※ 다음 행렬의 행렬식(determinant)을 구하시오.

1. $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & 3 & 4 \\ 5 & 1 & 2 \end{pmatrix}$

2. $B=\begin{pmatrix} 7 & 2 & 3 \\ 1 & 0 & 3 \\ 0 & 4 & 2 \end{pmatrix}$

3. $C=\begin{pmatrix} 3 & 2 & 7 \\ 9 & 1 & 0 \\ 3 & -1 & 2 \end{pmatrix}$

4. $D=\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{pmatrix}$

5. $E=\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{pmatrix}$

6. $F=\begin{pmatrix} 6 & 7 & 2 \\ 1 & 4 & 3 \\ -1 & 1 & 4 \end{pmatrix}$

7. $G=\begin{pmatrix} 5 & 0 & 0 \\ 6 & 3 & 2 \\ 4 & 5 & 7 \end{pmatrix}$

8. $H=\begin{pmatrix} 9 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

9. $I=\begin{pmatrix} 2 & -1 & 7 \\ 0 & 8 & 4 \\ 3 & 6 & 4 \end{pmatrix}$

※ 다음 행렬의 소행렬식(minor determinant)을 구하시오.

1. $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & 3 & 4 \\ 5 & 1 & 2 \end{pmatrix}$

2. $B=\begin{pmatrix} 7 & 2 & 3 \\ 1 & 0 & 3 \\ 0 & 4 & 2 \end{pmatrix}$

3. $C=\begin{pmatrix} 3 & 2 & 7 \\ 9 & 1 & 0 \\ 3 & -1 & 2 \end{pmatrix}$

4. $D=\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{pmatrix}$

5. $E=\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{pmatrix}$

6. $F=\begin{pmatrix} 6 & 7 & 2 \\ 1 & 4 & 3 \\ -1 & 1 & 4 \end{pmatrix}$

7. $G=\begin{pmatrix} 5 & 0 & 0 \\ 6 & 3 & 2 \\ 4 & 5 & 7 \end{pmatrix}$

8. $H=\begin{pmatrix} 9 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

9. $I=\begin{pmatrix} 2 & -1 & 7 \\ 0 & 8 & 4 \\ 3 & 6 & 4 \end{pmatrix}$

※ 다음 행렬의 수반행렬을 구하시오.

1. $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & 3 & 4 \\ 5 & 1 & 2 \end{pmatrix}$

2. $B=\begin{pmatrix} 7 & 2 & 3 \\ 1 & 0 & 3 \\ 0 & 4 & 2 \end{pmatrix}$

3. $C=\begin{pmatrix} 3 & 2 & 7 \\ 9 & 1 & 0 \\ 3 & -1 & 2 \end{pmatrix}$

4. $D=\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{pmatrix}$

5. $E=\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{pmatrix}$

6. $F=\begin{pmatrix} 6 & 7 & 2 \\ 1 & 4 & 3 \\ -1 & 1 & 4 \end{pmatrix}$

7. $G=\begin{pmatrix} 5 & 0 & 0 \\ 6 & 3 & 2 \\ 4 & 5 & 7 \end{pmatrix}$

8. $H=\begin{pmatrix} 9 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

9. $I=\begin{pmatrix} 2 & -1 & 7 \\ 0 & 8 & 4 \\ 3 & 6 & 4 \end{pmatrix}$

※ 다음 행렬의 역행렬(inverse matrix)을 구하시오.

1. $A=\begin{pmatrix} 1 & 2 & 1 \\ -1 & 3 & 4 \\ 5 & 1 & 2 \end{pmatrix}$

2. $B=\begin{pmatrix} 7 & 2 & 3 \\ 1 & 0 & 3 \\ 0 & 4 & 2 \end{pmatrix}$

3. $C=\begin{pmatrix} 3 & 2 & 7 \\ 9 & 1 & 0 \\ 3 & -1 & 2 \end{pmatrix}$

4. $D=\begin{pmatrix} 3 & 2 & -1 \\ 2 & -1 & 2 \\ 1 & -3 & -4 \end{pmatrix}$

5. $E=\begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{pmatrix}$

6. $F=\begin{pmatrix} 6 & 7 & 2 \\ 1 & 4 & 3 \\ -1 & 1 & 4 \end{pmatrix}$

7. $G=\begin{pmatrix} 5 & 0 & 0 \\ 6 & 3 & 2 \\ 4 & 5 & 7 \end{pmatrix}$

8. $H=\begin{pmatrix} 9 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 8 \end{pmatrix}$

9. $I=\begin{pmatrix} 2 & -1 & 7 \\ 0 & 8 & 4 \\ 3 & 6 & 4 \end{pmatrix}$

소행렬식 M_{ij}

$$M_{ij} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix}$$

$$= \begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ \begin{vmatrix} a_{12} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ \begin{vmatrix} a_{11} & a_{12} \\ a_{22} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (a_{22}a_{33} - a_{23}a_{32}) & (a_{21}a_{33} - a_{23}a_{31}) & (a_{21}a_{32} - a_{22}a_{31}) \\ (a_{12}a_{33} - a_{13}a_{31}) & (a_{11}a_{33} - a_{13}a_{31}) & (a_{11}a_{32} - a_{12}a_{31}) \\ (a_{11}a_{23} - a_{12}a_{22}) & (a_{11}a_{23} - a_{13}a_{21}) & (a_{11}a_{22} - a_{12}a_{21}) \end{pmatrix}$$

여인수 A_{ij}

$$A_{ij} = (-1)^{i+j} M_{ij}$$

$$A_{ij} = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}$$

$$= \begin{pmatrix} (+1)\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & (-1)\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & (+1)\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ (-1)\begin{vmatrix} a_{12} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (-1)\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ (+1)\begin{vmatrix} a_{11} & a_{12} \\ a_{22} & a_{23} \end{vmatrix} & (-1)\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (a_{22}a_{33} - a_{23}a_{32}) & -(a_{21}a_{33} - a_{23}a_{31}) & (a_{21}a_{32} - a_{22}a_{31}) \\ -(a_{12}a_{33} - a_{13}a_{31}) & (a_{11}a_{33} - a_{13}a_{31}) & -(a_{11}a_{32} - a_{12}a_{31}) \\ (a_{11}a_{23} - a_{12}a_{22}) & -(a_{11}a_{23} - a_{13}a_{21}) & (a_{11}a_{22} - a_{12}a_{21}) \end{pmatrix}$$

수반행렬 $adj\,(A_{ij})$

$$
\begin{aligned}
adj(A_{ij}) &= \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T \\
&= \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} \\
&= \begin{pmatrix} (+1)\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & (-1)\begin{vmatrix} a_{12} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{12} \\ a_{22} & a_{23} \end{vmatrix} \\ (-1)\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (-1)\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ (+1)\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & (-1)\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix} \\
&= \begin{pmatrix} (a_{22}a_{33}-a_{23}a_{32}) & -(a_{12}a_{33}-a_{13}a_{31}) & (a_{11}a_{23}-a_{12}a_{22}) \\ -(a_{21}a_{33}-a_{23}a_{31}) & (a_{11}a_{33}-a_{13}a_{31}) & -(a_{11}a_{23}-a_{13}a_{21}) \\ (a_{21}a_{32}-a_{22}a_{31}) & -(a_{11}a_{32}-a_{12}a_{31}) & (a_{11}a_{22}-a_{12}a_{21}) \end{pmatrix}
\end{aligned}
$$

역행렬 A^{-1}

$$A^{-1} = \frac{1}{|A|} adj(A)$$

$$= \frac{1}{|A|} \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix}^T$$

$$= \frac{1}{|A|} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

$$= \frac{1}{|A|} \begin{pmatrix} (+1)\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & (-1)\begin{vmatrix} a_{12} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{12} \\ a_{22} & a_{23} \end{vmatrix} \\ (-1)\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & (-1)\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ (+1)\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & (-1)\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & (+1)\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$= \frac{1}{|A|} \begin{pmatrix} (a_{22}a_{33} - a_{23}a_{32}) & -(a_{12}a_{33} - a_{13}a_{31}) & (a_{11}a_{23} - a_{12}a_{22}) \\ -(a_{21}a_{33} - a_{23}a_{31}) & (a_{11}a_{33} - a_{13}a_{31}) & -(a_{11}a_{23} - a_{13}a_{21}) \\ (a_{21}a_{32} - a_{22}a_{31}) & -(a_{11}a_{32} - a_{12}a_{31}) & (a_{11}a_{22} - a_{12}a_{21}) \end{pmatrix}$$