s+1)}$$

sol)

$$|GH(s)|=\frac{K}{|s|^2|\tau s+1|} \qquad \measuredangle GH(s)=-2\measuredangle(s)-\measuredangle(\tau s+1)=-(2\theta_1+\theta_2)$$

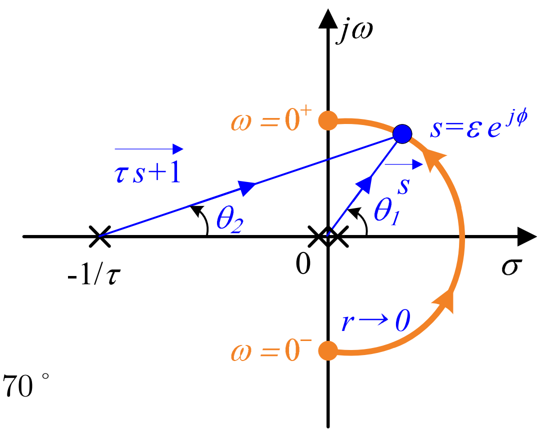


1) small semi-circle at the origin

$(s=\varepsilon\, e^{j\phi},\quad \varepsilon\to 0,\quad \phi=-90^\circ\to+90^\circ)$

$$|GH| = \frac{K}{0^2\cdot 1/\tau}=\infty$$

$$\measuredangle\, GH(s)=+180^\circ\ \to\ -180^\circ$$



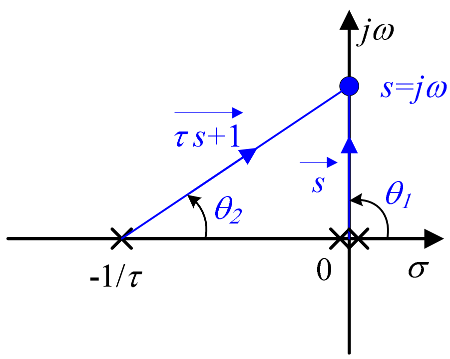
2) $+jw$ axis $(s=jw,\ w=0^+\to\infty)$

$$|GH|=\infty\to 0 \qquad \measuredangle\, GH(s)=-180^\circ\ \to\ -270^\circ$$

3) $-jw$ axis ※ $s=jw$와 대칭

4) infinite semi-circle

$$|GH|=\frac{K}{\infty^2\cdot\infty}=0 \quad \measuredangle\, GH(s)=-270^\circ\ \to\ +270^\circ$$



$N=2 \qquad \Rightarrow \quad Z=N+P=2$

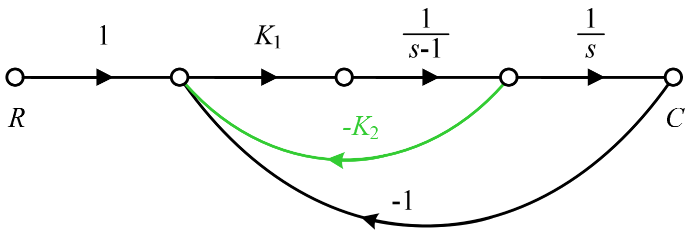
Hence, “unstable“

Example 9.5) System with a pole in rhp

$$\Delta = 1 - \left(\frac{-K_1K_2}{s-1} + \frac{-K_1}{s(s-1)} \right)$$

$$= 1 + \frac{K_1(1+K_2 s)}{s(s-1)}$$

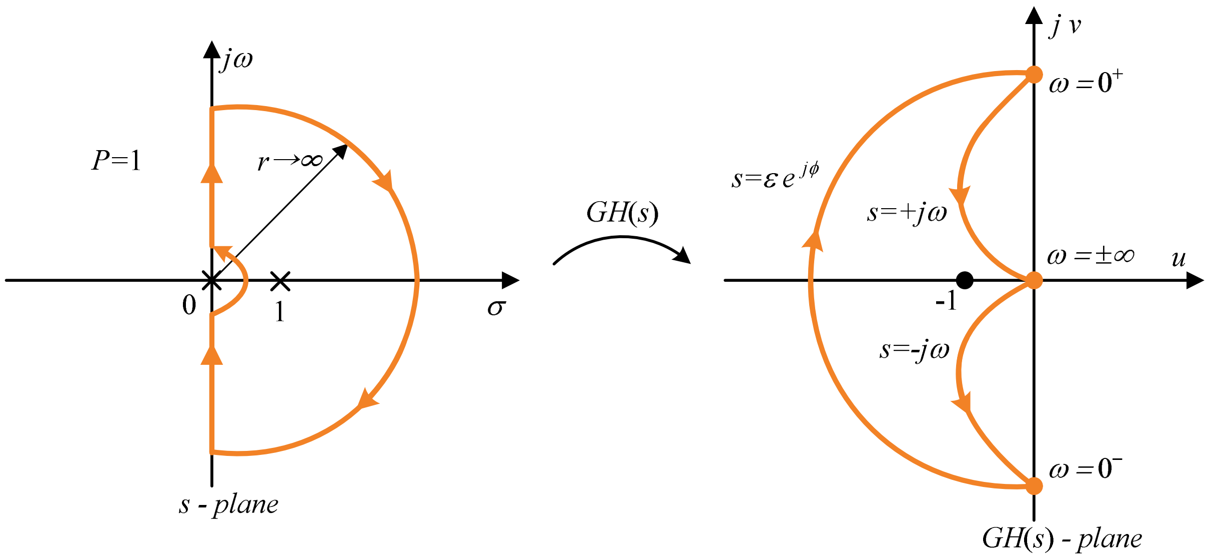
$$\Rightarrow \; GH(s) = \frac{K_1(1+K_2 s)}{s(s-1)}$$



※ **When** $K_2 = 0$

$$GH_1(s) = \frac{K_1}{s(s-1)}$$

$$|GH(s)| = \frac{K_1}{|s||s-1|} \qquad \measuredangle GH_1(s) = -\measuredangle(s) - \measuredangle(s-1) = -(\theta_1 + \theta_2)$$

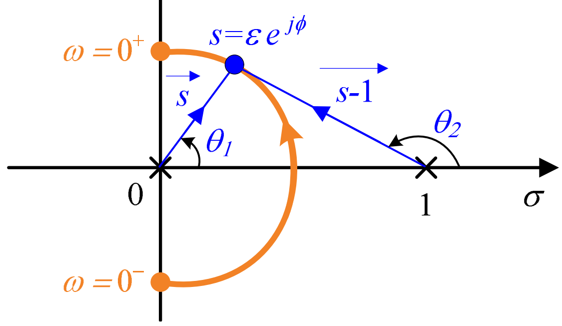


1) small semi-circle at the origin

$(s = \varepsilon e^{j\phi}, \quad \varepsilon \rightarrow 0, \quad \phi = -90^\circ \rightarrow +90^\circ)$

$$|GH| = \frac{K_1}{0 \cdot 1} = \infty$$

$$\measuredangle GH(s) = -90^\circ \rightarrow -270^\circ$$



2) $+jw$ axis $(s = jw, \; w = 0^+ \rightarrow \infty)$

$$|GH| = \infty \rightarrow 0$$

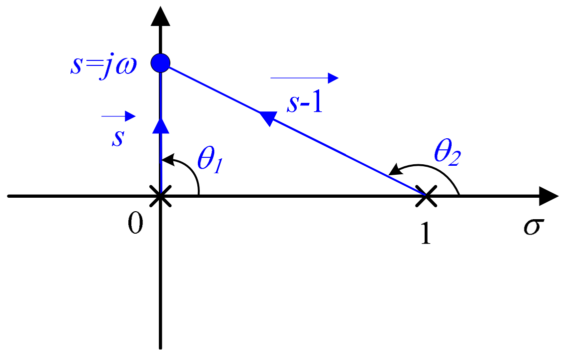
$$\measuredangle GH(s) = -270^\circ \rightarrow -180^\circ$$

3) $-jw$ axis ※ $s=jw$와 대칭

4) infinite semi-circle

$$|GH| = \frac{K_1}{\infty \cdot \infty} = 0$$

$$\measuredangle GH(s) = -180^\circ \rightarrow +180^\circ$$



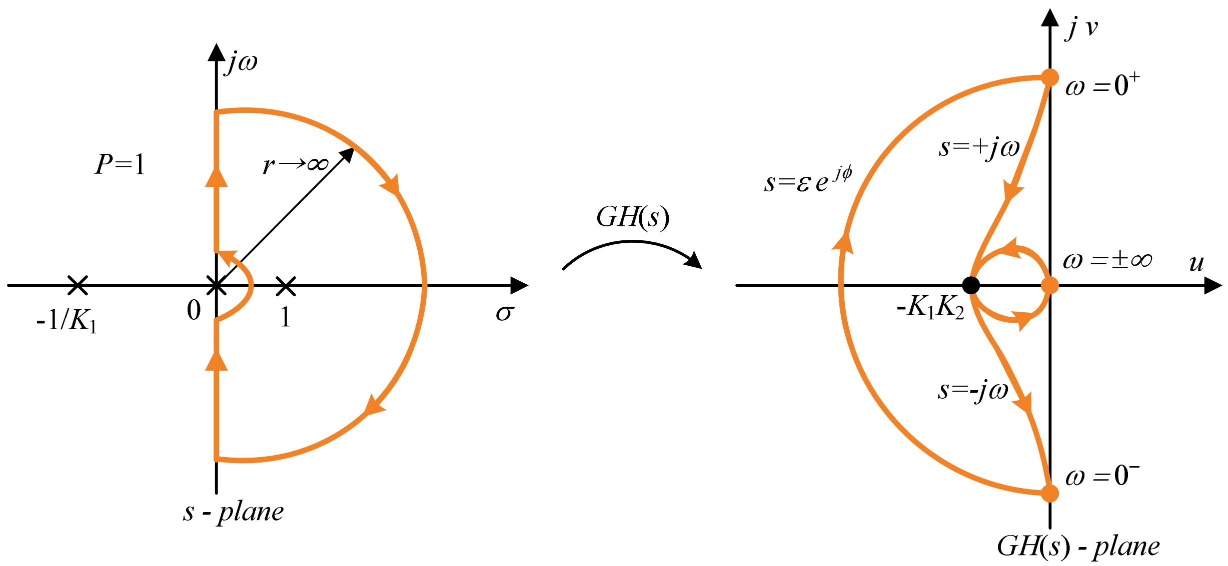
$N=1 \qquad \Rightarrow \quad Z=N+P=2$

Hence, “unstable“

※ When $K_2 \neq 0$

$$GH(s) = \frac{K_1(1+K_2 s)}{s(s-1)}$$

$$|GH_2(s)| = \frac{K_1|1+K_2 s|}{|s||s-1|} \qquad \measuredangle GH_1(s) = \measuredangle(1+K_2 s) - \measuredangle(s) - \measuredangle(s-1) = \theta_1 - (\theta_2 + \theta_3)$$

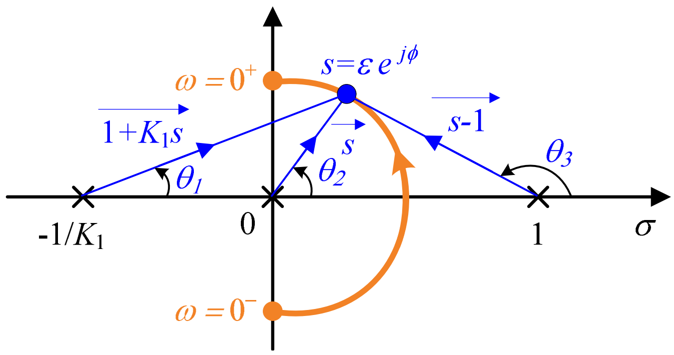


1) small semi-circle at the origin

$(s = \varepsilon e^{j\phi}, \quad \varepsilon \rightarrow 0, \quad \phi = -90^\circ \rightarrow +90^\circ)$

$$|GH| = \infty$$

$$\measuredangle GH(s) = -90^\circ \rightarrow -270^\circ$$

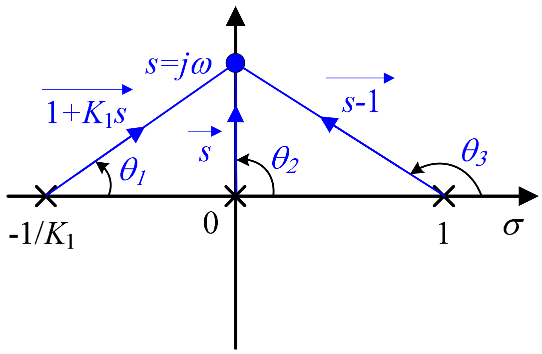


2) $+jw$ axis $(s=jw, \ w=0^+ \rightarrow \infty)$

$$|GH| = \infty \rightarrow 0$$

$$\measuredangle GH(s) = -270^\circ \rightarrow -180^\circ$$

3) $-jw$ axis ※ $s=jw$와 대칭



4) infinite semi-circle

$$|GH| = \frac{\infty}{\infty \cdot \infty} \rightarrow \frac{0}{1} = 0$$

$$\measuredangle GH(s) = -90^\circ \rightarrow +90^\circ$$

※ Intersection with real axis

$$GH(jw) = \frac{K_1(1+jwK_2)}{jw(jw-1)} = \frac{jK_1(1+jwK_2)(jw+1)}{-w(w^2+1)}$$

$$= \frac{K_1 w(1+K_2)}{-w(w^2+1)} + j\frac{K_1(1-K_2w^2)}{-w(w^2+1)}$$

$$\Rightarrow Im(GH) = 0 \text{ at } 1-K_2w^2 = 0 \quad \Rightarrow w^2 = \frac{1}{K_2}$$

$$\Rightarrow GH(jw)\Big|_{w^2=1/K_2} = \frac{-K_1(1+K_2)}{1+\dfrac{1}{K_2}} = -K_1K_2$$

1) If $K_1K_2 < 1$, then $\Rightarrow N=1$ $\quad Z=N+P=2$ $\quad$ “unstable“

2) If $K_1K_2 > 1$, then $\Rightarrow N=-1$ $\quad Z=N+P=0$ $\quad$ “stable“

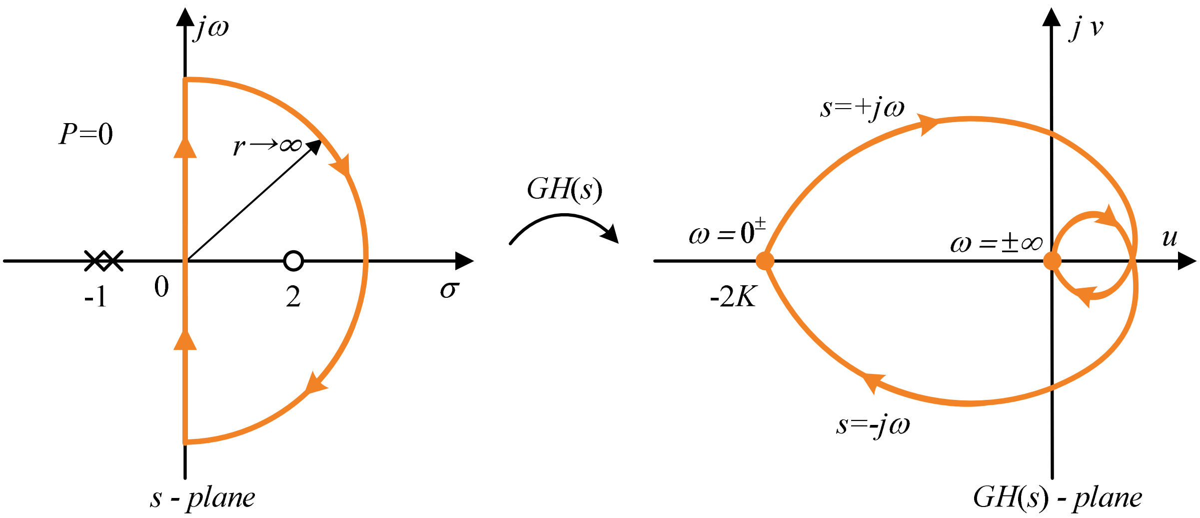
※ If $K_1K_2 > 1$, then “marginally stable“

Example 9.6) System with a zero in rhp

$$GH(s) = \frac{K(s-2)}{(s+1)^2}$$

sol)

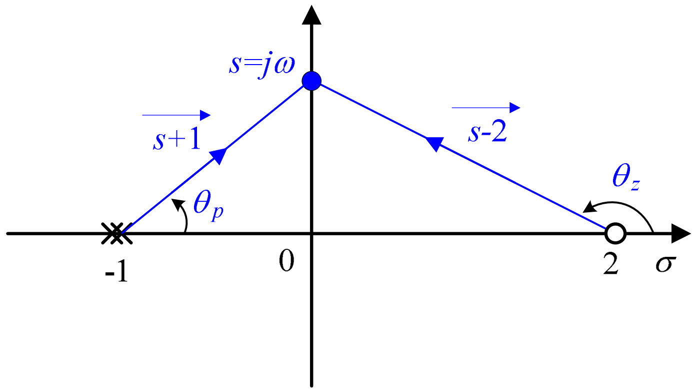
$$|GH(s)| = \frac{K|s-2|}{|s+1|^2} \qquad \measuredangle GH_1(s) = \measuredangle(s-2) - 2\measuredangle(s+1) = \theta_z - 2\theta_p$$



1) $+jw$ axis $(s = jw,\ w = 0^+ \rightarrow \infty)$

$|GH| = 2K \rightarrow 0$

$\measuredangle GH(s) = \underset{w=0^+}{180^\circ} \rightarrow 0^\circ \rightarrow \underset{w=+\infty}{-90^\circ}$



2) $-jw$ axis ※ $s = jw$와 대칭

3) infinite semi-circle

$$|GH| = \frac{\infty}{\infty \cdot \infty} \rightarrow \frac{0}{1} = 0$$

$$\measuredangle GH(s) = -90^\circ \rightarrow +90^\circ$$

1) If $K < 1/2$, then $\Rightarrow N = 0 \qquad Z = N + P = 0$ "stable"

2) If $K > 1/2$, then $\Rightarrow N = 1 \qquad Z = N + P = 1$ "unstable"

※ If $K = 1/2$, then "marginally stable"

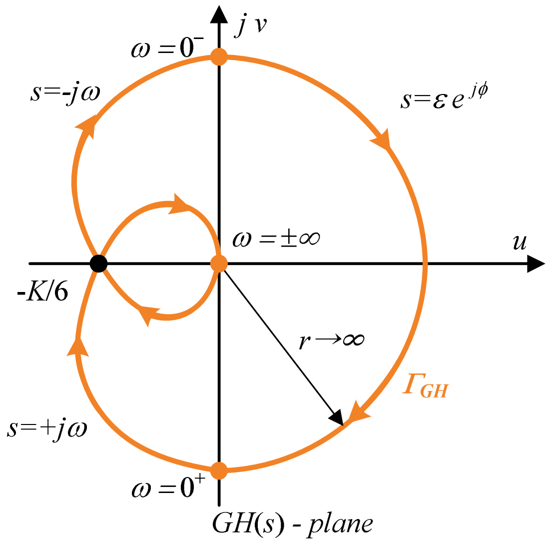
9.4 Relative stability and Nyquist criterion

(-1,0) point on the polar plot of $GH(jw)$

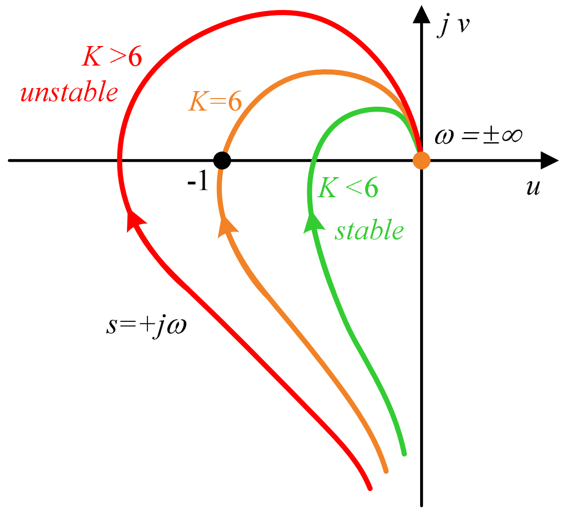
$\Rightarrow$ $|GH(jw)| = 1$ and $\measuredangle(GH) = -180^\circ$ on the Bode plot

Example) Return to Example 9.3)

$$GH(s) = \frac{K}{s(s+1)(s+2)}$$



<Nyquist plot>



<Polar plot>

※ Relative stability measure

: the margin between (-1, 0) point and $-K/6$

1) gain margin $\equiv \dfrac{1}{d}$

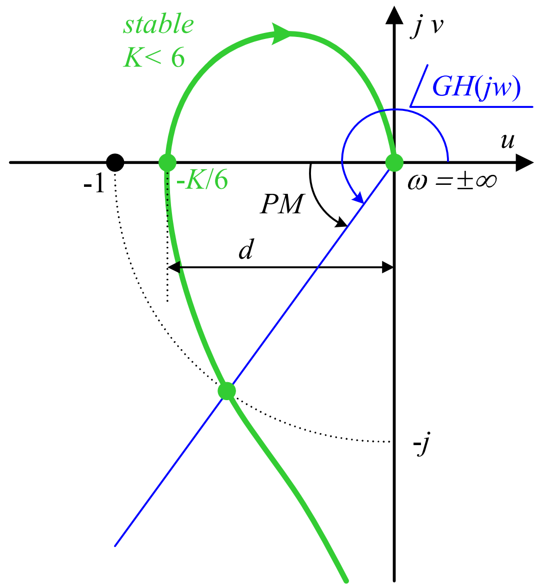
$d \equiv |GH(jw)|$ when $\measuredangle(GH) = -180^\circ$

$\Rightarrow$ logarithmic measure

$$GM = 20\log\left(\frac{1}{d}\right) = -20\log(d)$$

(임계안정까지 K의 증가에 대한 여유)

2) phase margin $\equiv \left| \measuredangle(GH)_{|GH|=1} - 180^\circ \right|$



Example) $K = 0^+ \rightarrow 6$

$GM = -20\log(K/6) = \infty\,(dB) \rightarrow 0\,(dB)$

$PM = 90^\circ \rightarrow 0^\circ$

Example) $K=3,\ GH(s)=\dfrac{3}{s(s+1)(s+2)} \quad \Rightarrow\ GM=?\ \ PM=?$

1) GM

$d=0.5 \qquad \Rightarrow\ GM=-20\log(2^{-1})=20\log(2)\cong 6\,dB$

2) PM when $|GH(jw)|=1$

$$|jw|\,|1+jw|\,|2+jw|=3$$

$$\Rightarrow\ w\sqrt{1+w^2}\,\sqrt{4+w^2}=3$$

$$\Rightarrow\ w^2(1+w^2)(4+w^2)=9$$

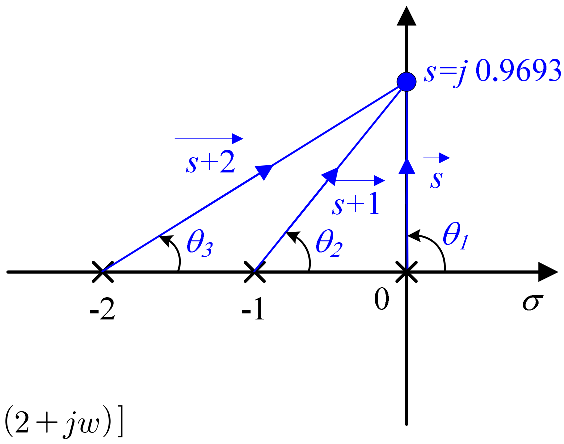
$$\Rightarrow\ w^6+5w^4+4w^2-9=0$$

$$\Rightarrow\ w_{1\sim6}=\pm0.9693,\ \pm0.2504\pm j1.7414$$

Hence, $w_{1\sim6}=+0.9693$

$$\begin{aligned}\measuredangle(GH)|_{w=0.9693}&=-[\measuredangle(jw)+\measuredangle(1+jw)+\measuredangle(2+jw)]\\&=-[90^\circ+44.1069^\circ+25.8571^\circ]\\&=-159.964^\circ\end{aligned}$$

Hence, $PM=180^\circ-159.964^\circ=20.036^\circ$



※ Bode plot of $GH(jw)$

1) GM

$\measuredangle(GH)=-180^\circ$

$\Rightarrow\ 20\log|GH|=-6dB$

Hence,

$GM=-20\log(d)=6dB$

2) PM

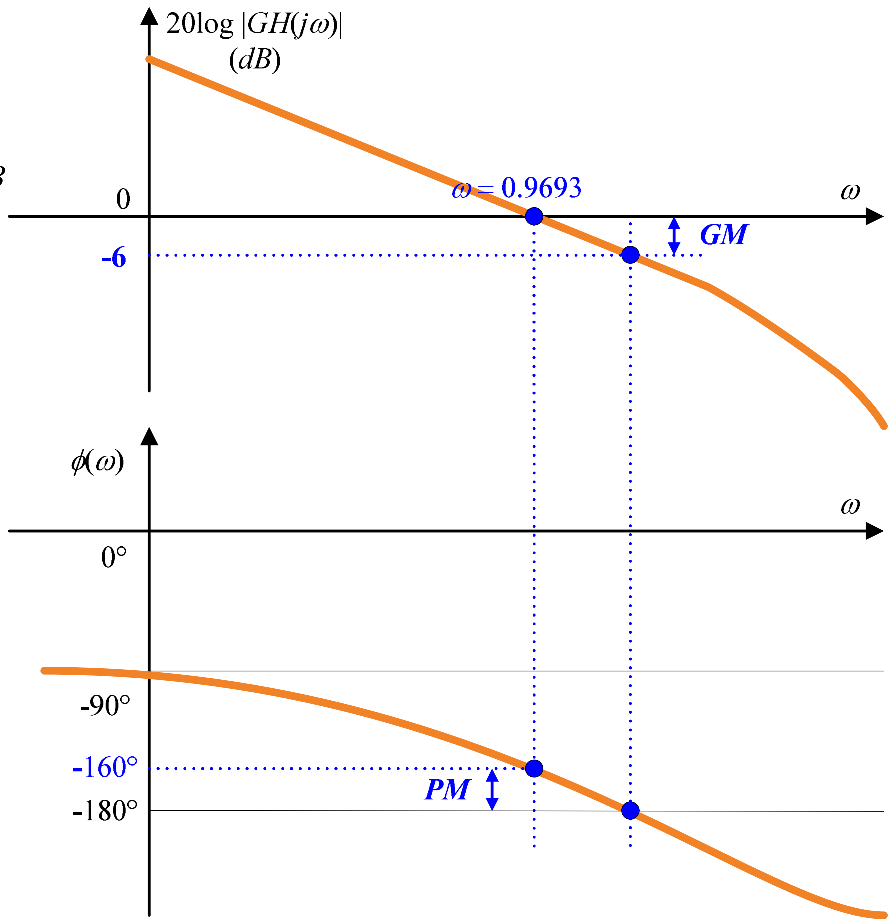
$20\log|GH|=0\,dB$

(that is, $|GH|=1$)

$\Rightarrow\ \measuredangle(GH)\cong-160^\circ$

Hence,

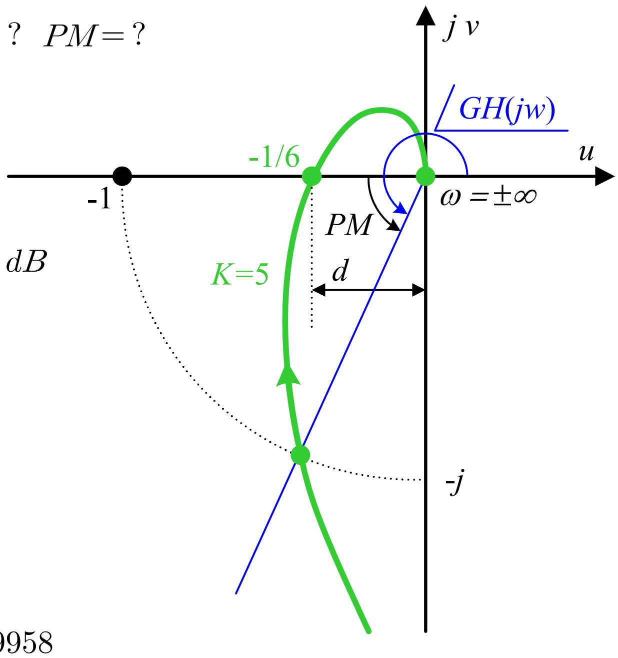
$$\begin{aligned}PM&=180^\circ-160^\circ\\&=20^\circ\end{aligned}$$



Example) $GH(s) = \dfrac{5}{s(s+1)(s+5)} \quad \Rightarrow \quad GM = ? \quad PM = ?$

sol)

Polar plot of $GH(s)$



1) $GM = -20\log(1/6) = 20\log(6) = 15.56\ dB$

2) PM

$|GH(jw)| = 1$

$\Rightarrow \quad |jw|\ |1+jw|\ |5+jw| = 5$

$\Rightarrow \quad w\sqrt{1+w^2}\ \sqrt{25+w^2} = 5$

$\Rightarrow \quad w^2(1+w^2)(25+w^2) = 25$

$\Rightarrow \quad w^6 + 26w^4 + 25w^2 - 25 = 0$

$\Rightarrow \quad w_{1\sim 6} = \pm 0.7793, \quad \pm j1.2842, \quad \pm j4.9958$

Hence, $w_{1\sim 6} = +0.7793$

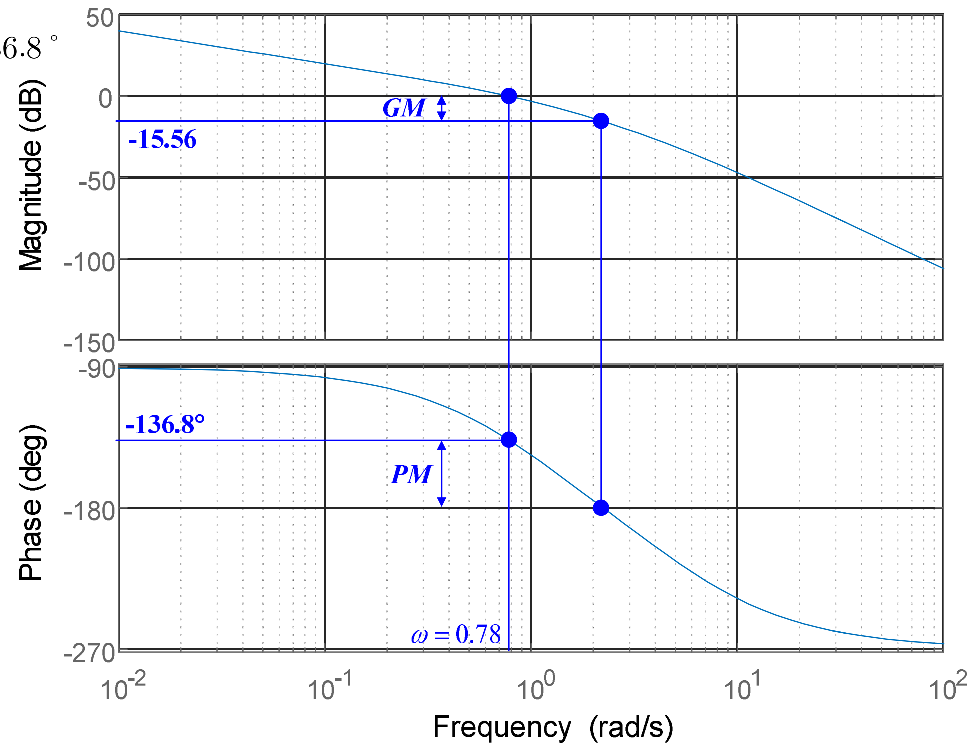
$$\begin{aligned} \measuredangle(GH)|_{w=0.9693} &= -[\measuredangle(jw) + \measuredangle(1+jw) + \measuredangle(5+jw)] \\ &= -[90^\circ + 37.93^\circ + 8.86^\circ] \\ &\cong -136.8^\circ \end{aligned}$$

Hence, $PM = 180^\circ - 136.8^\circ = 43.2^\circ$

※ Bode plot of $GH(jw)$

1) $GM = 15.56\ dB$

2) $PM = 180^\circ - 136.8^\circ$
$= 43.2^\circ$



※ Relation of PM and ζ for 2nd order system

$$GH(s)=\frac{w_n^2}{s(s+2\zeta w_n)} \quad \Rightarrow \quad T(s)=\frac{GH(s)}{1+GH(s)}=\frac{w_n^2}{s^2+2\zeta w_n s+w_n^2}$$

sol)

$$|GH(jw)|_{w=w_c}=\frac{w_n^2}{w_c\sqrt{w_c^2+4\zeta^2 w_n^2}}=1 \qquad \text{where } w_c \text{ is cutoff freq.}$$

$$\Rightarrow w_c\sqrt{w_c^2+4\zeta^2 w_n^2}=w_n^2 \qquad \Rightarrow w_c^2\left(w_c^2+4\zeta^2 w_n^2\right)=w_n^4$$

$$\Rightarrow w_c^4+4\zeta^2 w_n^2 w_c^2-w_n^4=0 \qquad \Rightarrow \left(\frac{w_c^2}{w_n^2}\right)^2+4\zeta^2\left(\frac{w_c^2}{w_n^2}\right)-1=0 \quad \leftarrow x^2+4\zeta^2 x-1=0$$

$$\Rightarrow \frac{w_c^2}{w_n^2}=-2\zeta^2\pm\sqrt{4\zeta^4+1}=-2\zeta^2+\sqrt{4\zeta^4+1}$$

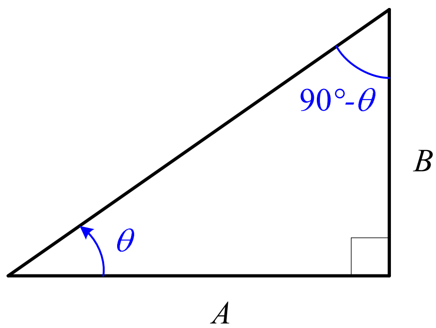
Phase Margin

$$\Phi_{pm}=180^\circ+\measuredangle(GH)\big|_{w=w_c}=180^\circ-90^\circ-\tan^{-1}\frac{w_c}{2\zeta w_n}$$

$$\text{where } \tan^{-1}\frac{w_c}{2\zeta w_n}=\tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}=\tan^{-1}\frac{B}{A}\equiv\theta$$

$$\Phi_{pm}=90^\circ-\tan^{-1}\frac{B}{A}=\tan^{-1}\frac{A}{B}$$

$$=\tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}$$



Approximation for $\zeta\le 0.7$

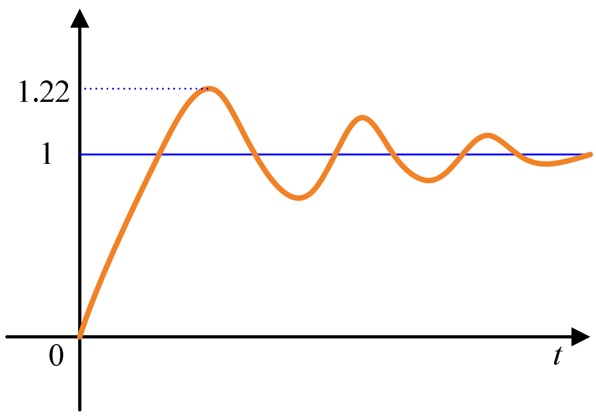
$$\zeta=0.01\,\Phi_{pm} \qquad \text{Eq. (9.58)}$$

Example) Previous example

$$GH(s)=\frac{5}{s(s+1)(s+5)}, \quad \Phi_{pm}=43^\circ$$

$$\Rightarrow \zeta=0.01\,\Phi_{pm}=0.43$$

$$\Rightarrow M_{pt}=1.22 \text{ from Fig. 5.8}$$



9.6 System Bandwidth

the speed of unit step response $(t_r,\ t_p)$ $\propto$ w_B

settling time T_s $\propto$ $1/w_B$

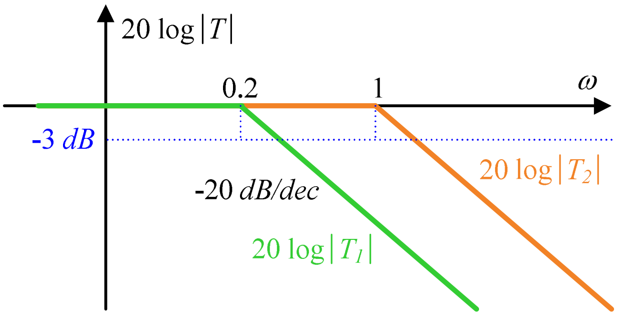
$$\Leftrightarrow \quad t_r = \frac{2.16\zeta + 0.6}{w_n}$$

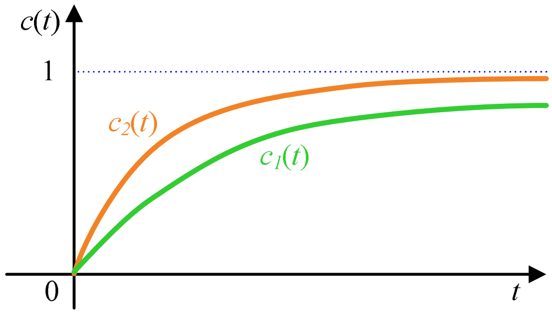
$$\frac{w_B}{w_n} = -1.19\zeta + 1.8508$$

$$T_s = \frac{4}{\zeta w_n}$$

Hence, "large Bandwidth"

Example) $T_1(s) = \dfrac{1}{1+5s}$, $\quad T_2(s) = \dfrac{1}{1+s}$





$w_{B1} = 1,\ w_{B2} = 0.2$

Example) $T_1(s) = \dfrac{100}{s^2 + 10s + 100}$, $\quad T_2(s) = \dfrac{900}{s^2 + 30s + 900}$

$\zeta = 0.5$

$w_{n1} = 10, \quad w_{n2} = 30 \qquad \Rightarrow \ w_{B1} \cong 15,\ w_{B2} \cong 40$