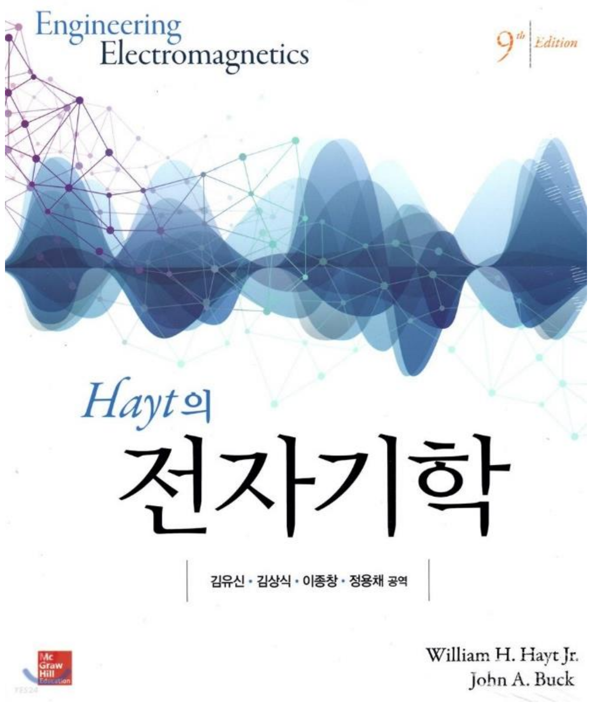
전기자기학 I

(강의자료 #2)



교과목명 : 전기자기학 I

담당교수 : 이 수 형

E-mail : soohyong@uu.ac.kr

교재명 : Hayt의 전자기학

Ch. 2. 쿨롱의 법칙과 전계세기

CH. 2 : 쿨롱의 법칙과 전계세기

1) 쿨롱의 실험 법칙

2) 전계의 세기

3) 연속적인 체적전하분포에 의한 전계

4) 선전하에 의한 전계

5) 면전하에 의한 전계

6) 전계의 유선(streamline)과 스케치

2.1 쿨롱의 실험 법칙 - 정의

- 쿨롱의 법칙 (Coulomb's Law)

진공 또는 자유공간 내에서 두 개의 하전 입자 사이의 힘은 각각의 입자가 가지고 있는 전하량에 비례하고 그들 사이의 거리의 제곱에 반비례한다.

$$F = k\frac{Q_1 Q_2}{R^2}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}$$

뉴턴(newton, N)

Q_1, Q_2 : 물체들이 갖는 전하량, 쿨롱 (coulomb, C)

R : 물체 사이의 거리, 미터(meter, m)

k : 비례상수

$k = \frac{1}{4\pi\varepsilon_0}$ 자유 공간의 유전율(permittivity), (farad/meter, F/m)

$\varepsilon_0 = 8.854 \times 10^{-12} \approx \frac{1}{36\pi} \times 10^{-9}$ [F/m]

- 전하의 단위; C(쿨롱, Coulomb)
  - 전자의 전하량 $e = -1.602 \times 10^{-19}\ \mathrm{C}$
  - $1\ \mathrm{C} = -6 \times 10^{18} e\mathrm{C} = (6 \times 10^{18})(1.602 \times 10^{-19})\ \mathrm{C}$
  - $F = \frac{1}{4\pi\varepsilon_0}\frac{1C \times 1C}{(1m)^2} = 9 \times 10^9 N \approx 100$만톤

2.1 쿨롱의 실험 법칙 - 정의

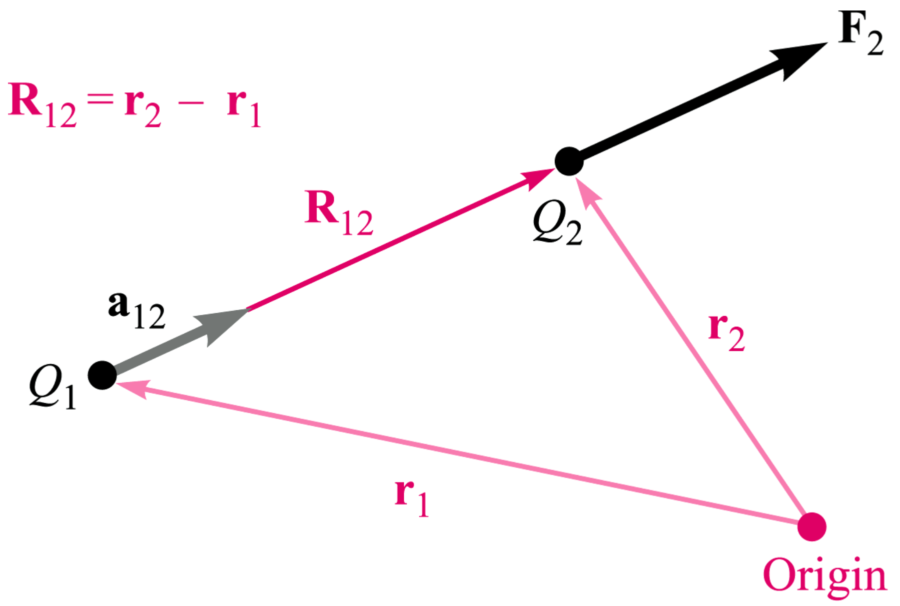
- 전기력의 벡터 표시
  - Q_1, Q_2의 부호가 같으면 Q_2에 작용하는 힘 $\mathbf{F}_2$의 방향은 벡터 $\mathbf{R}_{12}$의 방향과 같다.

❖ $\mathbf{F}_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R_{12}^2}\mathbf{a}_{12}$

✓ 여기서, $\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{r}_2-\mathbf{r}_1}{|\mathbf{r}_2-\mathbf{r}_1|}$

❖ $\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R_{21}^2}\mathbf{a}_{21}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{R_{12}^2}(-\mathbf{a}_{12}) = -\mathbf{F}_2$$



2.1 쿨롱의 실험 법칙 - 예제

- (예제 2.1) 점 $M(1,2,3)$에 $Q_1 = 3 \times 10^{-4}$[C], 점 $N(2,0,5)$에 $Q_2 = -10^{-4}$[C]인 전하가 자유공간에 놓여져 있다. Q_1에 의해서 Q_2에 가해지는 힘을 구하라.

➢ $\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = (2-1)\mathbf{a}_x + (0-2)\mathbf{a}_y + (5-3)\mathbf{a}_z = \mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z$

➢ $\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3}$

$$\therefore \mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\mathbf{a}_{12}$$

$$= \frac{3\times10^{-4}(-10^{-4})}{4\pi(1/36\pi)10^{-9}\times9}\left(\frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3}\right)$$

$$= -30\left(\frac{\mathbf{a}_x - 2\mathbf{a}_y + 2\mathbf{a}_z}{3}\right) = -10\mathbf{a}_x + 20\mathbf{a}_y - 20\mathbf{a}_z \text{ [N]}$$

➢ $\mathbf{F}_1 = -\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}\mathbf{a}_{21} = -\frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\mathbf{a}_{12}$

2.1 쿨롱의 실험 법칙 - 예제

- (ex 1) $Q_1 = 2mC$ at $P_1(3,-2,-4)$, $Q_2 = -5\mu C$ at $P_2(1,-4,2)$
  - (a) $\mathbf{R}_{12}$ (b) $|\mathbf{F}_1|$
  - (a) $\mathbf{R}_{12} = \mathbf{r}_2 - \mathbf{r}_1 = -2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z$

$$\mathbf{a}_{12} = \frac{\mathbf{R}_{12}}{R_{12}} = \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{(-2)^2+(-2)^2+6^2}} = \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{44}}$$

$$\therefore \mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\mathbf{a}_{12}\ [\mathrm{N}]$$

$$= \frac{2\times10^{-3}\times(-5)\times10^{-6}}{4\pi(1/36\pi)10^{-9}\times44} \cdot \frac{-2\mathbf{a}_x - 2\mathbf{a}_y + 6\mathbf{a}_z}{\sqrt{44}}\ [\mathrm{N}]$$

$$= 2.045\frac{2\mathbf{a}_x + 2\mathbf{a}_y - 6\mathbf{a}_z}{\sqrt{44}}\ [\mathrm{N}]$$

$$= 0.616\mathbf{a}_x + 0.616\mathbf{a}_y - 1.848\mathbf{a}_z\ [\mathrm{N}]$$

  - (b) $|\mathbf{F}_1| = |\mathbf{F}_2| = \sqrt{0.616^2 + 0.616^2 + (-1.848)^2} = 2.04\ [\mathrm{N}]$

2.1 쿨롱의 실험 법칙 – 예제

- (응용예제 2.1) $Q_A = 20\mu C$ at $A(-6,4,7)$, $Q_B = 50\mu C$ at $B(5,8,-2)$
  - (a) $\mathbf{R}_{AB}$ (b) R_{AB} (c) $\mathbf{F}_A$

2.2 전계의 세기

- 전하 Q_1의 위치를 고정하면 제2의 전하 Q_2는 어떠한 위치에서도 힘을 받는다.

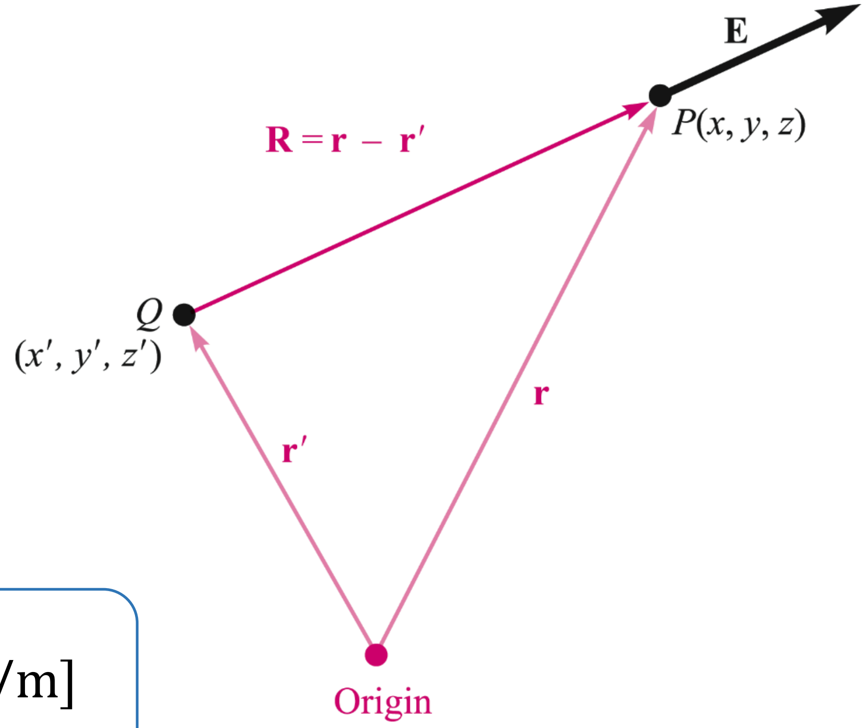
  → Q_1에 의한 힘의 장(계)(force field)이 존재

  → Q_2를 Q_t로 대체하면 $\mathbf{F}_t = \frac{Q_1 Q_t}{4\pi\varepsilon_0 R_{1t}^2}\mathbf{a}_{1t}$

- 시험전하의 단위전하당 작용하는 힘

  - $\frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2}\mathbf{a}_{1t}$ → 전계세기(electric field intensity) $\mathbf{E}$
  - 전계의 세기 (the electric field intensity: 전하 → 전기장)

$$\therefore \mathbf{E} = \frac{\mathbf{F}_t}{Q_t} = \frac{Q_1}{4\pi\varepsilon_0 R_{1t}^2}\mathbf{a}_{1t} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \quad [\mathrm{V/m}]$$



2.2 전계의 세기

- (ex 2) Q_1 at $O(r = 0, \theta = 0, \phi = 0) \rightarrow r = r_1$인 곳에서의 $\mathbf{E}$?

$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R = \frac{Q_1}{4\pi\varepsilon_0 r_1^2}\mathbf{a}_r$$

- (ex 3) Q at $O(x = 0, y = 0, z = 0) \rightarrow P(x = x_1, y = y_1, z = z_1)$에서의 $\mathbf{E}$?

$$\mathbf{R} = x_1\mathbf{a}_x + y_1\mathbf{a}_y + z_1\mathbf{a}_z$$

$$\mathbf{a}_R = \frac{x_1\mathbf{a}_x + y_1\mathbf{a}_y + z_1\mathbf{a}_z}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$$

$$\therefore \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R = \frac{Q}{4\pi\varepsilon_0(x_1^2 + y_1^2 + z_1^2)} \cdot \frac{x_1\mathbf{a}_x + y_1\mathbf{a}_y + z_1\mathbf{a}_z}{\sqrt{x_1^2 + y_1^2 + z_1^2}}$$

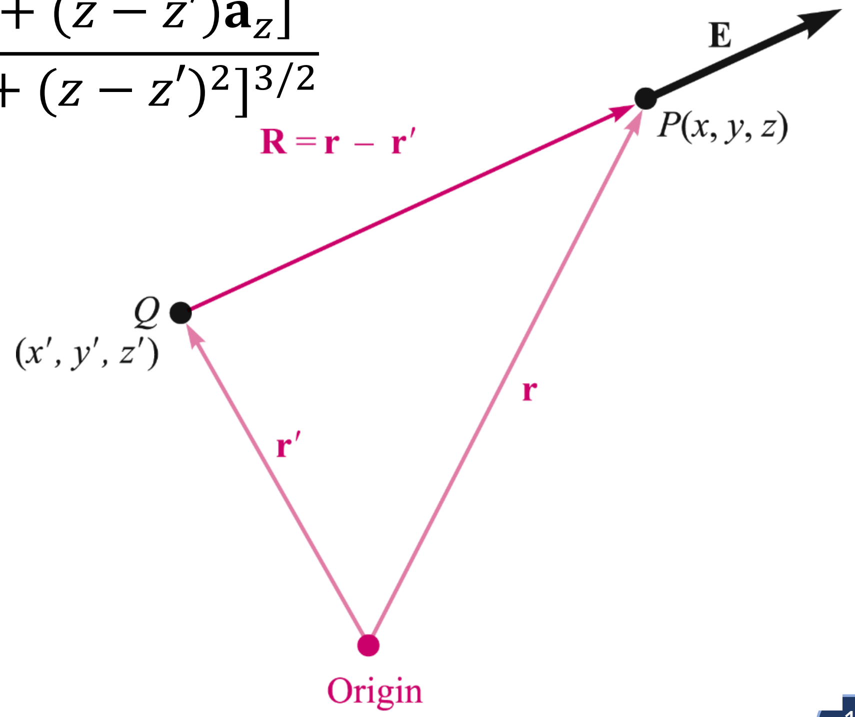
2.2 전계의 세기

- (ex 4) Q at $O(x',y',z') \rightarrow P(x,y,z)$에서의 $\mathbf{E}$?

$$\mathbf{R} = \mathbf{r}_P - \mathbf{r}_O = (x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z$$

$$\mathbf{a}_R = \frac{\mathbf{R}}{R} = \frac{(x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

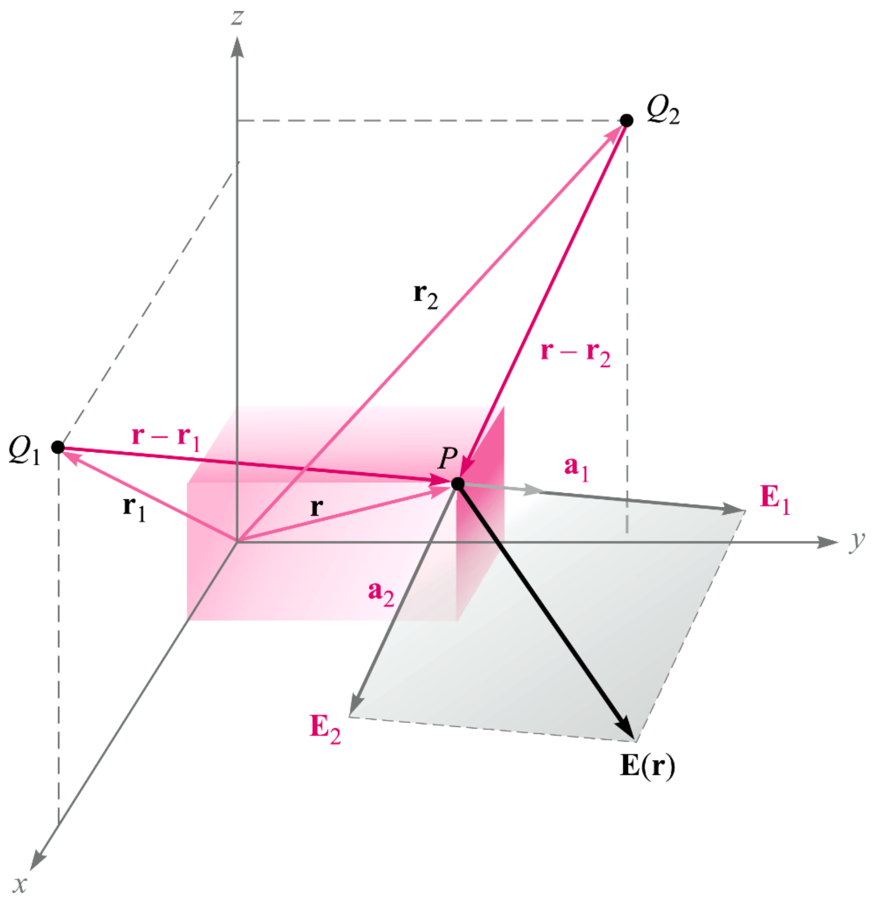
$$\therefore \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R = \frac{Q}{4\pi\varepsilon_0 R^3}\mathbf{R} = \frac{Q[(x-x')\mathbf{a}_x + (y-y')\mathbf{a}_y + (z-z')\mathbf{a}_z]}{4\pi\varepsilon_0[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}}$$



2.2 전계의 세기

- 중첩의 원리 (Superposition)
  - ➢ 두 점전하에 의한 전계의 세기는 각각 단독으로 있을 때 작용하는 힘을 더한 것과 같다.

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{Q_1}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2$$



2.2 전계의 세기

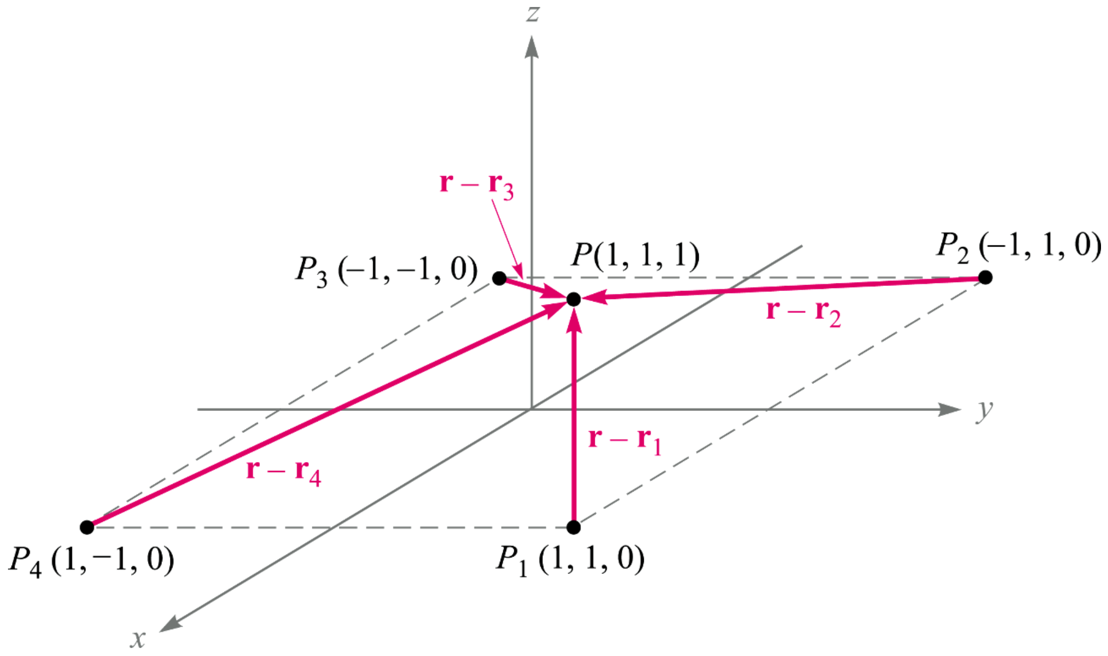
- Q_1 at r_1, Q_2 at r_2, $\cdots$, Q_n at r_n $\rightarrow r$인 곳에서의 $\mathbf{E}$?
  - $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \cdots + \mathbf{E}_n$

$$= \frac{Q_1}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}_1|^2}\mathbf{a}_1 + \frac{Q_2}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}_2|^2}\mathbf{a}_2 + \cdots + \frac{Q_n}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}_n|^2}\mathbf{a}_n$$

$$= \sum_{m=1}^{n} \frac{Q_m}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}_m|^2}\mathbf{a}_m$$

2.2 전계의 세기

- (예제 2.2) 3 nC at $P_1(1,1,0)$, 3 nC at $P_2(-1,1,0)$, 3 nC at $P_3(-1,-1,0)$, 3 nC at $P_4(1,-1,0)$, → $\mathbf{E}$ =? at $P(1,1,1)$



$$\mathbf{E} = \sum_{m=1}^{4} \mathbf{E}_m$$

$$= \sum_{m=1}^{4} \frac{Q_m}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_m|^2} \mathbf{a}_m$$

$$= \sum_{m=1}^{4} \frac{Q_m}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}_m|^2} \frac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|} = \sum_{m=1}^{4} \frac{Q_m}{4\pi\varepsilon_0} \frac{\mathbf{r} - \mathbf{r}_m}{|\mathbf{r} - \mathbf{r}_m|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0} \left[\frac{\mathbf{r} - \mathbf{r}_1}{|\mathbf{r} - \mathbf{r}_1|^3} + \frac{\mathbf{r} - \mathbf{r}_2}{|\mathbf{r} - \mathbf{r}_2|^3} + \frac{\mathbf{r} - \mathbf{r}_3}{|\mathbf{r} - \mathbf{r}_3|^3} + \frac{\mathbf{r} - \mathbf{r}_4}{|\mathbf{r} - \mathbf{r}_4|^3} \right]$$

2.2 전계의 세기

- $\mathbf{E} = \sum_{m=1}^{4} \mathbf{E}_m = \frac{Q}{4\pi\varepsilon_0}\left[\frac{\mathbf{r}-\mathbf{r}_1}{|\mathbf{r}-\mathbf{r}_1|^3} + \frac{\mathbf{r}-\mathbf{r}_2}{|\mathbf{r}-\mathbf{r}_2|^3} + \frac{\mathbf{r}-\mathbf{r}_3}{|\mathbf{r}-\mathbf{r}_3|^3} + \frac{\mathbf{r}-\mathbf{r}_4}{|\mathbf{r}-\mathbf{r}_4|^3}\right]$

where, $\mathbf{r}-\mathbf{r}_1 = \mathbf{a}_z$ $\quad |\mathbf{r}-\mathbf{r}_1| = 1$

$\mathbf{r}-\mathbf{r}_2 = 2\mathbf{a}_x + \mathbf{a}_z$ $\quad |\mathbf{r}-\mathbf{r}_2| = \sqrt{2^2+1^2} = \sqrt{5}$

$\mathbf{r}-\mathbf{r}_3 = 2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z$ $\quad |\mathbf{r}-\mathbf{r}_1| = \sqrt{2^2+2^2+1^2} = 3$

$\mathbf{r}-\mathbf{r}_4 = 2\mathbf{a}_y + \mathbf{a}_z$ $\quad |\mathbf{r}-\mathbf{r}_1| = \sqrt{2^2+1^2} = \sqrt{5}$

$$\therefore \mathbf{E} = \frac{3\times10^{-9}}{4\pi\times8.854\times10^{-12}}\left[\frac{\mathbf{a}_z}{1^3} + \frac{2\mathbf{a}_x+\mathbf{a}_z}{\sqrt{5}^3} + \frac{2\mathbf{a}_x+2\mathbf{a}_y+\mathbf{a}_z}{3^3} + \frac{2\mathbf{a}_y+\mathbf{a}_z}{\sqrt{5}^3}\right]$$

$$= 6.82\mathbf{a}_x + 6.82\mathbf{a}_y + 32.8\mathbf{a}_z\ [\mathrm{V/m}]$$

- (ex 7) $\mathbf{E}$ at $M(3,-4,2)$?
  - (a) $Q_1 = 2\mu C$ at $P_1(0,0,0)$

$$\mathbf{r}_M - \mathbf{r}_1 = 3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z$$

$$|\mathbf{r}_M - \mathbf{r}_1| = \sqrt{3^2 + (-4)^2 + 2^2} = \sqrt{29}$$

$$\therefore \mathbf{E}_{1M} = \frac{Q}{4\pi\varepsilon_0 R_{1M}^2}\mathbf{a}_R = \frac{Q_1}{4\pi\varepsilon_0|\mathbf{r}_M - \mathbf{r}_1|^2}\frac{\mathbf{r}_M - \mathbf{r}_1}{|\mathbf{r}_M - \mathbf{r}_1|}$$

$$= \frac{2\times10^{-6}}{4\pi\cdot8.854\times10^{-12}\cdot29}\frac{3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{29}}$$

$$= 620\frac{3\mathbf{a}_x - 4\mathbf{a}_y + 2\mathbf{a}_z}{\sqrt{29}}\ [\mathrm{V/m}]$$

$$= 345\mathbf{a}_x - 460\mathbf{a}_y + 230\mathbf{a}_z\ [\mathrm{V/m}]$$

2.2 전계의 세기

- (b) $Q_2 = 3\mu C$ at $P_2(-1,2,3)$

$$\mathbf{r}_M - \mathbf{r}_2 = 4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z$$

$$|\mathbf{r}_M - \mathbf{r}_2| = \sqrt{4^2 + (-6)^2 + (-1)^2} = \sqrt{53}$$

$$\therefore \mathbf{E}_{2M} = \frac{Q}{4\pi\varepsilon_0 R_{2M}^2}\mathbf{a}_R = \frac{Q_1}{4\pi\varepsilon_0|\mathbf{r}_M - \mathbf{r}_2|^2}\frac{\mathbf{r}_M - \mathbf{r}_2}{|\mathbf{r}_M - \mathbf{r}_2|}$$

$$= \frac{3\times 10^{-6}}{4\pi\cdot 8.854\times 10^{-12}\cdot 53}\frac{4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z}{\sqrt{53}}$$

$$= 509\frac{4\mathbf{a}_x - 6\mathbf{a}_y - \mathbf{a}_z}{\sqrt{53}}\ [\mathrm{V/m}]$$

$$= 280\mathbf{a}_x - 419\mathbf{a}_y - 69.9\mathbf{a}_z\ [\mathrm{V/m}]$$

2.2 전계의 세기

- (c) $Q_1 = 2\mu C$ at $P_1(0,0,0)$ and $Q_2 = 3\mu C$ at $P_2(-1,2,3)$

$$\mathbf{E}_M = \mathbf{E}_{1M} + \mathbf{E}_{2M}$$

$$= \left(345\mathbf{a}_x - 460\mathbf{a}_y + 230\mathbf{a}_z\right) + \left(280\mathbf{a}_x - 419\mathbf{a}_y - 69.9\mathbf{a}_z\right)$$

$$= 625\mathbf{a}_x - 879\mathbf{a}_y + 160.1\mathbf{a}_z \ [\mathrm{V/m}]$$

2.2 전계의 세기

- (응용예제 2.2) $Q_A = -0.3\mu C$ at $A(25, -30, 15)$ cm,
  $Q_B = 0.5\mu C$ at $B(-10, 8, 12)$ cm,
  - $\mathbf{E} = ?$ at (a) 원점 (b) $P(15, 20, 50)$ cm

- (응용예제 2.3)
  - (a) $\sum_{m=0}^{5} \frac{1+(-1)^m}{m^2+1}$ (b) $\sum_{m=1}^{4} \frac{(0.1)^m+1}{(4+m^2)^{1.5}}$

2.3 연속적인 체적전하분포에 의한 전계

- 전하가 점이 아닌 부피를 가지고 분포 → 체적전하밀도(ρ_v [C/m^3])로 표시

$$\rho_v = \frac{Q}{v} \rightarrow \rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} \rightarrow \rho_v = \lim_{\Delta v \to 0} \frac{\Delta Q}{\Delta v} = \frac{dQ}{dv}$$

$$\therefore Q = \int_{vol} dQ = \int_{vol} \rho_v \, dv \text{ [C]}$$

$$\rightarrow \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_r$$

$$= \frac{Q}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{\int_{vol} \rho_v(\mathbf{r}')dv'}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \int_{vol} \frac{\rho_v(\mathbf{r}')dv'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

2.3 연속적인 체적전하분포에 의한 전계

- (예제 2.3) 체적전하밀도가 $\rho_v = -5 \times 10^{-6} e^{-10^5 \rho z}$ $[\mathrm{C/m^3}]$이고, 길이 2 cm, 반지름 1 cm인 전자빔의 총전하량 Q?

$$Q = \int_{vol} dQ = \int_{vol} \rho_v \, dv$$

$$= \int_{vol} -5 \times 10^{-6} e^{-10^5 \rho z} \rho d\rho d\phi dz$$

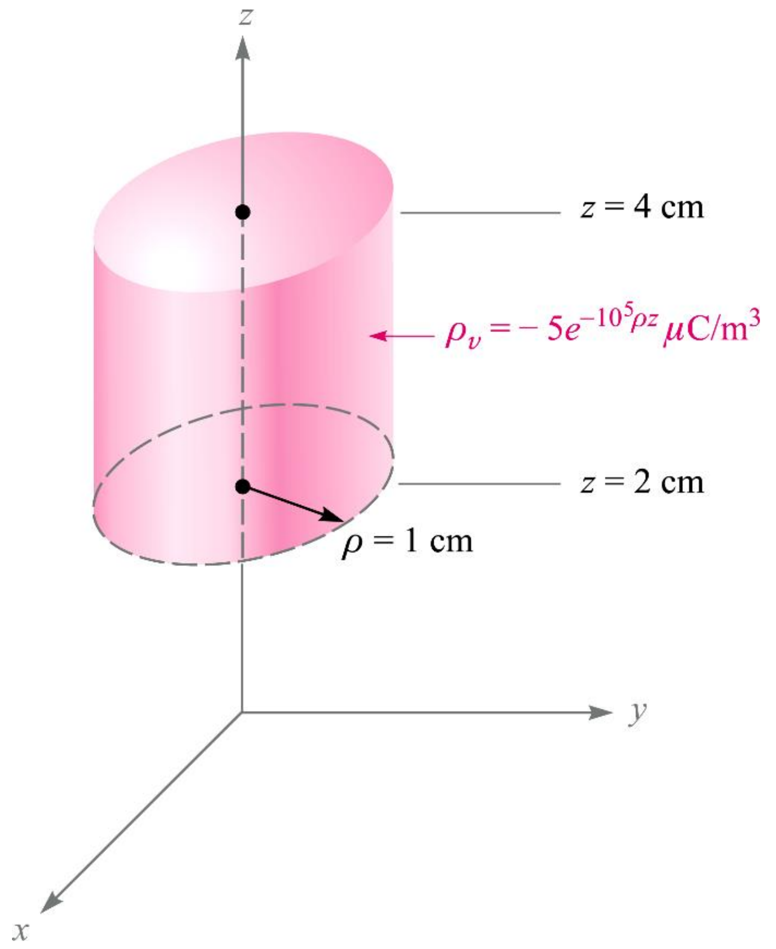
$$= \int_{z=0.02}^{z=0.04} \int_{\phi=0}^{\phi=2\pi} \int_{\rho=0}^{\rho=0.01} -5 \times 10^{-6} e^{-10^5 \rho z} \rho d\rho d\phi dz$$

$$= -5 \times 10^{-6} \int_{z=0.02}^{z=0.04} \int_{\rho=0}^{\rho=0.01} e^{-10^5 \rho z} \rho d\rho \left(\int_{\phi=0}^{\phi=2\pi} d\phi \right) dz$$

$$= -5 \times 10^{-6} \int_{z=0.02}^{z=0.04} \int_{\rho=0}^{\rho=0.01} e^{-10^5 \rho z} \cdot 2\pi \rho d\rho dz$$

$$= -10^{-5}\pi \int_{\rho=0}^{\rho=0.01} \rho \left(\int_{z=0.02}^{z=0.04} e^{-10^5 \rho z} \right) dz d\rho$$

$$= -10^{-5}\pi \int_{\rho=0}^{\rho=0.01} \rho \left[\frac{1}{-10^5 \rho} e^{-10^5 \rho z} \right]_{0.02}^{0.04} d\rho$$



2.3 연속적인 체적전하분포에 의한 전계

- (예제 2.3) 체적전하밀도가 $\rho_v = -5\times10^{-6}e^{-10^5\rho z}$ $[\mathrm{C/m^3}]$이고, 길이 2 cm, 반지름 1 cm인 전자빔의 총 전하량 Q?

$$Q=\int_{vol} dQ=\int_{vol}\rho_v\,dv=-10^{-5}\pi\int_{\rho=0}^{\rho=0.01}\rho\left[\frac{1}{-10^5\rho}e^{-10^5\rho z}\right]_{0.02}^{0.04}d\rho$$

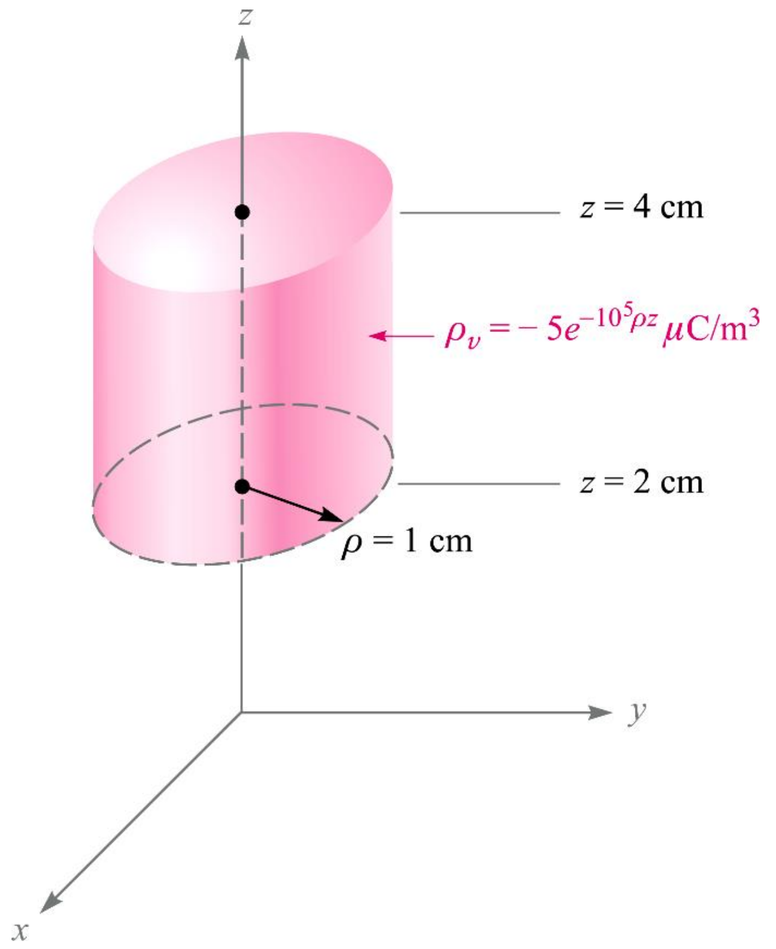
$$=10^{-10}\pi\int_{\rho=0}^{\rho=0.01}\left(e^{-4000\rho}-e^{-2000\rho}\right)d\rho$$

$$=10^{-10}\pi\left[\frac{e^{-4000\rho}}{-4000}-\frac{e^{-2000\rho}}{-2000}\right]_0^{0.01}$$

$$=10^{-10}\pi\left\{\left(\frac{e^{-40}}{-4000}-\frac{e^{-20}}{-2000}\right)-\left(\frac{1}{-4000}-\frac{1}{-2000}\right)\right\}$$

$$\approx10^{-10}\pi\left(\frac{1}{4000}-\frac{1}{2000}\right)=10^{-10}\pi\cdot\left(-\frac{1}{4000}\right)$$

$$=-\frac{\pi}{40}10^{-12}=-\frac{\pi}{40}pC=-0.0785pC$$



2.3 연속적인 체적전하분포에 의한 전계

- (응용예제 2.4) $Q = ?$
  - (a) $0.1 \le |x|, |y|, |z| \le 0.2$; $\rho_v = \frac{1}{x^3 y^3 z^3}$
  - (b) $0 \le \rho \le 0.1,\ 0 \le \phi \le \pi,\ 2 \le \rho \le 4$; $\rho_v = \rho^2 z^2 \sin 0.6\phi$
  - (c) 우주; $\rho_v = \frac{e^{-2r}}{r^2}$

2.4 선전하에 의한 전계

- 전하가 점이 아닌 라인 위에 분포 → 선전하밀도(ρ_L [C/m])로 표시

$$\rho_L = \frac{Q}{l} \rightarrow \rho_L = \frac{\Delta Q}{\Delta} l \rightarrow \rho_L = \frac{\Delta Q}{\Delta l} = \frac{dQ}{dl}$$

$$\therefore Q = \int_l dQ = \int_l \rho_L \, dl$$

$$\rightarrow \mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_r = \frac{Q}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{\int_l \rho_L\,(\mathbf{r}')dl'}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^2}\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \int_l \frac{\rho_L(\mathbf{r}')dl'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

2.4 선전하에 의한 전계

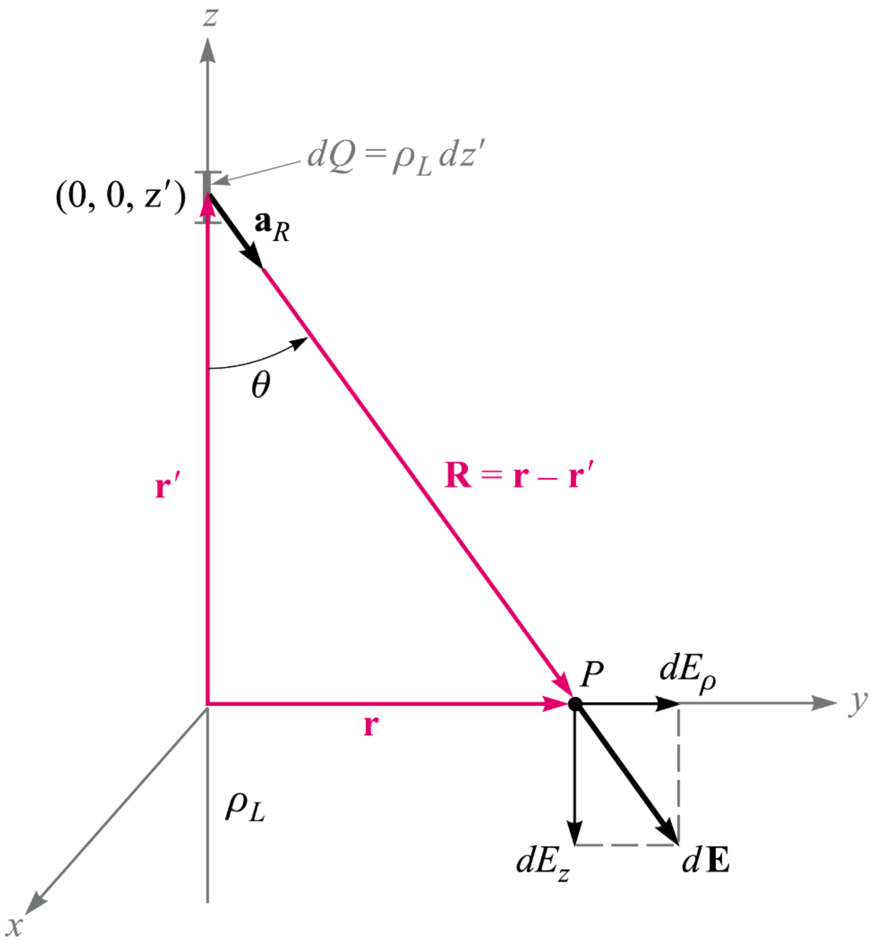
- z축의 $-\infty$에서 $+\infty$까지 선전하밀도 ρ_L의 전하가 균일하게 분포되어 있을 때, z축에서 ρ만큼 떨어진 곳에서의 전계세기?

$$d\mathbf{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$= \frac{dQ(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$= \frac{\rho_L dz'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$dQ = \rho_L dz'$$
$$\mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho$$
$$\mathbf{r}' = z'\mathbf{a}_z$$
$$\mathbf{R} = \mathbf{r}-\mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z$$



2.4 선전하에 의한 전계

$$d\mathbf{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\mathbf{a}_R = \frac{dQ(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3} = \frac{\rho_L dz'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$dQ = \rho_L dz'$$

$$\mathbf{r} = y\mathbf{a}_y = \rho\mathbf{a}_\rho, \quad \mathbf{r}' = z'\mathbf{a}_z$$

$$\mathbf{R} = \mathbf{r}-\mathbf{r}' = \rho\mathbf{a}_\rho - z'\mathbf{a}_z$$

$$d\mathbf{E} = \frac{\rho_L dz'(\rho\mathbf{a}_\rho - z'\mathbf{a}_z)}{4\pi\varepsilon_0(\rho^2+z'^2)^{3/2}}$$

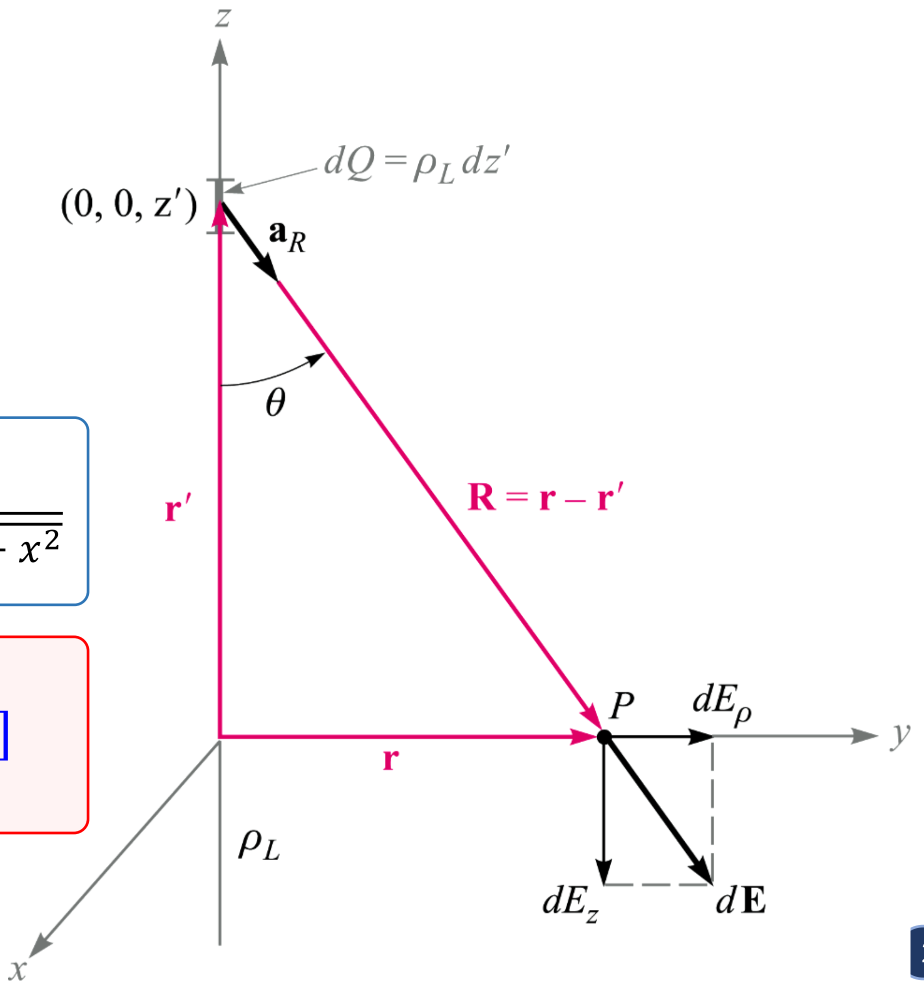
$$dE_\rho = \frac{\rho_L \rho dz'}{4\pi\varepsilon_0(\rho^2+z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty}\frac{\rho_L \rho dz'}{4\pi\varepsilon_0(\rho^2+z'^2)^{3/2}}$$

$$= \frac{\rho_L \rho}{4\pi\varepsilon_0}\frac{1}{\rho^2}\left(\frac{z'}{\sqrt{\rho^2+z'^2}}\right)_{-\infty}^{\infty}$$

$$\int\frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}}$$

$$E_\rho = \frac{\rho_L}{2\pi\varepsilon_0\rho} \quad [\mathrm{V/m}]$$



2.4 선전하에 의한 전계

$$d\mathbf{E} = \frac{dQ}{4\pi\varepsilon_0 R^2}\mathbf{a}_R = \frac{dQ(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3} = \frac{\rho_L dz'(\mathbf{r}-\mathbf{r}')}{4\pi\varepsilon_0|\mathbf{r}-\mathbf{r}'|^3}$$

$$dQ = \rho_L dz'$$

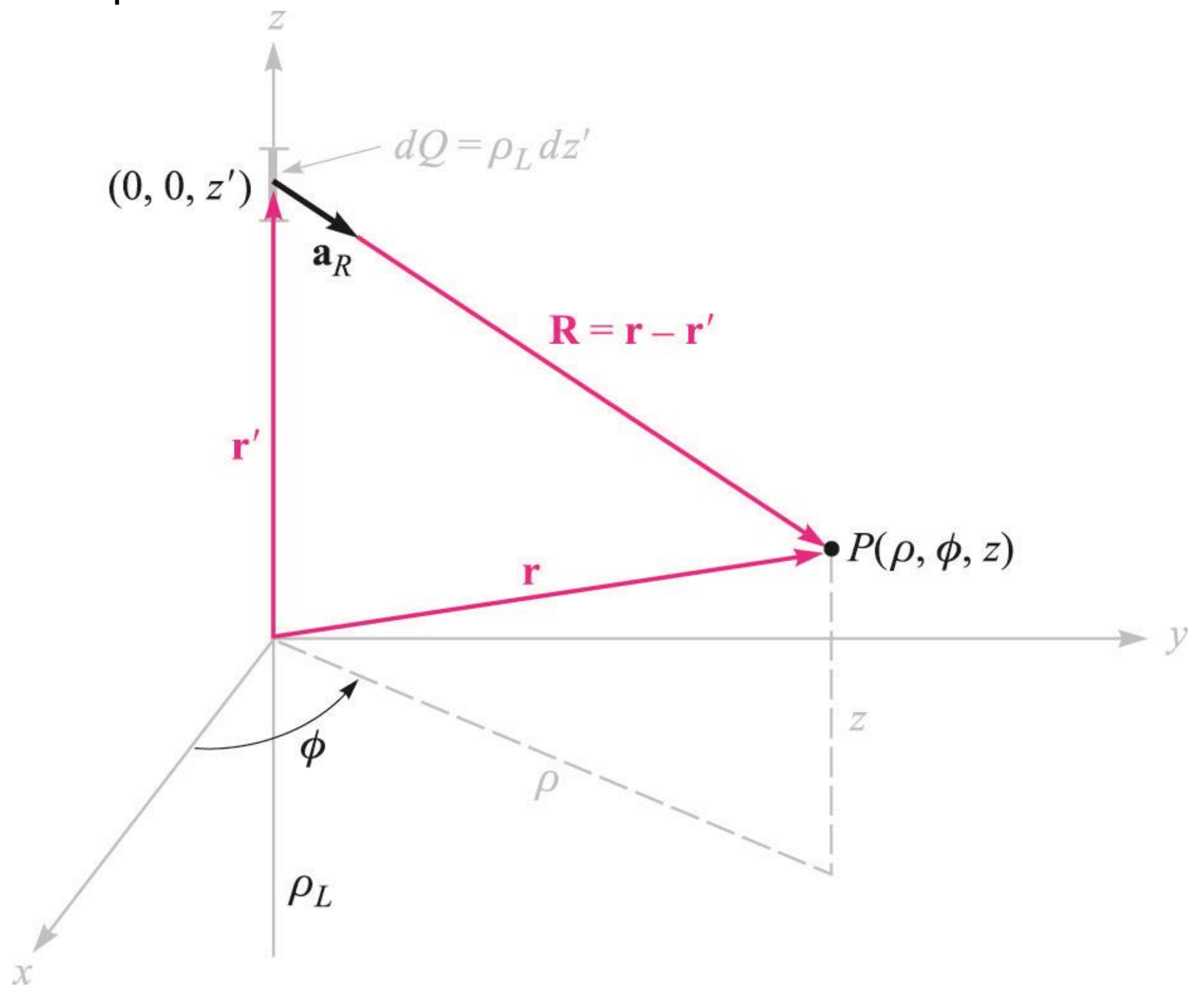
$$\mathbf{r} = \rho\mathbf{a}_\rho + z\mathbf{a}_z$$

$$\mathbf{r}' = z'\mathbf{a}_z$$

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \rho\mathbf{a}_\rho + (z-z')\mathbf{a}_z$$

$$R = \sqrt{\rho^2 + (z-z')^2}$$

$$\mathbf{a}_R = \frac{\rho\mathbf{a}_\rho + (z-z')\mathbf{a}_z}{\sqrt{\rho^2 + (z-z')^2}}$$



2.4 선전하에 의한 전계

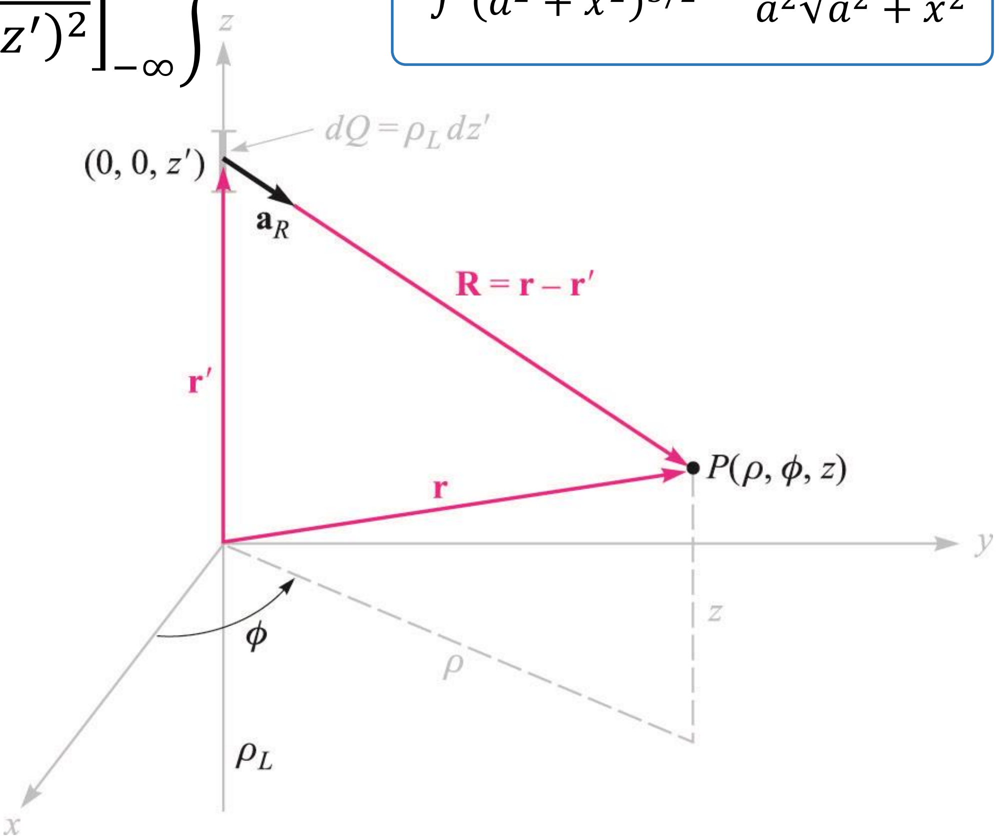
$$\mathbf{E}=\int_{-\infty}^{\infty}\frac{\rho_L dz'[\rho\mathbf{a}_\rho+(z-z')\mathbf{a}_z]}{4\pi\varepsilon_0[\rho^2+(z-z')^2]^{3/2}}=\frac{\rho_L}{4\pi\varepsilon_0}\left\{\int_{-\infty}^{\infty}\frac{\rho dz'\mathbf{a}_\rho}{[\rho^2+(z-z')^2]^{3/2}}+\int_{-\infty}^{\infty}\frac{(z-z')dz'\mathbf{a}_z}{[\rho^2+(z-z')^2]^{3/2}}\right\}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0}\left\{\left[\rho\frac{1}{\rho^2}\frac{-(z-z')}{\sqrt{\rho^2+(z-z')^2}}\mathbf{a}_\rho\right]_{-\infty}^{\infty}+\left[\frac{\mathbf{a}_z}{\sqrt{\rho^2+(z-z')^2}}\right]_{-\infty}^{\infty}\right\}$$

$$\int\frac{dx}{(a^2+x^2)^{3/2}}=\frac{x}{a^2\sqrt{a^2+x^2}}$$

$$=\frac{\rho_L}{4\pi\varepsilon_0}\left[\frac{2}{\rho}\mathbf{a}_\rho\right]=\frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho$$

$$\mathbf{E}=\frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho\ \ [\mathrm{V/m}]$$



2.4 선전하에 의한 전계

- (응용예제 2.5) $\rho_L = 5\ [\text{nC/m}]$ at x축 & y축
  - $\mathbf{E}$ =? at (a) $P_A(0,0,4)$ (b) $P_B(0,3,4)$

2.5 면전하(판전하)에 의한 전계

- 전하가 점이 아닌 판에 분포 → 면적전하밀도(ρ_S [C/m^2])로 표시

$$\rho_S = \frac{Q}{S} \rightarrow \rho_S = \frac{\Delta Q}{\Delta}S \rightarrow \rho_S = \frac{\Delta Q}{\Delta S} = \frac{dQ}{dS}$$

$$\therefore Q = \int_S dQ = \int_S \rho_S \, dS$$

2.5 면전하(판전하)에 의한 전계

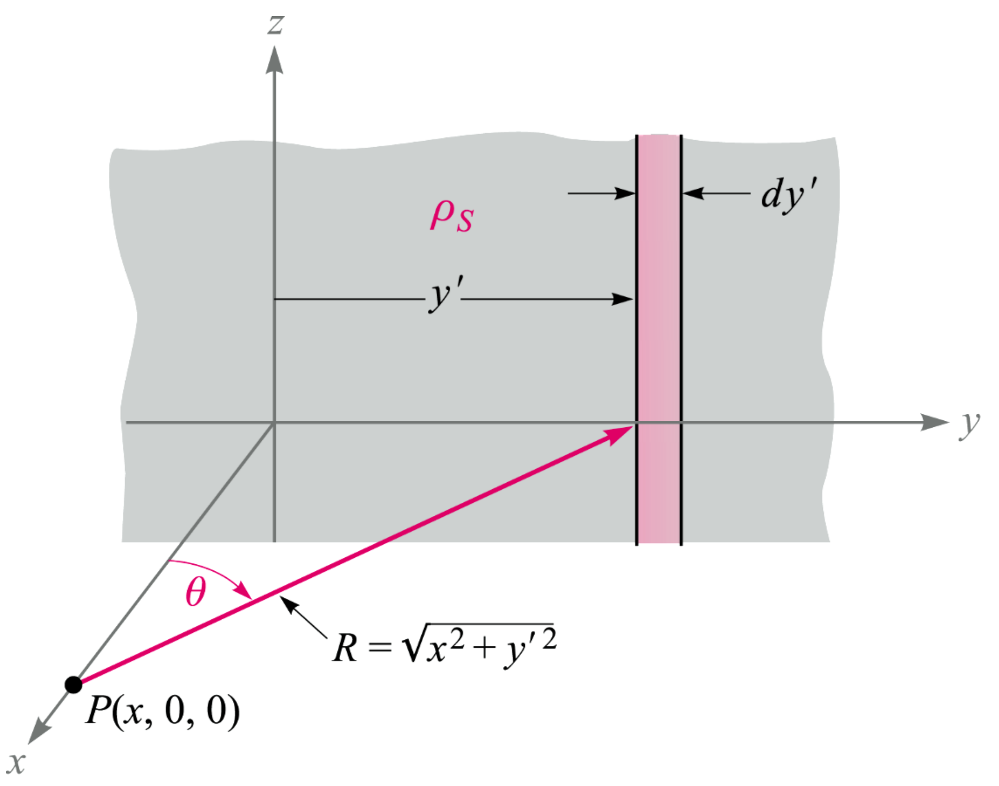
$$d\mathbf{E} = \frac{\rho_S dy'}{2\pi\varepsilon_0 R}\mathbf{a}_r \qquad \left(R = \sqrt{x^2 + y'^2}\right)$$

$$dE = d\mathbf{E}\cdot\mathbf{a}_x = \frac{\rho_S dy'}{2\pi\varepsilon_0\sqrt{x^2 + y'^2}}\cos\theta = \frac{\rho_S}{2\pi\varepsilon_0}\frac{x dy'}{x^2 + y'^2}$$

$$E_x = \frac{\rho_S}{2\pi\varepsilon_0}\int_{-\infty}^{\infty}\frac{x dy'}{x^2 + y'^2} = \frac{\rho_S}{2\pi\varepsilon_0}\left[\tan^{-1}\frac{y'}{x}\right]_{-\infty}^{\infty} = \frac{\rho_S}{2\pi\varepsilon_0}$$

$$\int\frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\mathbf{E} = \frac{\rho_S}{2\pi\varepsilon_0}\mathbf{a}_N \quad [\mathrm{V/m}]$$



2.5 면전하(판전하)에 의한 전계

- 음의 전하밀도 $-\rho_S$를 갖는 또 다른 두번째 무한 넓이의 대전판이 $x = a$인 평면상에 위치한다면

i) $x > a$ $\quad \mathbf{E}_+ = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x \quad \mathbf{E}_- = -\frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x \quad \therefore \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 0$

ii) $x < 0$ $\quad \mathbf{E}_+ = -\frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x \quad \mathbf{E}_- = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x \quad \therefore \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = 0$

iii) $0 < x < a$ $\quad \mathbf{E}_+ = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x \quad \mathbf{E}_- = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_x \quad \therefore \mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \frac{\rho_S}{\varepsilon_0}\mathbf{a}_x$

- (응용예제 2.6) $\rho_S = 3\ \mathrm{nC/m^2}$ at $z = 1$ & $\rho_S = -8\ \mathrm{nC/m^2}$ at $z = 4$
  - $\mathbf{E} = ?$ at (a) $P_A(2,5,-5)$ (b) $P_B(4,2,-3)$ (c) $P_C(-1,-5,2)$ (d) $P_D(-2,4,5)$

2.6 전계의 유선(streamline)과 스케치

- 전기장의 선; 전기력선 또는 전력선(lines of electric force)
  - 선속 또는 속선(flux line)
  - 방향선(directional line)
  - 유선(streamline)

- 전기력선의 표시방법
  - + 전하가 있는 곳으로부터 먼 곳으로 화살표
  - 먼 곳에서 - 전하가 있는 곳으로 화살표
  - → 전기력선 사이의 간격과 전기장의 세기는 반비례